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Comparison of the mks and mksC Systems of Units for the Description of Both Dynamic and Electromagnetic Quantities

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Abstract

The mks System is capable of making a concise description of the units of all dynamical variables in terms of the three fundamental quantities of distance (m), inertial mass (kg) and time (s). It is shown in Table 1 how these units can be represented by three-dimensional vectors in which the exponents of each of the fundamental units are located in the respective three positions. The mks System cannot be used directly to define the units of quantities employed in electromagnetic laws, however. Instead, a new (mksC) system is developed in which the Coulomb is added as a fourth component. Many examples are given in Table 2, and a corresponding set of four-dimensional vectors is listed in each case. Because of a degree of freedom in the electromagnetic laws, the Coulomb can effectively be replaced by the Nm =J. This choice allows each of the electromagnetic quantities to be described exclusively in terms of the mks System (also shown in Table 2).

The key advantage of this approach is that it is completely compatible with the relativistic Uniform Scaling procedure which determines conversion factors for the numerical results obtained in one rest system to those of any other. There are two fundamental quantities required for this purpose, one for kinetic scaling (Q) and the other (S) for gravitational scaling. Since the conversion factors for m, kg and s are known in terms of Q and S, it is therefore possible to determine the corresponding factors for all electromagnetic quantities based on their adjusted mks composition.

Keywords: mks System of Units, Electromagnetic Quantities, mksC system, Vector Notation for Units, Uniform Scaling Method

1. Introduction

An important advantage of the mks system is that it allows the units of many properties to be expressed completely in terms of the three fundamental quantities of distance, inertial mass and time, i.e. meter (m). kilogram (kg) and second (s). For example, energy is defined as the product of inertial mass and the square of the speed of the object. Accordingly, the corresponding unit is expressed as kg m^2 s^{-2} in the mks system. It is especially noteworthy that only *integral factors* of the three fundamental quantities appear in each property unit; they can be both positive or negative or zero. As discussed in the following, this characteristic allows one to express the units in terms of three-dimensional vectors, in which the elements are the powers of m. kg and s, respectively. For example, the vector for energy is (2, 1, -2).

2. Vector Notation for the Units of Mechanical Properties

The three fundamental properties in the mks system are represented by (1,0,0) for distance/m, (0,1,0) for inertial mass/kg, and (0,0,1) for time/s. Since speed/velocity is the ratio of distance travelled in a certain amount of time, the corresponding mks unit is m/s and therefore (1,0,-1) in vector notation. Energy is defined as the product of inertial mass and the square of speed, so its vector unit is obtained by adding (0,1,0) for mass to twice the vector for speed. i.e. 2 (1.0.-1) and therefore to the vector given above as (2,1.-2). A list of the vectors for key mechanical properties is given in Table 1. The meaning of the conversion factors listed therein will be discussed subsequently.

Table 1: List of non-electromagnetic properties and their units in the mks/Nms systems. Note that the mks and mksC systems are identical if no electromagnetic properties are involved, i.e. the fourth column in the mksC vector is zero. Vector notation is listed as well in each case. Conversion factors for measured values in two different rest frames are given in the second column

Property	Conversion Factor	mks Unit
Distance	Q (1,0)	m (1,0,0)
Inertial mass	Q/S (1,-1)	kg (0,1,0)
Time	Q/S (1,-1)	s (0,0,1)
Speed	S (0,1)	m/s (1,0,-1)
Energy	QS (1,1)	kg m^2/s^2 (2,1,-2)
Frequency	S/Q (-1,1)	1/s (0,0,-1)
Acceleration	S^2/Q (-1,2)	m/s^2 (1,0,-2)
Momentum	Q (1,0)	kg m/s (1,1,-1)
Angular Momentum	Q^2 (2,0)	kg m^2/s (2,1,-1)
Force	S (0,1)	kg m/s^2 (1,1,-2)
Pascal	S/Q^2 (-2,1)	kg /m s^2 (-1,1,-2)
Grav. Constant	S^2Q (1,2)	m^3/s^2 (3,0,-2)
Grav. Mass	1 (0,0)	1 (0,0,0)

For example, since frequency is the reciprocal of time, its vector is just equal to -1 (0,0,1) = (0,0,-1). Acceleration is equal to the ratio of speed to time, so its vector unit (1,0,-2) is obtained by subtracting the vector for time, i.e. (0,0,1), from the vector for speed (1,0,-1). Momentum is the product of inertial mass and speed, so its vector unit is (1,1,-1). Angular momentum is the product of distance and (linear) momentum, so its vector is obtained by adding (1,0,0) to (1,1,-1), which gives (2,1,-1) as shown in Table 1. Force is the ratio of momentum to time and thus its unit vector is (1,1,-2). This can also be obtained from the definition as the product of inertial mass and acceleration: (0,1,0) + (1,0,-2), whereby of course such consistency is required in the overall theory. The unit of pressure = force per square meter is the Pascal. Its unit is therefore obtained as (1,1,-2) – 2 (1,0,0) = (-1,1,-2). Note that this is consistent with the Ideal Gas Law, since it states that the product of pressure and volume (m^3) is equal to energy with its vector unit of (2,1,-2).

The three-dimensional vectors are also capable of describing gravitational interactions. The Universal Gravitation Constant G has a unit of m^3/s^2. According to Newton's Inverse Square Law (ISL), the acceleration due to gravity g is equal to $G\mu_0/r^2$. The gravitational mass μ_0 differs fundamentally from inertial mass. Its value is equal to the rest mass of the object, i.e. the value of the object's inertial mass when measured by an observer who is stationary with respect to the object. It does not change with the relative speed of the observer, contrary to the case for inertial mass. Its value is therefore expressed as a multiple of the dimensionless vector (0,0,0), as shown in Table 1. The corresponding vector notation for G is (3,0,-2). The vector representation of g is therefore obtained as the sum of (3,0,-2) + (-2,0,0) = (1,0,-2), the same as for (kinetic) acceleration (Table 1). The value of the corresponding gravitational force for a given object according to the ISL is therefore equal to the product of g with its inertial mass μ. Thus, it has the same unit as given for force in Table 1, i.e. kg m s^{-2}. The corresponding three-dimensional vector is therefore (1,1-2). Finally, the energy of the object is equal to the product of the gravitational force with its distance h above the location of the active mass. so it has the unit of energy, namely m^2 kg s^{-2}, corresponding to the vector (2,1,-2).

3. Expanded Vector Notation for Electromagnetic Properties

The situation with properties that appear in the laws of electricity and magnetism is not so
easily incorporated in the mks system of units as those needed to describe gravitational interactions. Coulomb's Law given below is a prime example. It equates the force F in N between two electric charges q_1 and q_2 which are separated by a distance of r meters:

$$F= q_1q_2/4\pi\varepsilon_0 r^2.$$

Note that there is also factor of ε_0 in the denominator. It is referred to as the electric permittivity of free space. The unit of charge is the Coulomb (Coul), so the unit for ε_0 must be equal to the product of $Coul^2/Nm^2 = Coul^2 s^2/kg\ m^3$. The way around this notational problem is to introduce a fourth fundamental quantity in addition to the meter, kilogram and second, namely the Coulomb (Coul). The revised set of units will be referred to as the mksC system. Accordingly, the Coulomb is equated to the vector (0,0,0,1) whereas the vectors for the three mks properties are simply enlarged by including 0 in their respective fourth positions. The corresponding mksC (four-dimensional) vector for ε_0 is thus determined to be (-3,-1,2,2). Thus, the units for electrostatic force F in the above equation are $Coul^2/m^2$ (m^{-3},kg^{-1}, s^2, $Coul^2$) = $1/m^{-1}$ kg^{-1} s^2 = m kg s^{-2}= N, as required.

The four-dimensional notation for a large number of electromagnetic properties is illustrated in Table 2 [1]. Electrical current i is the ratio of electric charge flowing in given period of time.

The mksC unit is therefore Coul/s = A (Ampere) and the corresponding vector notation is thus (0,0,-1,1). A key quantity for magnetic interactions is permeability μ_0. It satisfies the Law of Biot and Savart [2] given below, which follows from Maxwell's equations (c is the speed of light in free space and thus has the unit of m/s in the mks system:

$$\varepsilon_0\ \mu_0\ c^2.$$

Consequently, μ_0 must have the unit of $(\varepsilon_0 c^2)^{-1}$, i.e. (m^3kg /$Coul^2$ s^2)(s^2/m^2) = kg m/$Coul^2$. The corresponding vector notation is thus (1,1,0,-2).

Another basic quantity in electromagnetic theory is the Tesla, which is the unit of magnetic field B. It is equal to N/A m, and thus (kg m/s^2) (Coul $m/s)^{-1}$ = kg/Coul s, which in vector notation is (0,1,-1,-1). Magnetic flux has a unit of Weber = Nms/Coul= kg m^2/Coul s = m^2 Tesla. The corresponding vector is (2,1,-1,-1). Magnetic Intensity H is equal to B/μ_0 = (kg/Coul s) ($Coul^2$/kg m) =Coul/m s. Its mksC vector is thus (-1,0,-1,1).

The determination of the units of electrostatic properties is generally less complicated than for those involving magnetism. For example, the unit of electric field is equal to the ratio F/Coul = m kg s^{-2} $Coul^{-1}$ and thus its vector notation is (1,1.-2,-1). The unit for electric dipole moment is Coul m, while that for Volt is Coul/ε_0 m = kg m^2 s^{-2} $Coul^{-1}$, corresponding to (2,1,-2,-1) in the mksC vector notation. The unit for electrical resistance R (Ohm) is kg m^2/ $Coul^2$s, which is (2,1-1,-2) in vector notation. Note that the key relationship V=iR is fulfilled when mksC definitions are used, namely (Coul/s) (kg m^2 s^{-1} $Coul^{-2}$)= kg m^2 s^{-2} $Coul^{-1}$. The unit for electrical conductance (siemans) is just the reciprocal of R and thus (-2-1,1,2) in vector notation. The Farad = Coul/Volt is the unit of capacitance and therefore (-2,-1,2,2) in mksC vector notation.

The henry is the unit of electrical inductance. It has the mksC unit of m^2 kg/$Coul^2$ and thus corresponds to the vector (2,1,0-2). Electric polarization P and electric displacement D have the same unit of Coul/m^2 and therefore fit directly into the mksC scheme. Polarizability has the unit of s^2Coul^2/kg and therefore (0,-1,2,2) in vector notation. Charge density ρ is defined as the ratio of electric charge to volume. Its mksC unit is thus Coul/m^3, or (-3,0,0,1) in vector notation. The unit of magnetization M is Coul/m s or (-1,0,-1,1). Other quantities such as electric (H/M) and magnetic susceptibility (P/E ε_0) are dimensionless as can be seen by comparing the respective units in Table 2. Finally, the quadrupole moment is defined as the product of charge and the square of distance, so it corresponds to mksC vector (2,0,0,1), whereas the magnetic dipole moment has the unit of m^2 Coul/s, corresponding to (2,0,-1,1) and also to Nm/Tesla (kg m^2 s^{-2}) (Coul s/kg) = m^2Coul/s).

Vector notation is listed as well in each case. Conversion factors for measured values in two different rest frames are given in the second column.

Table 2: List of electromagnetic quantities and their units in the Nms and mksC systems

Property	Conversion Factor	mks Unit	mksC Unit
Electric Charge	QS (1,1)	Nm (2,1,-2)	Coul (0,0,0,1)
Electrical Current i	S^2 (2,0)	Nm/s (2,1-3)	Coul/s = A (0,0.-1,1)
Permittivity ε_0	S (1,0)	N (1,1,-2)	s^4A^2/kgm^3 (-3,-1,2,2)
Magnetic Field B	$(QS)^{-1}$ (-1,-1)	s/m^2 (-2,0,1)	Tesla=N/Am (0,1,-1,-1)
Magnetic Flux	Q/S (1.-1)	s (0,0,1)	Wb (2,1,-1,-1)
Permeability μ_0	S^{-3} (0,-3)	s^2/Nm^2 (-3,-1,4)	N/A^2 (1,1,0,-2)
Magnetic Intensity H	S^2/Q (-1.2)	N/s (1,1-3)	A/m (-1,0,-1,1)
Electric Field E	1/Q (-1,0)	1/m (-1,0,0)	Coul/ ε_0 m^2 (1.1.-2.-1)
Electric Dipole Moment	Q^2S (2,1)	Nm^2 (3,1,-2)	Coul m (1,0,0,1)
Magnetic Dipole Moment	Q^2S^2 (2,2)	Nm^3/s (4,1,-3)	m^2A = Nm/Tesla (2,0,-1,1)
Volt	1 (0,0)	1 (0,0,0)	Coul/ ε_0 m (2,1,-2,-1)
Electrical Resistance	S^{-2} (0,-2)	s/Nm (-2,-1,3)	$kgm^2/sCoul^2$ = Ohm (2,1,-1,-2)
Electrical Conductance	S^2 (0,2)	Nm/s (2,1,-3)	siemans = mho (-2,-1,1,2)
Electrical Inductance	Q/S^3 (1,-3)	s^2/Nm (-2,-1,4)	$m^2kg/Coul^2$ = henry (2,1,0,-2)
Capacitance	QS (1,1)	Nm (2,1,-2)	Coul/Volt=Farad (-2,-1,2,2)
Charge density ρ	S/Q^2 (-2,1)	N/m^2 (-1,1,-2)	Coul/m^3 (-3,0,0.1)
Quadrupole Moment	Q^3/S (3,-1)	Nm^3 (4,1,-2)	Coul m^2 (2,0,0,1)
Electric Polarization P	S/Q (-1,1)	N/m (0,1,-2)	Coul/m^2 (-2,0,0,1)
Electric Displacement D	S/Q (-1,1)	N/m (0,1,-2)	Coul/m^2 (-2,0,0,1)
Magnetization M	S^2/Q (-1,2)	N/s (1,1,-3)	A/m (-1,0, -1,1)
Magnetic Susceptibility χ_m	1 (0,0)	1 (0,0,0)	H/M (0,0,0,0)
Electric Susceptibility χ_e	1 (0,0)	1 (0,0,0)	P/ε_0 E (0,0,0,0)
Polarizability	SQ^3 (3,1)	Nm^3 (4,1,-2)	s^2Coul^2/kg (0,-1,2,2)
Coefficient of Potential	$(QS)^{-1}$ (-1,-1)	$(Nm)^{-1}$ (-2,-1,2)	Volt/Coul (2,1,-2,-2)

4. Conversion of the mksC units to the mks System

It is important to recognize that electric charge is NOT a measurable quantity. Applications of Coulomb's Law can only be based on direct measurements of the force F between two electrons and the distance r separating them from one another. A determination of the charge of an object needs to take account of the value of ε_0, but this is not a measurable quantity either. All that one can conclude from this is that the unit for the ratio of two charges to ε_0 is equal to Nm^2.

As a consequence, there is a *degree of freedom* available for choosing the unit of electric charge itself [2, 3]. One can take advantage of this situation by assuming that the unit of charge is some combination of mks units. In that way, the units of each of the electromagnetic properties can be expressed entirely within the (three-dimensional) mks system.

In past work [1], it has been assumed that electric charge has a unit of Nm =J. When this is done consistently, a unit for each such property is described uniquely within the mks system.

As a prime example, the unit for electric charge is assumed to be Nm = m^2 kg s^{-2}. The associated three-dimensional vector representation is (2,1.-2), as shown in Table 2. Accordingly, the corresponding mks unit for ε_0 is N, so that the ratio of the units of two charges to ε_0 is Nm^2, as required by the above analysis.

In this manner, the mks vector for electric charge is obtained by adding the vector Δ4 = (2,1,-2,-1) to the corresponding mksC vector for electric charge (0,0,0,1). The vector sum is thus (2,1,-2,0), which upon elimination of the fourth position gives the mks vector for Nm^2 = J, again as required. This procedure can be generalized to all properties. One merely needs to know the value in the fourth position (λ) of a given property, which must be an integer (positive or negative) according the general mksC definition, and add λ Δ4 to the property's mksC vector. This eliminates the fourth position in the resulting vector, allowing it to be transferred without further change to the corresponding three-dimensional mks vector. Take permittivity ε_0 as an example. It has a value of 2 in the fourth position (see Table 2), so according to the above procedure, 2 Δ4 must be added. The result is (4,2,-4-2) + (-3.-1,2,2) = (1,1,-2,0), which corresponds to the ε_0 mks vector (N) after elimination of 0 in the fourth position. The value of λ in the case of Tesla is -1. Thus, the mks unit (s/m^2) is obtained by adding - Δ4 to the mksC vector, i.e. (-2,-1,2,1) + (0,1,-1,-1) = (-2,0,1,0). The mks unit for Weber is also obtained by adding - Δ4 to its mksC vector, with the result (0,0,1,0) or 1 s. This is consistent with the early assignment of Wb/m^2 as the unit of magnetic field before it was changed to Tesla, i.e. Wb/m^2 becomes s/m^2 after transformation by the above procedure. Inspection of Table 2 shows that adding the appropriate factor of Δ4 to the mksC vector for each property leads in all cases to the corresponding mks three-dimensional vector.

5. Relationship to the Relativistic Uniform Scaling Method

A key advantage of being able to express electromagnetic properties exclusively in the mks system of units is that it allows for direct application of the Uniform Scaling Method [4, 5] to convert the numerical values of a given property in one rest system to the corresponding values in another. Results of experiments show that the values of properties change when the object is either accelerated or changes its position in a gravitational field. Moreover, the conversion factors for each property can be expressed as a product Z of integral multiples of two quantities, Q for kinetic variations and S for gravitational changes. An observer in one rest frame obtains the numerical values of measurements in his rest frame by multiplying the result in the other rest frame by Z [4, 5]. The conversion factors for distance, inertial mass and time are Z = Q, Q/S and Q/S, respectively. Corresponding values of Z are given in Tables 1 and 2 for a large number of properties. For example, the mks unit for energy is m^2 kg s^{-2}, so the value of Z in this case is equal to (Q^2 Q/S) $(S/Q)^2$ = QS. The Uniform Scaling Method is used in the application of the GPS Navigation System to adjust clock rates on satellites orbiting the Earth [6].

The analogous procedure is not possible for electromagnetic properties, however, since there is no way to evaluate Z directly for the Coul unit. By expressing the electromagnetic properties exclusively [1] in the mks system of units, it is possible to compute the value of each conversion factor on the basis of the composition of each property in terms of the three fundamental units. For example, the mks unit for the Tesla is s/m^2, so Z = (Q/S) Q^{-2} = $(QS)^{-1}$ in this case.

Finally, examination of the conversion factors in Tables 1 and 2 shows that they can also be expressed as two-dimensional vectors, i.e. with only two positions filled with integers.

6. Conclusion

The mks system of units is based on the fact that all purely dynamical properties can be expressed as products of the three fundamental quantities of distance (m). inertial mass (kg) and time (s). These relationships are closely allied with fundamental laws of physics. For example, the force F in Newton (N) is defined as the product of an object's inertial mass and the amount of its acceleration d^2x/dt^2 relative to a given position, i.e. N = m kg s^{-2} (see Table 1). A simple means of expressing these relationships is through the use of three-dimensional vectors for which the three positions are filled with integers corresponding to the respective factors of m, kg and s in the property's composition; for example, the vector for N is (1,1,-2). It is also applicable to classical gravitation theory. The unit for the Universal Gravitation Constant is m^3s^{-2}, for example. It is necessary to distinguish between gravitational and inertial mass, however. The former is a dimensionless quantity and is thus represented by the null vector (0,0,0), whereas the latter's vector is (0,1,0).

This approach nonetheless does not apply directly to electromagnetic quantities. It can be extended, however, by introducing a fourth fundamental quantity which allows, in combination with m. kg and s, for the unique representation of all electromagnetic properties. This possibility is realized in the present work by employing the Coulomb (Coul) for this purpose. The resulting system of units is referred to as the mksC system. The unit for Ampere (A) is Coul/s. The unit of magnetic field Tesla is Ns/Coul m, whereas the unit for electric permittivity ε_0 is $Coul^2$ s^2/ kg m^3. Many more examples are listed in Table 2.

There is an alternative means of including electromagnetic properties into a general scheme of units, however. Because a degree of freedom exists in the laws of electricity and magnetism, it is possible to assign an arbitrary mks unit to the Coulomb and still maintain consistency with these laws. In the present work, Coul has been set equal to the NmJ. The corresponding unit for ε_0 is set to N, which therefore satisfies the requirement of Coulomb's Law that the product of the units of two electric charges to that of ε_0 is Nm^2. Similar mks assignments are therefore also assumed for all other electromagnetic properties. For example, the unit for the Ampere is Nm/s and that for Tesla is s/m^2. As shown in Table 2, the resulting units for all other such properties then conform to the three-vector mks scheme, e.g. (-2,0,1) for Tesla and (1,1,-2) for ε_0. There is a simple procedure for transforming the mksC vector for a given electromagnetic property over to the mks System of units. It is always possible to add a multiple of the Δ4 vector (2,1,-2,-1) to the mksC vector for a property to obtain a resultant with a null component in the fourth position which effectively then corresponds to a three-dimensional mks vector.

The key advantage of employing exclusively mks vectors is that one can then employ the Uniform Scaling Method directly to determine the conversion factor of each property in terms of the two fundamental quantities: Q (kinetic scaling) and S (gravitational scaling. Since one knows the conversion factors for distance (Q) and both inertial mass and time (Q/S), it is therefore always possible to determine the conversion factor for any other property based on its mks composition. This is not the case for a property representation which contains the Coulomb in its definition, which therefore rules out direct application of Uniform Scaling for electromagnetic properties within the framework of the mksC System. The Q,S values can also be described in terms of two-dimensional vectors, as shown in both Tables 1 and 2.

7. References

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