Can Einstein's General Theory of Relativity Be Replaced By a Far Simpler Theory Which Only Relies On the Uniform Scaling of Physical Properties?

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Abstract

In 1960 Schiff published a paper which questioned to what extent the full formalism of Einstein's General Theory of Relativity (GRT) is required in the calculation of three key experimental effects (the gravitational red shift, the deflection of light rays that pass close to the Sun, and the precession of the perihelion of Mercury's orbit around the Sun), but rather "may be correctly inferred from weaker assumptions that are well established by other experimental evidence." He noted that the method he employed was not capable of describing the third of the above effects, however. In the present work it will be shown that the latter deficiency has been removed by expanding his scaling procedure to cover the acceleration due to gravity g in Newton's theory of gravitation, thus further strengthening his argument against the essentiality of GRT. In addition, the scaling procedure has been extended to include other key physical quantities such as energy, momentum and force and even the Universal Gravitation Constant G. The significance of these theoretical developments for the terrestrial experiments of Pound et al. is also discussed.

Keywords: Schiff's Scaling Procedure, Displacement of Star Images, Advancement Angle of the

Precession of Mercury's Perihelion, Newton's Classical Law of Gravitation, Pound's Experimental Verification of the Scaling of Velocity

I. Introduction

According to Pais [1], the first attempt to describe the trajectory of light as it passes by a heavy object such as the Sun was made by von Soldner in 1801. He based his theory on Newton's corpuscular theory of light which he enunciated in the late 17th century. Einstein took a fundamentally different approach in 1907 on the basis of his Equivalence Principle [3]. This led eventually to his General Relativity Theory [4]. Schiff later came up with a conceptually simpler theory [5] which led to nearly the same results as GRT. He assumed that light travels in a straight line as it passes the Sun, whereas the prevailing view based on GRT is that space and time are intertwined and that light follows a curved trajectory as a consequence. In the following it will be discussed how Newton's Laws of Motion can be applied to this general question.

II. Combining Einstein's Equivalence Principle with Newton's Theory of Gravity

One of the main goals of gravitational physics is to determine the path pf light as it passes a massive body such as the Sun. The approach pursued by von Soldner in 1801 [1,2] was based squarely on Newton's view that light is composed of particles. Pais [1] points out that von Soldner made use of Newton's scattering theory in his calculations. He did not know what the mass of the light particles is, but his results depended very little on this question. Thereupon, he obtained a value of 0".84. What is clear, however, is that this angle corresponds to the attraction of the light particles *toward the Sun*.

Einstein's quest to determine the trajectory of light began with his 1907 paper [3] in which he intoduced his Equivalence Principle. He concluded that light waves located close to the surface of the Sun have frequencies which are subject to a *gravitational red shift* when they arrive at the Earth's surface. Accordingly, the frequency ν of a given light wave near the Sun is found to have a smaller value $\nu' = S^{-1}\nu$ at the surface of the Earth, where $S = 1 + gh/c^2$ (g is the acceleration due to gravity, h is the distance separating the observer from the Earth's center of mass and c is the speed of light). He based this result on the Doppler effect [6] and his belief that gravitational acceleration and kinetic acceleration are equivalent. It has been shown recently [7], however, that this equivalence does not occur in reality, but Einstein's above formula is nonetheless quite useful. It can be obtained by assuming that the unit of energy depends on the gravitational potential energy difference between the object and the observer. The energy of the object is mc^2 in the rest frame in which it is located, where m is the inertial mass of the object. This result is based directly on Einstein's energy-mass equivalence relation. In the units of the observer, the object's energy is measured instead to be $mgh + mc^2$ according to Newtonian gravitational theory. The conversion factor S for these two rest frames is obtained by dividing the two numerical values, i.e. $S = (mgh + mc^2)/mc^2 = 1\ gh/c^2$. One therefore assumes that the same conversion factor holds for frequencies.

The above conversion factor is only valid in a narrow region of space where g is effectively constant. To obtain a general value, it is necessary to integrate over the entire region of space which separates the object from the observer. When this is done, the result is $S=A(r_o)/A(r_p)$, where $A(r) = 1 + GM_0/c^2r$; r_o and r_p are the respective distances of the observer o and object p from the gravitational source, $G=6.6743 \pm 0.00015) \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ is the universal gravitational constant and $M_0 = 1.99 \times 10^{30}$ kg is the corresponding rest/gravitational mass of the source such as the Sun.

III. Use of Schiff's Scaling Method to Compute the Trajectory of Light

The speed of light also varies with its distance r from the Sun, in the same way as light frequencies and energy, namely as Sc, where $S=1/ A(r) = (1+ GM_O/c^2r)^{-1}$. Einstein made a special point of this fact in his 1907 paper [3,6], remarking that this behavior is different than expected from the special theory of relativity [8]. As a result, the speed of light *decreases* as it draws closer to the Sun. Shapiro et al. [9,10] verified this general expectation in their experiments with radio waves passing close to the planets Venus and Mercury, thereby producing time delays in echoes arriving back at the origin..

The latter authors also noted that *light travels in a straight line* as it passes through empty space. This conclusion stands in stark contrast to the claim in GRT that light follows a curved path in the vicinity of massive objects such as the Sun [4]. It also contradicts the assumption of von Soldner [2], who calculated that light should be constantly coming closer to the Sun on its way from outer space. He did not know that the gravitation mass of light is equal to zero. As a consequence, one must conclude on the basis of Newton's gravitational theory that the Sun does not exert a force on light rays. Light is also expected on this basis to be unable to exert a gravitation force on the Sun (and all other objects). This is in agreement with Newton's Third Law of Motion, i.e. for every action there is an equal reaction in the opposing direction.

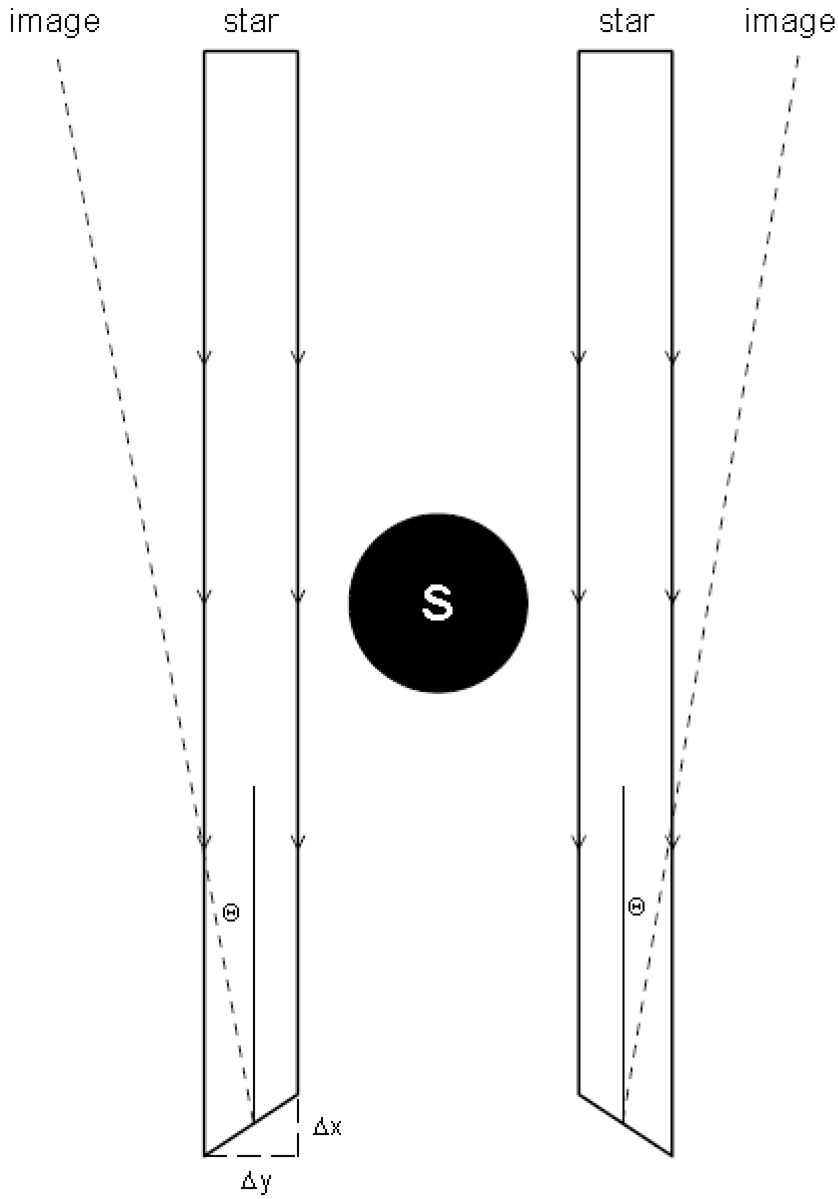
The qualitative conclusion from the above remarks is that light coming from infinity will always travel in a straight line toward the Sun [11], gradually decreasing in speed until it reaches the closest point to the Sun and thereafter increasing gradually on its way to infinity. The diagram in Fig. 1 shows a parallel series of light rays emanating from a star as they pass downward to the observer located on the Earth's surface [12]. This doesn't explain the conventional belief that the light trajectory is bent during this process, however. For this purpose, it is important to note that the line connecting the end points of the light rays constitutes a *wave front*. Because the waves that are located farther from the Sun travel at a greater speed, the wave front is rotated away from the Sun. The corresponding rotational angle Θ is defined by Huygens' Principle, as shown in Fig. 1. The difference in the respective distances travelled by the two waves is equal to Δx and the perpendicular distance separating the waves is equal to Δy, whereupon $\Theta = \Delta x / \Delta y$ in radians. Note that Einstein also used Huygens' Principle to compute the angle of rotation. In actuality, however, it can be seen that *Huygens' Principle is not used in the above procedure to compute the value* $\Theta = \Delta x / \Delta y$. It is just a race between two parallel light waves separated by a distance Δy, whereby the one farther from the Sun moves at faster speed.

There is a straightforward interpretation of this result. When light approaches us from a distant object, we assume that it is *located on the normal to the wave front.* If the latter has rotated, this causes one to conclude that the object is not located in its actual position, but rather has moved away from it. *This is clearly an illusion*. An experiment with light refraction has been suggested which is intended to verify this interpretation [see Fig. 3 of ref. 13]. The darkness associated with a solar eclipse makes it possible to recognize this effect whereas it is otherwise hidden from view. Einstein's prediction of such an event made him a "legend' in 1919 [14].

The fact is that Einstein based his theory [4] on his claim that space is not flat, as Newton had claimed, but rather is curved. Nonetheless, it is clear from Fig. 1 that the "light bending" effect can be discerned from a far different assumption, namely that light from a star travels in a perfectly straight line, which in turn leads to a rotation of the wave front of the light which reaches the observer on the Earth's surface. A key property of light in this regard is its *null gravitational mass*. At least according to Newton's theory of gravity, this means *that the Sun can exert no force on the ligh*t that would cause it to deviate from its straight-line trajectory. It bears worth repeating that Shapiro's experimental results [9, 10] are also consistent with this conclusion.

In 1960, Schiff, who was an acknowledged expert on Einstein's General Relativity Theory [4], published a paper [5] in which he computed the trajectory of light in the manner outlined above, with one exception. He assumed that the radial component of the light velocity must be scaled with an additional factor of S. He also used Huygens' Principle to evaluate the angle of deflection of light. He carried out his calculations analytically, but the same results have also been obtained using a numerical approach [13]. The light is assumed to follow a straight-line trajectory at each stage of its motion between a star and the Earth's surface. Schiff obtained nearly the same angle of deflection as Einstein did with GRT [4, 15] (1".7517), which is very close to the best estimate based on experimental data [16]. Schiff also noted that only half the correct angle is obtained when the same scaling is used for all components of the light velocity. This is the same (incorrect) value obtained by Einstein [17] in his work prior to the introduction of GRT

To illustrate the simplicity of Schiff's scaling method, consider a light ray in Fig. 1 whose closest approach to the solar midpoint is y. The speed of light is always c, but it is necessary to resolve it into its transverse and radial components at each point in the pathway of the ray. If we define the angle α as $\cos^{-1}(y/r)$ (r being the current distance of the ray to the solar midpoint), the value of the radial component of the light velocity is $c \sin \alpha$, while the corresponding transverse component is $c \cos \alpha$. According to Schiff's procedure, the scaled values of these two quantities are $S^2 c \sin \alpha$ and $Sc \cos \alpha$, respectively. In a given time interval Δt, the distance travelled by the light ray is $\Delta x= Sc\Delta t (\cos^2 \alpha + S^2 \sin^2 \alpha)^{0.5}$. The angle α varies from $\pi/2$ when the ray is infinitely far away to 0 when it reaches its closest point of approach to the solar midpoint, i.e. y= r. Thus, Δx varies from $S^2 c\Delta t$ at large distances to $Sc\Delta t$ at the ray's closest distance from the Sun and then gradually back to $S^2 c\Delta t$ as it moves toward infinity beyond this point. The total distance travelled by the ray is simply the sum of the individual Δx values, which we will refer to as X (y). The value of the deflection angle Θ in Fig. 1 is computed by repeating this procedure for a different value of y for the same time difference. If we refer to the two values of y as Y_1 and Y_2, the value of Θ is $\tan^{-1} [X(Y_2) - X(Y_1)]/(Y_2 – Y_1) = \tan^{-1} \Delta X/\Delta Y$. In the calculations reported in Ref. [14], the light rays start at a distance of 10^{12} m above the Earth's surface and stop at a position which is only slightly above this. The value obtained is $\Delta X = 0.008492807$ m $\Delta Y =$ 1000 m. Note that these results are independent of the total time difference employed as long as it is sufficient to allow the rays to reach the Earth's surface.. When this procedure is carried out, the resulting value for Θ is 1".7517 [5, 14], in very good agreement with the GRT value [15]. If the additional scaling of the perpendicular component is ignored, so that $\Delta x= Sc\Delta t$ at each value of r, only half of the above value for Θ results, the same as Einstein calculated in 1911 [17].



$$\Theta = \frac{\Delta x}{\Delta y} = \int \frac{dc'dt}{dy} = \int \frac{1}{c'}\frac{dc'}{dy}dx$$

Fig. 1 Schematic diagram showing light rays emitted by stars to follow straight-line trajectories as they pass near the Sun. Because of gravitational effects, the speed of the light rays c' is known to increase with gravitational potential, with the effect that the corresponding Huygens wave front gradually rotates away from the Sun. As discussed in the text, the normal to a given wave front points out the direction from which the light appears to have come, causing the star images to be displaced by an angle Θ during solar eclipses.

IV. Adaptation of Schiff's Scaling Method for Predictions of Planetary Orbits

Despite the success of his scaling method in describing the gravitational deflection of light, Schiff also expressed disappointment in not being able to devise a similar approach to deal with anomalous characteristics of planetary orbits [5]. The precession of the perihelion of Mercury had been quantitatively described using Einstein's GRT[4]. It was pointed out some 50 years later [19] that this failure of Schiff's theory can be overcome by scaling the acceleration of gravity (g) as well as velocities and distances.

Ascoli has argued [20, 21] that g depends on the speed u_P of the object (P) relative to the observer (M), specifically that $g(M) = g(P)\, \gamma^{-2}(u_P)$. In this equation, $g(P) = GM_O/r_P^2$ is the local value of the acceleration due to gravity according to Newton's classical theory (r_P is the distance between the object P and the active source of gravitation/Sun) and $\gamma(u_P) = (1- u_P^2c^{-2})^{-0.5}$. Note that u_P=c when the object is light, so that $\gamma^{-2} = 0$ in this case. This is consistent with the assumption in Sect. III that light is not affected by the gravitational attraction of the Sun and therefore travels in a straight line.

The Uniform Scaling Method [22, 23] is consistent with Ascoli's position. One simply must take account of the fact that the unit of distance varies with u_P. Specifically, the observer on the Earth's surface finds that the distance r_P varies in direct proportion to $Q=\gamma(u_P)$. Since according to Newton's formula, g (P) is inversely proportional to r_P^2, while the gravitational mass M_s is completely independent of u_P, the observer M co-moving with the Sun finds that $g(M) = Q^{-2} g(P) = \gamma^{-2} g(P)$, in agreement with Ascoli's position.

Moreover, the value of g(M) is also subject to the gravitational scaling assumed by Schiff [5]. Accordingly, the scaled value of the distance is Sr_P since the vector is directed perpendicularly outward from the Sun. Employing this factor therefore requires that $g(M) = Q^{-2}S^{-2} g(P)$, when account is taken of the inverse-square dependence of r_P in g (P).

The calculation of Mercury's orbit is done has been done using numerical methods [19].

It is assumed that the initial velocity $\boldsymbol{u_o}$ and position $\boldsymbol{P}$ of the object are known relative to a primary (stationary) observer O located at infinity (A_o=1). In a time interval Δt the current value of the velocity **u** is added g Δt to obtain the velocity **u'** at the end of the cycle. The position of the planet is then updated by **u'** Δt and the procedure is continued from this point. At each stage the velocity and g value are scaled according to the method discussed above. The calculation continues until the planet moves from one perihelion to the next. A value for the desired precession angle Θ obtained in this manner is found to be somewhat smaller than the correct value. The latter has been obtained in closed form by Einstein using GR [4]. The discrepancy has been removed, however, by changing the scaling of g(M), namely as $Q^{-2}S^{-3}$ g(P); see Sect. VI for an explanation of this change based on the scaling of G. The result is 43".0033/cy, in good agreement with both the currently accepted experimental value for this quantity of 43".2 ± 0".9/cy [24] and that computed by Einstein from GTR of 43".0076/cy [4, 25].

The value of the precession angle Θ of the perihelion of Mercury's orbit around the Sun obtained from the present treatment is 43".0033/cy [19], in good agreement with both the currently above values. Einstein's closed expression [4, 25, 26] indicates that the precession angle in general is proportional to M_s and inversely proportional to both r and $(1-e^2)$ [e is the eccentricity of the orbit]. Tests have been carried out for different values of the latter three quantities, and very good agreement with the predictions of GTR has been found in all cases. Indeed, since the amount of computer time required increases with r, most of the tests carried out are for a hypothetical planet with one-thousandth of Mercury's radius and therefore a period of revolution around the Sun of only 240 s. When the solar mass is increased by a factor of 10.0, it is found that the value of Θ is 10.0012 times greater. If the mean radius is cut in half, Θ is found to increase by a factor of 1.9990. Similarly good agreement with GTR is obtained if the radius is changed by factors of 10 and 100. Finally, when e is changed from its experimental value of 0.2056 for Mercury to 0.10, the value of Θ is found to be 0.9677 times smaller, as compared to the predicted factor of 0.9674.

The A_p factors have been computed in the present treatment in two different ways in each time-step: either as $A_p = 1 + GM_0/c^2r_p$, or by making use of the proportionality relationship [19]

$$\frac{\gamma(u_p)}{A_p} = \frac{\gamma(u_o)}{A_o}$$

after using the above definition to obtain an initial value for A_p only. The corresponding two values of Θ agree to within a factor of 1.000093, with that obtained with the latter definition being higher. This result thus clearly supports the conclusion that the whole concept of gravitational scaling is rooted in the conservation of energy principle (see Sect. II).

V. Comparison of the Adaptation of Schiff's Scaling Method to General Relativity Theory

In the Introduction to his 1960 paper, Schiff [5] cautioned experimentalists of the need to understand the extent to which their results actually "support the full structure of general relativity theory, and do not merely verify the equivalence principle and the special theory of relativity." Implicit in this remark was his feeling that there might be a possibility of explaining the results of all relativity experiments without the use of GRT [18]. At the time of his writing, he was unable to extend his scaling theory to deal successfully with the observed precession of the perihelion of Mercury's orbit around the Sun. It was in fact a simple matter to achieve this objective, namely to also scale g in a proper manner, as demonstrated in Sect. IV [19].

There are thus two *qualitatively* different relativistic theories of gravity which obtain *quantitatively* the same results for the key effects that are mentioned in Schiff's paper.

The issue that distinguishes them most clearly is their respective claims of how light travels in a gravitational field. The adapted Schiff theory relies on the undisputed fact that light has a null gravitational mass. According to Newton's theory of gravitation, this means that the Sun cannot exert a force on light, which therefore implies that a straight-line trajectory must be observed. This conclusion is verified in Shapiro's experiments [9,10]. They found that the echoes of radio waves that collide with planets are received back at the laboratory in which they were emitted.

This could not happen if light follows other than a straight-line path in both directions. The curved trajectory claimed in Einstein's GRT is not supported by any direct experimental evidence. Instead, the supporters of GRT [27] rely on the fact that the aforementioned tests of relativity are satisfied by this theory, while at the same time ignoring the fact that the same results are obtained by the scaling theory as well. .

Schiff and others [28, 29] also suggested an experiment to test GRT which would distinguish between the two theories. It was noted that application of Thomas precession [30] for satellites leads to the following prediction, namely that the component of spin in the plane of the satellite's orbit will precess at the rate (M is the gravitational mass of the planet/Earth and **r** is the satellite's radial distance from the planet's center of mass):

$$\omega_T = (GM/2c^2r^3)\ \mathbf{v} \times \mathbf{r}.$$

By contrast, a GRT calculation [28] gives to a good approximation

$$\omega_T = -3\ GM/2c^2r^3)\ \mathbf{v} \times \mathbf{r}.$$

i.e. an effect which is 3 times larger and in the opposite sense. The adapted Schiff method is in quantitative agreement with the former result for ω_T, so it is critical that the proposed experiment be carried out.

VI. Pound's Terrestrial Experiment for the Free Fall of Light

The Uniform Scaling Method [23, 24] operates on the principle of the complete objectivity of the measurement process. The only reason two observers can differ on the results of any measurement, excluding technical errors, is because they using different units in which to express their respective numerical results. The parameter S introduced in Sect. II is a key element in applying the method. The gravitational scale factors are always an integral multiple of S. A detailed example of how the procedure is carried out for energy has been given there.

The scaling of velocity, distance and acceleration due to gravity is an essential feature of the successful description of the effects of both the deflection of light effect and the anomalous precession of planetary orbits discussed in Sects. III and IV.

The scale factors required in general for one observer located at different gravitational potential than that where measurements have been carried out to accomplish this objective are shown in Table I below, whereby the factor S is defined by the procedure detailed in Sect. II.

Table I Comparison of measured values of properties at two different gravitational potentials in terms of the parameter S and corresponding results after free fall

Property	Value at P	Scaled value for observer M	Free-fall value for M	Ratio
Energy	E	SE	SE	1
Frequency	ν	Sν	Sν	1
Period	t	$S^{-1}t$	$S^{-1}t$	1
Planck's Constant	h	h	h	1
Speed of light	c	Sc	c	S^{-1}
Distance	X	X	$S^{-1}X$	S^{-1}
Acceleration	a	S^2a	Sa	S^{-1}
Inertial mass	m	$S^{-1}m$	Sm	S^2
Gravitational mass	m_0	m_0	m_0	1
Grav. Constant	G	S^2G	$S^{-1}G$	S^{-3}
Force	F	SF	S^2F	S
Momentum	p	p	Sp	S

The key relations such as Einstein's $E=mc^2$, Planck's $E=h\nu$ and de Broglie's $p=h/\lambda$ are each satisfied at every level. This is in recognition of the Addendum to Galileo's Relativity Principle [22, 23, 31]: The laws of physics are the same in all inertial systems but the units in which they are expressed can and do vary in a systematic manner from one rest frame to another. Standard definitions such p=mv and F=ma=p/t are also satisfied in each case. The Sν scaling of frequencies (Einstein's gravitational red shift [3]) has been verified by comparing the rates of atomic clocks which were separated over a long period of time; one was located on a mountain top and while the other remained in the valley. The Sc scaling of light speed was verified in a terrestrial experiment carried out by Pound and coworkers [32]. They used the Mössbauer technique to measure the change in the speed of light as it fell through a distance of d= 22.5 m.

A blue shift of $gd/c^2 = 2.45x10^{-15}$ was measured, in quantitative agreement with the predicted value of S^{-1}.

An important observation in the present study is the fact that the value of the Universal Gravitation Constant G varies between different potentials. Observer M finds that the local value of G at position P is equal to S^2Q G in the units employed at his position (see Table I). Note that G must have the same units as c^2r in the definition of A (r) given in Sect. II. As a result, g=a at every level in Table I, as it must. This also explains the extra factor of S^{-1} in the scale factor for g in the treatment of the advancement of Mercury's perihelion. In the Newtonian formulation, $g= GM_0/r^2$, so it is consistent to include the free-fall factor of S^{-1} for G in the scale factor for g as well as the other factors for the square of distance.

VII. Conclusion

The Uniform Scaling Method has been shown to lead to the same level of accuracy as Einstein's General Theory of Relativity for key effects such as the apparent deflection of stars during solar eclipses and the precession of the perihelion of Mercury's orbit around the Sun. It does so in a far simpler manner than GRT. It would be very useful if students of GRT would be given an exercise to calculate the angle of deflection using the scaling technique discussed in Sect. III. All that is required is that the parameter S defined in Sect. II be computed at each stage of the motion of light waves as they pass close to the Sun and continue on their way to the Earth's surface. The clear assumption is that the light waves follow a completely straight line path in the process, contrary to what is claimed by the proponents of GRT. This procedure only needs to be carried out for a pair of parallel rays passing the Sun at two different distances. The Uniform Scaling Method simply assumes that the speed of light decreases as they come closer to the Sun, with the result that the ray farther away from the Sun will cover a larger distance than the other in the same amount of time. As shown in Fig. 1, this leads to a rotation of the wave front of light by an angle Θ which is identical to the deflection angle measured experimentally.

The scaling technique to illustrate the precession of Mercury's perihelion in Sect. IV starts with a standard computer program which simply reproduces Newton's method for determining the elliptical orbits of planets as they move around the Sun. Only a few statements need to be added to account for the scaling of velocity components, distances and the acceleration due to gravity g. The results of Pound et al.'s terrestrial experiment to demonstrate the effect of gravity on gamma rays as they fall through space are easily computed using the scaling technique, as shown in Table I in Sect. VI. The gravitational red shift, also as demonstrated by a terrestrial experiment, is also quantitatively described by the Uniform Scaling Method.

General Relativity and Uniform Scaling do not always agree, however. The prime example is whether they expect that light travels a curved path or always moves along a straight-line path.. Shapiro's experiments (Fourth Test of General Relativity) with echoes of radio waves passing close by planets indicate that they do in fact follow straight-line paths, in support of the Uniform Scaling position. Schiff noted that the two methods disagree with regard to an application of Thomas precession. He suggested an experiment with satellites to resolve this issue, but this has apparently not yet been done.

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