The Central Role of the Relativistic Velocity Transformation in Modern Physics Theory

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Abstract

The Relativistic Velocity Transformation (RVT) is a very valuable tool for understanding the results of experiments. It was first used in 1907 by von Laue to explain the Fresnel-Fizeau light-damping phenomenon associated with the passage of light through a tube filled with a non-dispersive liquid. It is also essential for the prediction of characteristics of modern-day high-energy collision and decay processes. The purpose of the present work is to show that the RVT successes are in no way attributable to the Lorentz Transformation (LT), which is the cornerstone of Einstein's Special Theory of Relativity (SR) published in 1905. It is shown instead that the space-time transformation (VT) introduced in 1887 by Voigt can be used directly to derive the RVT. This is an important observation since it easily proven the LT is not internally consistent and therefore is not a viable component of relativity theory.

There are nonetheless experiments which cannot be understood on the basis of the RVT, but rather require the use of the classical (Galilean) velocity transformation (GVT). It is pointed out the ranges of applicability for the GVT and RVT are mutually exclusive, and that it is a straightforward manner to distinguish between them on the basis of the specific characteristics of the experiments being carried out. The "distance rephrasing procedure" is introduced to prove that the GVT can be used successfully in examples involving light pulses, contrary to what is claimed in SR. Finally, the Law of Causality is shown to lead to a strict proportionality between the timing results of two observers in relative motion to one another. This relationship is referred to as "Newtonian Simultaneity" since it requires that events which are simultaneous for one observer will also be simultaneous for the other, unlike the prediction of remote non-simultaneity (RNS) which follows from the LT. The Newton-Voigt space-time transformation (NVT) employs the Newtonian Simultaneity proportional relationship explicitly and is also consistent with Galileo's Relativity Principle (RP). It therefore serves as a viable replacement for the LT. Newtonian Simultaneity is also a key element of the Uniform Scaling Method discussed in previous work.

Keywords: Relativistic Velocity Transformation (RVT), Galilean Velocity Transformation (GVT), Law of Causality, Newtonian Simultaneity, Newton-Voigt Space-time Transformation (NVT), Galileo's Relativity Principle (RP)

I. Introduction

There is a prevailing attitude among many physicists that the Lorentz Transformation (LT), which is the cornerstone of Einstein's Special Theory of Relativity (SR),[1] is an indispensable guide for explaining and predicting the results of experiments. Nonetheless, it is easily shown that the LT is not physically valid. For example, its conclusion that both FitzGerald length contraction (FLC) and time dilation (TD) are compatible with the SR assumption that the speed of light has the same value in all rest frames is clearly impossible to realize in actual practice.[2]

Nonetheless, attempts to point out the inadequacies of SR have been met with considerable opposition in mainstream physics journals.[3] One of the main reasons for the staunch belief in the LT in some quarters despite its obvious failures is that its supposed successful application in high-speed particle interactions is mischaracterized; results of this nature are invariably obtained instead by virtue of the Relativistic Velocity Transformation (RVT).[4] It is true that Einstein derived the RVT in his original work[1] on the basis of the LT, but the RVT can also be obtained without invoking the LT, so the above experimental results cannot be taken to be direct confirmations of SR in general.

II. VELOCITY TRANSFORMATIONS

One of the earliest laws of physics deals with the combination of velocities. As a simple example, consider the case of a car leaving the origin of the coordinate system with speed v in the x direction. The driver reports that there is a train moving at speed w relative to him in the same direction. The speed of the train relative to the origin can then be assumed to have a value of v+w, that is, the sum of the other two speeds. The above law is generally referred to as the Galilean velocity transformation (GVT), but it is quite doubtful that it is due to Galileo. In more traditional mathematical terms, it is simply an application of *vector addition* in this case of speeds.

There was confusion among physicists in the latter half of the 19th century, however, because of their inability to explain the results of a number of experiments that had been recently carried out with light waves.[5] It had started with the Fresnel light-drag experiment, which not only showed that light is slowed as it moves through a transparent medium but, by extrapolation of the value of the medium's refractive index n to a unit value, that the observed light speed in the laboratory should be completely independent of the speed v of the medium in the limit of free space [$c(v) = c$]. Maxwell's theory of electricity and magnetism published in 1864 also indicated that the speed of light has the same constant value c in each rest frame in which it is observed. This result is clearly at odds with the traditional application of the GVT which indicates that speeds should be additive and therefore that $c + v \neq c$. This led to a frantic search for an "ether" which serves as a rest frame for the light waves analogous to that known for sound waves. Michelson and Morley[6] used their newly developed interferometer to test this theory, but it merely verified the conclusion that the speed of light is independent of the rest frame through which it moves, in particular that it is directionally independent at all times of the year.

Voigt[7,8] then stepped into the fray with what in retrospect must be seen as both a daring and ingenious proposition. He speculated in 1887 that the problem lay with the Galilean transformation itself [given below in eqs. (1a-d)]. He attempted to resolve the issue by using nothing more than a free parameter and a little algebra. The resulting transformation was ultimately rejected on other physical grounds, namely it violates Galileo's Relativity Principle (RP), but it is nonetheless deserving of more than just a footnote in history. This is because it introduced for the first time the concept of *space-time mixing,* which remains to the present day to be a dogmatic principle of theoretical physics. It contradicts one of Newton's[9] most cherished beliefs, which held sway with the physics community for several centuries, namely that space and time are completely separate entities, one measured with a yardstick and the other with a clock. The consequences of this aspect of Voigt's conjecture will be discussed in the following.

A. Derivation of the Voigt transformation

The starting point of Voigt's derivation is the Galilean transformation (GT). It relates the measured values of space (x,y,z) and time (t) for a given object obtained by two observers in relative motion to one another. It is assumed that the two observers are separating with constant speed v along the common x,x' axis of the their respective coordinate systems. The relationship between their measured values is given below in terms of their respective coordinates, x,y,z,t and x',y',z',t', whereby it is assumed that the two systems are coincident at t=t'=0.

$$t' = t \tag{1a}$$

$$x' = x - vt \tag{1b}$$

$$y' = y \tag{1c}$$

$$z' = z\,. \tag{1d}$$

By construction, the velocity of the object in each coordinate system is obtained by division of the respective space and time coordinates at any instant. From eqs. (1a-b) one obtains the key relationship between the measured speeds of the object when it moves along the x,x' axis:

$$\frac{x'}{t'} = u'_x = \frac{x}{t} - v = u_x - v\,. \tag{2}$$

There is thus a linear relation connecting the two values of the speed of the object. More generally, the GT predicts that the corresponding velocities **u** and **u'** are related by vector addition when the object travels in a direction which is not parallel to the separation velocity of the two observers. Voigt[7] introduced a free parameter a into eq. (1a):

$$t' = t + ax\,. \tag{3}$$

Combining this relation with eq. (1b) of the GT[8], one concludes that $a = -vc^{-2}$ in eq. (3).

The above derivation can be extended to apply to motion of the light waves in an arbitrary direction by assuming instead of eqs. (1c-d) that $y' = \gamma^{-1}y$ and $z' = \gamma^{-1}z$ $[\gamma=(1-v^2c^{-2})^{-0.5}]$. The corresponding transformation is thus:

$$t' = t - vc^{-2}x \tag{4a}$$

$$x' = x - vt \tag{4b,1b}$$

$$y' = \gamma^{-1} y \quad (4c)$$

$$z' = \gamma^{-1} z\,. \quad (4d)$$

It can be seen that this set of equations reduces to the GT of eqs. (1a-d) in the limit of null relative velocity of the two observers, i.e. if we ignore the fact that the equations are useless in this case (with $v = 0$).[8] More significant is the fact that the same equations reduce to the GT when c is assumed to have an infinite value. One can say then without qualification that the classical transformation (GT) contains the implicit assumption that the speed of light is infinite. This is a moot point, however, since the value of c has been determined to be 299792458 ms^{-1}.

B. Taking the Relativity Principle into Account

The space-time transformation that Voigt[7] presented is successful in satisfying the light-speed constancy condition, but it fails on other grounds. This can be seen by evaluating the inverse transformation, obtained by Gauss elimination from eqs. (4a-d). According to Galileo's RP, the inverse transformation should be obtained by simply exchanging the primed and un-primed subscripts in the forward set of equations and substituting −v for v. This is a mathematical procedure that mimics the situation when the observers change positions; it will be referred to as *Galilean inversion* in the following. It is easily shown that the inverse of eqs. (4a-d) does not satisfy this requirement, however. For example, if the inverse equation for y' is applied to eq. (4c), the result is $y' = \gamma^{-2} y'$, an obviously unacceptable relationship. This proves that the Voigt transformation is not consistent with the RP and thus must be rejected as a physically valid set of equations.

It is nonetheless a simple matter to modify the transformation in a way which satisfies both the RP and the light-speed constancy condition. Before doing this, it is helpful to make a change in variables to *intervals* for two different events: $\Delta x = x_2 - x_1$, $\Delta x' = x'_2 - x'_1$ etc. This change allows each observer to choose his own coordinate system without the necessity of having it coincide at some point with the other coordinate system. Intervals are of course required in order to compute speeds, which remains the center of attention in this discussion. The Voigt transformation (VT) thus becomes:

$$\Delta t' = \Delta t - vc^{-2} \Delta x \quad (5a)$$

$$\Delta x' = \Delta x - v\Delta t \quad (5b)$$

$$\Delta y' = \gamma^{-1}\Delta y \tag{5c}$$

$$\Delta z' = \gamma^{-1}\Delta z\,. \tag{5d}$$

When the above equations are used to form the following linear combination of squared quantities, the result is:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \gamma^{-2}\left(\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2\right). \tag{6}$$

In order for eq. (6) to hold, it is necessary that both observers measure the speed of light to be equal to c so that both sides of the equation vanish in this case. This shows that Voigt's goal is achieved by the transformation in eqs. (5a-d). It is also clear that if each of the right-hand sides of the four equations is multiplied by the factor ε, the same objective is satisfied. The factor in eq. (6) simply becomes $(\varepsilon\gamma^{-1})^2$ and therefore this change does not alter the conclusion regarding light-speed constancy.

This circumstance thus opens up the possibility of eliminating the problem with the RP without changing the condition for the two measured values of the light speed. Lorentz[10] made this observation for a different reason, namely to define a space-time transformation that allows the electromagnetism equations to be invariant while also insuring that the RP be satisfied. Both Larmor[11] and Lorentz[12] independently realized at about the same time that this goal can be achieved by using the factor $\varepsilon=\gamma$ (v) to modify the VT in this manner. The resulting set of equations is known as the Lorentz transformation (LT) and is given below $[\eta = (1-vc^{-2}\Delta x/\Delta t)^{-1}]$:

$$\Delta t' = \gamma\left(\Delta t - vc^{-2}\Delta x\right) = \gamma\eta^{-1}\Delta t \tag{7a}$$

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right) \tag{7b}$$

$$\Delta y' = \Delta y \tag{7c}$$

$$\Delta z' = \Delta z\,. \tag{7d}$$

It is obvious that the inverse of eqs. (7c,d) is achieved by application of Galilean inversion.

The corresponding results for the inverse of eqs. (7a,b) can be derived with the help of the following identity:[13] $\eta\eta'=\gamma^2$ [note that $\eta'=(1+vc^{-2}\Delta x'/\Delta t')^{-1}$ is obtained by applying Galilean inversion to η]:

$$(\eta\eta')^{-1} = \left(1-\frac{v\Delta x}{c^2\Delta t}\right)\left(1+\frac{v\Delta x'}{c^2\Delta t'}\right) = \left(1-\frac{v\Delta x}{c^2\Delta t}\right)\left[1+\eta\frac{v}{c^2}\left(\frac{\Delta x}{\Delta t}-v\right)\right] \tag{8}$$

$$= 1 - \frac{v\Delta x}{c^2\Delta t} - \frac{v^2}{c^2} + \frac{v\Delta x}{c^2\Delta t} = 1 - \frac{v^2}{c^2} = \gamma^{-2}$$

Proof that the inverses of eqs. (7a,b) are consistent with the RP proceeds by applying Galilean inversion as follows:

$$\Delta t = \gamma\eta'^{-1}\Delta t' = \gamma\eta'^{-1}\gamma\eta^{-1}\Delta t = \gamma^2\left(\eta'\eta\right)^{-1}\Delta t = \Delta t \qquad (9)$$

$$\Delta x = \gamma\left(\Delta x' + v\Delta t'\right) = \gamma\left[\gamma\left(\Delta x - v\Delta t\right)\right] + \gamma v\gamma\eta^{-1}\Delta t =$$

$$\gamma^2\left[\left(\Delta x - v\Delta t\right) + v\eta^{-1}\Delta t\right] = \gamma^2\Delta x - \gamma^2 v\Delta t + \gamma^2 v\Delta t\left(1 - \left(\frac{v}{c^2}\right)\frac{\Delta x}{\Delta t}\right) = \qquad (10)$$

$$\gamma^2\left[\Delta x - v\Delta t + v\Delta t - \Delta x v^2 c^{-2}\right)] = \gamma^2\Delta x\left(1 - v^2c^{-2}\right) = \Delta x$$

III. THE THIRD POSTULATE: THE LAW OF CAUSALITY

The above discussion has shown that the LT satisfies both the RP and the light-speed equality condition experimental data seemed to require. It has been pointed out in the Introduction, however, that it is not a viable space-time transformation and therefore must be rejected. It makes the claim[2] that when an experiment is carried out to measure the speed of light,

it is possible that observers in different rest frames can agree on both the value of the light speed and the distance travelled by the light in the experiment while using clocks which run at different rates (time dilation). *This is utter nonsense.*

How did it come to this state of affairs? One answer can be found by noting that both Poincaré and Lorentz made an assumption about the group theoretical properties of the desired space-time transformation.[14] Einstein came to the same conclusion[15] and this led to the following general transformation:

$$\Delta t' = \varepsilon\gamma\,(\Delta t - vc^{-2}\,\Delta x) \qquad (11a)$$

$$\Delta x' = \varepsilon\gamma\,(\Delta x - v\,\Delta t) \qquad (11b)$$

$$\Delta y' = \varepsilon\Delta y \qquad (11c)$$

$$\Delta z' = \varepsilon\Delta z \qquad (11d)$$

In this set of equations, ε is a function of v only. Since the product of this transformation and its inverse should yield the identity,[15] it follows that $\varepsilon(v)\varepsilon(-v) = 1$. At the same time, the y,z transformations should not change when the sign of v is inverted. Therefore, $\varepsilon(v) = \varepsilon(-v)$. and hence, $\varepsilon(v) = 1$ is the unique solution. Substitution of this value leads directly to eqs. (7a-d) of the LT.

One thing that can be taken away from the discussion about the possible relevance of group theoretical relationships is that something else, a "Third Postulate," is clearly needed besides the RP and the equality of light-speed measurements to arrive at a physically tenable solution to the search for a truly relativistic space-time transformation. A hint at what this might be is the observation that Einstein's version of relativity theory[5] takes little or no account of Newton's longstanding contributions[9] to the classical theory of dynamics. More fundamentally, it virtually ignores the Law of Causality, which has played a key role in the development of science through the ages.

The Law of Causality basically says that nothing happens without something causing it to occur. Newton's First Law of Motion[9] (Law of Inertia) is a prime example. It says that a body will continue in a straight line at constant speed until it is subjected to an unbalanced external force. By extension, each of the physical properties of the same object such as a clock will remain constant indefinitely *unless some outside force is applied*. Accordingly, one can conclude that the rate of such a (inertial) clock will not change unless it is acted upon by some outside force (clock-rate corollary[16,17]). One can therefore conclude that the *ratio* of the rates of any two such clocks will also be a *constant.* In other words, when these clocks are used to measure an elapsed time, their respective values Δt and $\Delta t'$ will always be found to be in the same ratio, i.e. $\Delta t'=\Delta t/Q$, where Q is the rate ratio.

The LT is based on the use of inertial clocks in two different rest frames. One of its main characteristics [eq. (7a)] is that the elapsed time $\Delta t'$ measured on one such clock will depend on the relative speed v of the two rest frames and the location Δx of the object in one of the other rest frames as well as the time Δt measured on that clock, i.e. $\Delta t'=\gamma(v)(\Delta t-v\Delta x c^{-2})$. It can be seen that if both v and Δx have non-zero values, then $\Delta t'$ will not be proportional to Δt. This characteristic of the LT is known as *space-time mixing*. It stands in direct contradiction to the $\Delta t'=\Delta t/Q$ relation required by the Law of Causality. *This shows that the LT is not consistent with the Law of Causality.*

One of the consequences of the space-time mixing of the LT is that it allows the two observers mentioned above to disagree on whether two events occurred simultaneously or not.[18,19] This is clear from the same LT equation mentioned above. If both v and Δx are not equal to zero, it follows that when Δt=0 (note that Δt=0 means that the two events did occur simultaneously for the one observer), it cannot be that Δt'= 0 as well, i.e. that the two events were also simultaneous for the other observer. This situation is referred to as remote non-simultaneity (RNS). The distinction between the LT and the Δt'=Δt/Q condition required by the Law of Causality is quite clear because in the latter case when Δt'=0, so must also Δt. For this reason the latter proportionality relation is referred to as *Newtonian Simultaneity.* This is in recognition of the historical fact that Newton was a firm believer in absolute simultaneity, that is, that if two events occur simultaneously, they will also be found to be simultaneous in any other pair of rest frames throughout the universe.

The space-time mixing equation of the LT in eq. (7a) needs to be replaced in order to be consistent with the Δt' = Δt/Q relation deduced from the Law of Causality. In this respect it is important to see that the latter equation is related to eq. (7a) by the following proportionality relation:

$$\Delta t' = \Delta t / Q = \left(\frac{\eta}{\gamma Q}\right)\gamma\eta^{-1}\Delta t \qquad (12)$$

One can therefore take advantage of Lorentz's observation[10] that the equal-light-speed relation of both the LT and the original Voigt transformation can be preserved by multiplying each of the right-hand sides of these transformations by a constant factor. As a result, a different transformation that also satisfies the equal-light-speed condition can be obtained by multiplying each of eqs. (7a-d) with (η/γQ):

$$\Delta t' = \left(\frac{\eta}{\gamma Q}\right)\left(\gamma\left(\Delta t - vc^{-2}\Delta x\right)\right) = \left(\frac{\eta}{\gamma Q}\right)\gamma\eta^{-1}\Delta t = \frac{\Delta t}{Q} \qquad (13a)$$

$$\Delta x' = \left(\frac{\eta}{\gamma Q}\right)\gamma\left(\Delta x - v\Delta t\right) = \frac{\eta\left(\Delta x - v\Delta t\right)}{Q} \qquad (13b)$$

$$\Delta y' = \left(\frac{\eta}{\gamma Q}\right)\Delta y \qquad (13c)$$

$$\Delta z' = \left(\frac{\eta}{\gamma Q}\right)\Delta z \qquad (13d)$$

The same result is obtained if one multiplies each of the VT eqs. (5a-d) by a factor of η/Q. The resulting set of equations will be referred to as the Newton-Voigt transformation (NVT). Note that it contains the *Newtonian Simultaneity* relation explicitly in eq. (13a). The latter designation for the Δt'= Δt/Q relation is in recognition of the fact that it implies that each event in the universe occurs simultaneously for all observers in the universe, which conclusion stands in full agreement with the long-held view of Newton and classical physicists in general that space and time are completely separate entities.[20] The consistency of the NVT with the equal-light-velocity requirement is demonstrated by forming the following linear combination of squared quantities from the NVT:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \left(\frac{\eta}{\gamma Q}\right)^2 \left(\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2\right). \qquad (14)$$

IV. DERIVATIONS OF THE RVT

The RVT is derived by taking the ratio of the distance travelled by an object to the necessary elapsed time for this to occur. When this procedure is followed using the VT[2] of eqs. (5a-d), the result is:

$$u_x = \left(1 + vc^{-2}u_x'\right)^{-1}\left(u_x' + v\right) = \eta'\left(u_x' + v\right) \qquad (15a)$$

$$u_y = \gamma^{-1}\left(1 + vc^{-2}u_x'\right)^{-1} u_y' = \gamma^{-1}\eta' u_y' \qquad (15b)$$

$$u_z = \gamma^{-1}\left(1 + vc^{-2}u_x'\right)^{-1} u_z' = \gamma^{-1}\eta' u_z'. \qquad (15c)$$

In these equations, u_x=Δx/Δt, u_x'=Δx'/Δt', etc., and the definitions of γ, η and η' are the same as used in deriving the identity in eq. (8). The same procedure can also be used based on both the NVT and LT equations.

The RVT eliminates the "c=c+v" problem through the use of the η' function. If u_x'=c, then $\eta'=(1+c^{-1}v)^{-1}=c(c+v)^{-1}$. As a result, in eq. (15a), $u_x=c(c+v)^{-1}(c+v)=c$, in agreement with the light-speed constancy assumption. This is certainly not surprising, since the underlying condition in deriving the RVT is that for any choice of u_x', u_y', u_z' with a vector magnitude of c, the corresponding result for u_x, u_y, u_z must also have the same magnitude, but with a generally different direction than the original vector. It should be noted that the RVT results cannot be obtained by vector addition, contrary to the analogous situation with the GVT.

V. PROPERTIES OF THE RVT AND NEWTON-VOIGT TRANFORMATION

It remains to show that the RVT satisfies a number of essential requirements, particularly with regard to the light velocity equality in different rest frames and Galileo's RP. The former characteristic is considered below by forming the following linear combination of squared quantities contained in eqs. (15a-c):

$$u_x^2 + u_y^2 + u_z^2 - c^2 = \eta'^2\left[u_x'^2 + 2u_x'v + v^2 + \gamma^{-2}\left(u_y'^2 + u_z'^2\right)\right]$$

$$-\eta'^2 c^2\left(1 + \frac{2vu_x'}{c^2} + \frac{u_x'^2 v^2}{c^4}\right) =$$

$$\eta'^2\left[\left(\left(1-\frac{v^2}{c^2}\right)\left(u_x'^2 + u_y'^2 + u_z'^2\right) + \frac{v^2 u_x'^2}{c^2}\right) + 2vu_x' - c^2 - 2vu_x' - \frac{v^2 u_x'^2}{c^2}\right] = \quad (16)$$

$$\eta'^2\gamma^{-2}\left(u_x'^2 + u_y'^2 + u_z'^2 - c^2\right).$$

It is clear that when the speed of the object is equal to c in one rest frame, it will also be equal to c in the other, as required. A key aspect of eq. (16) is the fact that the $\eta'^2\gamma^{-2}$ factor on the left-hand side is positive definite. As a consequence, if the object's speed is less than c in one rest frame, it will also be less than c in the other. Moreover, if it is greater than c in one frame, it will also be greater than c in the other. The latter situation can only occur if the inertial mass of the object is equal to zero, which therefore is consistent with greater-than-c speeds of photons.

The RVT is also consistent with the RP. This is shown below by applying the Galilean inversion operation to the RVT eqs. (15a-c) and making use of the $\eta\eta' = \gamma^2$ identity derived in eq. (8).

$$u_y' = \gamma^{-1}\eta u_y = \gamma^{-1}\eta\gamma^{-1}\eta' u_y' = u_y' \quad (17)$$

$$u_x' = \eta\left(u_x - v\right) = \eta\left[\eta'\left(u_x' + v\right) - v\right] =$$

$$\gamma^2\left(u_x\text{'}+v\right)-\eta v=\gamma^2\left(u_x\text{'}+v\right)-\frac{v\gamma^2}{\eta\text{'}}= \tag{18}$$

$$\gamma^2 u_x\text{'}+\gamma^2 v-\gamma^2 v\left(1+\frac{vu_x\text{'}}{c^2}\right)=\gamma^2 u_x\text{'}\left(1-\frac{v^2}{c^2}\right)=u_x\text{'}$$

The NVT is consistent with the RP. This can be shown by applying Galilean inversion to each of its equations. This procedure leads to a key requirement for the quantity Q and its counterpart Q' when applied to eq. (13a):

$$\Delta t=\frac{\Delta t\text{'}}{Q\text{'}}=\left(\frac{1}{Q\text{'}}\right)\frac{\Delta t}{Q}=\frac{\Delta t}{QQ\text{'}} \tag{19}$$

It is clear that the only way to satisfy the RP is for Q' to be the reciprocal of Q. From a physical point of view, this condition simply reflects the fact that the two proportionality factors have the reciprocal relationship expected for comparison of elapsed times from the different vantage points of the two rest frames represented in the space-time transformation. The two quantities are most simply looked upon as *conversion* factors[21,22] between different units of time. The reciprocal condition is exactly the same as for all other physical properties, and also for other quantities such as currency values. For example, the conversion factor for changing from kilometers (km) to meters (m) is 1000, whereas the factor for the opposite change from m to km is 1/1000. In what follows it will therefore be assumed that QQ' = 1 in all applications of Galilean inversion. The proofs for the spatial components proceed as follows:

$$\Delta y=\left(\frac{\eta\text{'}}{\gamma Q\text{'}}\right)\Delta y\text{'}=\left(\frac{\eta\text{'}}{\gamma Q\text{'}}\right)\left(\frac{\eta}{\gamma Q}\right)\Delta y=\left(\frac{\eta\text{'}\eta}{\gamma^2}\right)\Delta y=\Delta y \tag{20}$$

$$\Delta x=\left(\frac{\eta\text{'}}{Q\text{'}}\right)\left(\Delta x\text{'}+v\Delta t\text{'}\right)=\left(\frac{\eta\text{'}}{Q\text{'}}\right)\left[\left(\frac{\eta}{Q}\right)\left(\Delta x-v\Delta t\right)+\frac{v\Delta t}{Q}\right]=$$

$$\gamma^2\left(\Delta x-v\Delta t\right)+\eta\text{'}v\Delta t=\gamma^2\left(\Delta x-v\Delta t\right)+\gamma^2\eta^{-1}v\Delta t)= \tag{21}$$

$$\gamma^2\left(\Delta x-v\Delta t\right)+\gamma^2 v\Delta t\left(1-\frac{v\Delta x}{c^2\Delta t}\right)=$$

$$\gamma^2\left(\Delta x - v\Delta t\right) + \gamma^2 v\Delta t - \frac{\gamma^2 v^2 \Delta x}{c^2} =$$

$$\Delta x\left(\gamma^2 - \frac{\gamma^2 v^2}{c^2}\right) = \Delta x\,.$$

Finally, the chronology of the relativistic space-time transformations will be reviewed below in terms of Lorentz's ε factor[10] discussed in Sect. II B. The Voigt transformation (VT) is characterized by a value of $\varepsilon=\gamma^{-1}$. The fact that it satisfies the Lorentz criterion shows that it does satisfy the equal light-velocity requirement, but it is deficient because of its lack of consistency with the RP. The LT is characterized by ε=1, so it also satisfies the light-speed condition. It is also consistent with Galileo's RP, however, and this fact has led physicists to look upon it as a perfectly reliable means of describing the relationships between the measured values of any physical property by observers in two different rest frames. It has been pointed out, however, that the LT is not consistent with the Law of Causality and is therefore unacceptable as a law of physics. Finally, the NVT is characterized by a value of $\varepsilon=\eta/\gamma Q$. It has furthermore been shown that it is consistent with both the RP and the Newtonian Simultaneity relation for measured times in the two rest frames, and is therefore also consistent with the Law of Causality. The experimental results which have been claimed as verifications of the LT invariably involve the RVT and thus do not require the LT at all, especially since the NVT is also consistent with the RVT.

VI. FAILURE OF EINSTEIN'S LIGHT SPEED POSTULATE

In formulating his version of relativity theory,[1] Einstein agonized[23] over the definition of a postulate which is consistent with the experimental observation of light-speed constancy. He concluded that the speed of light in free space has the same value c for all observers *independent of their state of motion as well as that of the source of the light.* It will be shown in the following how this postulate leads directly to the conclusion that the lightning strikes on a train could not possibly be simultaneous for both an observer there and one who is stationary on the platform.

A basic part of the theory has to do with how different people perceive how fast an object is moving. Consider again the example discussed at the beginning of Sect. II. You are standing on a street corner as a car passes you with a speed of v=50 km/h. The car driver reports that he sees a train moving in the same direction with speed w=30 km/h relative to him. You can safely assume on this basis that the train is moving with speed v+w=80 km/h relative to you as you stand on the corner. It is all very easy to understand.

Now change the example so that there is a light pulse instead of a train. The light pulse moves with speed w=c relative to the car. So the relative speed of the light to you on the corner will be v+c according the above example using a train. Einstein did not agree with this conclusion, however. He assumed[24] instead (light-speed postulate LSP) that the speed of light is *independent* of the speed of the observer or light source. He claimed that the procedure used above in the car-train example (the Galilean velocity transformation GVT) is only valid at low speeds much less than c.

There is a simple way to test Einstein's assumption, however. Just consider how far the light travels in a given time T relative to the car/light source on the one hand and relative to the street corner/origin on the other.[25] According to Einstein's LSP, in both cases the value of the distance of separation from the light pulse is found to be cT. *This result is clearly unacceptable,* since it is impossible that the light pulse could be the same distance from both *since their two positions are not coincident at time T.* For example, T could be as great as one year, so the distance separating the light source from the origin/street corner would be 1.0 light year (ly) in that case. This proves beyond any shadow of a doubt that Einstein's LSP in untenable.

The same procedure (*distance reframing*[25]) can be put to good use in another way in this example. The distance moved by the light source relative to the origin is vT, while that moved by the light pulse relative to the light source is cT. The total distance separating the light pulse from the origin is obtained by simply adding these two values, with the result vT+cT=(v+c)T. (Note that the addition of distances is commonplace in everyday activities such as measuring the width of a room, whereas there is no such intuitive principle for the addition of velocities.) By definition, the speed of the light pulse relative to the origin is obtained by dividing the above value by the elapsed time T, which upon cancellation gives v+c. This is exactly the value that is obtained when the GVT is applied directly. In summary, the *distance reframing procedure* contradicts the long-held position of the physics community that the motion of the light pulse relative to two different rest frames is governed by Einstein's LSP, while at the same time verifying that the GVT is completely accurate in this example as well as in any conceivable variation involving other moving objects than light.

Relegation of the GVT to the realm of low-energy physics has its price, however. Belief in the LT and Einstein's LSP forces one to accept the doctrine of remote non-simultaneity (RNS).[18] Accordingly, two events which occur simultaneously for an observer in one rest frame may not necessarily be simultaneous for someone who is in motion relative to him. Einstein was aware that there is no experimental verification for RNS,[26] even though what Poincaré[27] had to say on the subject is just as true, namely that there is also no proof from experiment that all events must occur at the same time for all observers in the universe.

In order to deal with his own uncertainty on this subject, Einstein came up with an example[24] which is intended to demonstrate without doubt that RNS is a fact of nature. He asked his readers to consider the case in which two lightning strikes occur on a passing train. They are measured to occur simultaneously for an observer O_p who is at rest on the station's platform. He argued that if the two strikes occurred on opposite sides of the position M on the platform which both were separated by a distance of L from O_p, then light emanating from them would necessarily arrive at M simultaneously. The time T_p required for this to occur is L/c, where c is the speed of light in free space.

He further assumed that the passing train was moving at a constant speed v relative to the platform as the lightning strikes occurred. On the basis of his LSP, an observer O_t who is at rest on the train at the same position M when the two lightning strikes occur, cannot find that they would also occur simultaneously for him. This is because O_t must find that the light pulse moving in the opposite direction as the train would move a distance of cT toward him at any time T while he has moved a distance of vT during the same period. The light would therefore arrive at O_t's momentary position at time $T_1=L/(v+c) < T_p$. Meanwhile the light pulse travelling in the opposite direction would also move a distance of cT by virtue of the LSP, whereas O_t would have moved a distance of vT away from this pulse. The time required for this light pulse to "catch up" with O_t is thus $T_2= L/(c-v)>T_p$. Clearly, $T_2>T_1$, so the light pulses do not arrive simultaneously for O_t when the LSP is used, as Einstein wished to show.[24]

Let us now consider how the substitution of the GVT for the LSP in Einstein's example of two lightning strikes changes the result. Assume as before that the light from the two strikes reaches the observer O_p located at the midpoint M of the platform simultaneously at time T_p=L/c. After time T has elapsed, *the sources of the strikes* have moved to positions 2L+vT and vT, respectively, that is, by taking account of the speed of the train relative to the platform. The speed of the first light pulse relative to O_t is c+v in the negative direction according to the GVT, so at time T this pulse is located at 2L+vT–(v+c)T=2L-cT. Note that this is exactly the same trajectory for this light pulse as from the vantage point of O_p.

Meanwhile, the speed of the second pulse toward O_t is c-v according to the GVT. As a result it is located at vT+(c-v)T=cT at time T. The trajectory of this one is also identical to that measured by the stationary observer O_p on the platform. Therefore, the two light pulses will also meet for O_t when 2L-cT=cT. The corresponding time is $L/c=T_p$, the same as for O_p on the platform. In summary, the arrival time is simultaneous for O_t as well as for O_p when the GVT is applied. It is thus clear that there is no RNS in this procedure using the GVT, contrary to what one must assume when the LSP is assumed instead.

It is therefore clear from the above discussion that there are some experiments involving light which can be understood within the context of the GVT but not when the RVT is used in its place. The opposite is also true, however. Some experiments can be understood using the RVT, but not when the GVT is used instead. For example, the RVT performs well for the Fresnel-Fizeau light-drag experiment,[28] but not in the train example discussed above. It will be shown in the next section that the respective ranges of application of the GVT and RVT are *mutually exclusive.*

VII. DICHOTOMY OF THE APPLICATIONS OF THE GVT AND RVT

The goal is therefore to be able to decide on a definitive basis which of the two transformations is applicable in a given case. The solution is quite simple.[29] When *two observers in different rest frames* are to compare their measurements for the same light pulse, they must use the GVT to obtain the correct answer. By contrast, the RVT is valid when *only a single observer makes separate observations under two different conditions,* for example, namely v=0 and v≠0 for the relative speed of the medium in the Fresnel-Fizeau experiment.[28]

The RVT assumes that space and time are mixed, a concept first introduced by Voigt in 1887.[7] This position stands in stark contrast to the view of classical physicists such as Newton which holds that the two observers always agree on the amount of elapsed time in which measurements are made (Δt=Δt').

One can divide velocity measurements involving the speed of light into two distinct categories. In the first, Type A, there are *two observers in relative motion to one anoth*er, each of which carries out measurements of the speed of the same light pulse. They obtain different values which can be combined using the GVT and vector addition. It is possible for the speed of light to exceed a value of c in this case. The same procedure can be used for any object.

The second category of measurements, Type B, involves *only a single observer* who obtains measurements of the object *under two different circumstances*. The RVT must be used in order to relate these two values. It is therefore *not* possible for the speed of light to exceed a value of c in this case.

The phenomenon of stellar aberration refers to astronomical observations of the apparent movement of the positions of celestial objects at different times of the year. It is an example of Type A because there are two rest frames (Earth and Sun) relative to which the light speed is measured. The first coherent explanation for this effect is credited to James Bradley. Writing in 1727, he ascribed it to the finite velocity of light and the motion of the Earth relative to the Sun, and he used the classical theory of motion (GVT) to quantify his position. There was longstanding wide acceptance for his arguments, but they eventually met with considerable skepticism because they were thought to be incompatible with new experimental data obtained at the beginning of the next century. The latter results led to the development of numerous theories that posited the existence of an ether that was assumed to be essential to the true theory of the motion of light.

The matter came to a head in 1905 when Einstein published what has come to be known as the Special Theory of Relativity (SR).[1] He rejected the need for an ether to explain the outstanding questions, but assumed instead that "light in a vacuum always moves with a definite velocity, independent of the velocity of the emitting body." This conclusion is in conflict with Bradley's explanation of stellar aberration which assumed, in concert with the classical (Galilean) theory, that the speed of light emitted from the Sun depends on the state of motion of an observer located on the Earth's surface.

One can use the distance reframing procedure discussed in Sect. VI to prove that Bradley's interpretation is correct. Accordingly, in a given time period T, the Sun moves a distance of vT relative to the Earth whereas the light emitted from the Sun moves a perpendicular distance of cT in the same period. The total distance travelled by the light pulse is therefore obtained using the Pythagorean Theorem to have a value of $(v^2+c^2)^{0.5}$ T. Division by T gives the value of the light speed relative to the Earth to be $(v^2+c^2)^{0.5}$, which is greater than c. The aberration angle is thus found to be $\tan^{-1}$ (v/c). Use of the RVT instead[30,31] gives an incorrect value for this angle, namely $\tan^{-1}$ (γ v/c). It does so by assuming that the light pulse emanating from the Sun has a speed of c/γ rather than the correct value of c. For typical speeds of the Earth relative to the Sun, however, γ(v) differs from unity by on the order of only 10^{-8}, and this difference is therefore too small to be confirmed in actual observations.

The Fresnel light-drag experiment, on the other hand, is a concrete example of Type B. The experiment itself involves observations of the speed of light in transparent media. In the early 19th century, it was already clear that the value of the light speed varied when the speed of the medium v relative to the laboratory was increased. The measured value (c') was found to satisfy the formula given below (n is the refractive index of the medium):

$$c' = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right). \tag{22}$$

If n is changed to its free-space value (n=1), it is found that the v-dependence in eq. (22) disappears entirely, and one is led to conclude that c'=c(v) under this condition. This result is seen to be a verification of Einstein's LSP.[24] The RVT of eqs. (15a-c) leads to the same result for light moving in free space. Moreover, it also leads directly to eq. (22) when the light moves through a medium with refractive index n. This result was first obtained by von Laue[28] in 1907 and has been hailed as one of the first successes of Einstein's theory.[32]

The derivation proceeds by assuming that u_x'=c/n in eq. (15a). One then obtains in agreement with eq. (22):

$$u_x = \eta'\left(\frac{c}{n} + v\right) = \left(1 + \frac{v}{cn}\right)^{-1}\left(\frac{c}{n} + v\right) = \tag{23}$$

$$\left(1 - \frac{v}{cn}\right)\left(\frac{c}{n} + v\right) = \frac{c}{n} + v - \frac{v}{n^2} == \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right)$$

after making various approximations based on the condition that v<<c.

The crucial distinction in the Fresnel experiment is that there is *only one observer* in this case, as opposed to two in the example of stellar aberration. The quantities u_x and u_x' *refer to the same observer making separate observations under two different conditions,* namely v=0 and v≠0. The assumption of light-speed constancy is then suggested by the special case for the free-space value of n=1, in which case u_x=u_x'=c, as already discussed in connection with eq. (22). It is also clear that the GVT cannot be reasonably applied under this condition since it requires that two different observers are involved in making the speed determinations *at the same time.* In summary, the range of application of the two velocity transformations is indeed mutually exclusive. The RVT performs well for the Fresnel light-drag experiment (Type B), but not in the description of stellar aberration (Type A), whereas the opposite is the case for the GVT.

Another Type B example for which the RVT is essential involves the acceleration of electrons in electromagnetic fields. The objective in this case is to cause an electron to attain faster-than-c speed. As in the Fresnel light-drag experiment, there is but one observer who performs measurements under two different conditions, i.e. in this case before and after the field is applied. The assignments of velocities in the RVT in the two cases are made on this basis.

The value of v in the equations is taken to be the product of an acceleration **a** due to the field and a time difference Δt during which the field is applied. Einstein[1] predicted successfully that a massive particle such as the electron can never exceed or be equal to c. The assumption of light-speed constancy is justified because of the limiting case where the magnitudes of the two velocities each approach a value of c, i.e. one starts with the electron moving with a speed very close to c and ends up with a new velocity after application of the field with a magnitude which is only infinitesimally greater but is still less than c. This experiment cannot be explained on the basis of the GVT.

Another important example where the RVT is essential but for which the GVT cannot be used successfully is in deriving the theoretical explanation of the phenomenon of Thomas spin precession.[33,34] This case has some similarities to that discussed above regarding attempts to accelerate an electron to faster-than-c speed. The focus in both cases is on the state of motion of the electron in two different situations, before and after application of a field, so the application of the GVT is ruled out in this case as well. The derivation of Thomas spin precession is different, however, in that it uses the Lorentz transformation (LT) rather than the RVT. The result is the following expression for the angular velocity ω_T of the electron:

$$\omega_{\mathrm{T}} = \mathrm{c}^{-2} \gamma^2 \left(\gamma + 1\right)^{-1} \ \mathbf{a} \ \mathbf{x} \ \mathbf{v}, \tag{24}$$

where **v** and **a**, respectively, are the instantaneous velocity and acceleration of the electron at a given time. It has be shown[34] subsequently that the LT is not essential in this derivation; a different version of the space-time transformation than the LT achieves the same result.[10]

The Sagnac effect[35] is another example of a Type B experiment. It can be explained[36] entirely on the basis of Einstein's light-speed postulate and the RVT. Two light beams travelling in opposite directions on a circular platform of radius r rotating with frequency ω must travel different distances before interfering. Beam A must travel completely around to reach this point on the platform during one full revolution. The distance travelled is therefore assumed on the basis of the light-speed postulate to be $d_A=ct_A=2\pi r+r\omega t_A$, where t_A is the corresponding time of travel. The other beam (B) does not make it all the way around, so its distance travelled during one full revolution of the wheel before reaching the point of interference is $d_B=ct_B=2\pi r-r\omega t_B$. Solving for the respective elapsed times gives $t_A=2\pi r(c-r\omega)^{-1}$ and $t_B=2\pi r(c+r\omega)^{-1}$. The difference is thus $\Delta t=t_A-t_B=2\pi r(2r\omega)(c^2-r^2\omega^2)^{-1}\approx 4\pi r^2\omega c^{-2}=4A\omega c^{-2}$, which is the observed value in the laboratory (A is the area of the platform). An observer in another inertial system simply measures a different value for Δt because his proper clock runs at a different rate than that at rest in the laboratory, but the same value for the light speed is measured in both cases according to the light-speed constancy postulate.

The RVT is used extensively in the analysis of particles emitted by rapidly moving sources. Experiments of this kind are of Type B since they only involve a single observer (the laboratory) in which the particles are accelerated. For example, consider the case[37] in which a Σ^0 hyperon decays to a photon plus Λ particle. The variables which are to be inserted in eq. (15a) in one example are defined as follows: v is the speed of the Σ^0 particle in the laboratory rest frame, u_x' is the speed of Λ in this rest frame and u_x is the final speed of Λ after the decay has occurred. There is a collimating effect such that the higher the value of v, the more the particles get beamed forward in the laboratory rest frame. The GVT is unable to produce the correct values of u_x in this Type B example.

VIII. CONCLUSION

Up until the start of the 19th century, there was a general agreement among scientists that the motion of light and sound waves could both be successfully described by the classical (Galilean) velocity transformation (GVT). Fresnel predicted in 1818 on the contrary that light imparts elastic vibrations on an ether through which it passes. This assumption led him to conclude that when light passes through a tube filled with water, its velocity relative to the laboratory would depend on the speed of the liquid in a manner which cannot be explained on the basis of the GVT. His prediction was borne out by experiments carried out by Fizeau in 1851 and this led to a frantic search for the supposed ether, as well as for a replacement for the GVT.

A theory developed which claimed that the speed of the ether would vary with its orientation of relative to the Earth. This led to the conclusion that the velocity of light would be seasonally dependent, but the experiment carried out by Michelson and Morley in 1887 using a newly developed interferometer showed conclusively that there is no such effect. Voigt suggested on this basis that the questions regarding the speed of light could be answered by simply altering the classical space-time transformation (GT). The idea was to assume that two observers in different rest frames must agree that the value of the light speed in free space is equal to c. The result was the Voigt Transformation (VT) shown in eqs. (5a-d). It is important to see that the three equations of the RVT of eqs. (15a-c) can be derived from the VT by simply dividing each of the spatial variables in eqs. (5b-d) by the corresponding time defined in eq. (5a). Moreover, as von Laue later showed in 1907, the RVT can then be used to derive the expression for the Fresnel-Fizeau light-drag experiment shown in eq. (22).

From a historical perspective, the key point to be taken away from the above observations is that the LT given in eqs. (7a-d) is *in no way essential* in arriving at either the RVT or the Fresnel light-drag prediction. It is only true that the same procedure of dividing spatial by time variables in the LT also leads to the RVT, but that does not justify the oft-mentioned claim that experiments that can be explained on the basis of the RVT somehow also amount to verifications of the LT. This state of affairs is also relevant when considering the fact, as mentioned in the Introduction, that the LT is not internally consistent and therefore that it must be ruled out as a viable component of relativity theory. This is an example of the general axiom *that the use of a false premise in a logical argument can nonetheless lead to a correct conclusion.* The same holds true for the VT since it does not agree with the requirements of the RP, and yet still can be successfully used to derive the RVT. Indeed, there are an infinite number of space-time transformations that lead to the RVT when the same procedure used for the VT and LT is applied to them.

As discussed in Sect. VII, however, there are situations in which the GVT must still be used in place of the RVT. The conclusion made by Einstein that the GVT cannot be applied to the to the motion of light waves is easily disproven by considering how two observers in different rest frames view them. If a light source moves away from a street corner with speed v at the same time it emits a light pulse, the distance travelled by the light after a certain time T has passed is equal to cT for an observer moving with the light source, but cT + vT for his counterpart at rest on the street corner ("distance reframing" procedure). By definition, the speed of the light pulse for the latter is therefore c +v, which is exactly the value predicted by the GVT. Application of the RVT to the Fresnel-Fizeau light-damping experiment only involves a single observer in the laboratory who makes his determination under two different conditions, namely with the non-dispersive liquid at rest with respect to him, and at a later time when the liquid flows through the tube at speed v.

The ranges of application for the GVT and RVT, are mutually exclusive as a consequence (referred to as Types A and B, respectively). For example, when two light pulses approach each other head-on, this is a Type A experiment and therefore can only be successfully described by the GVT; the relative speed of the two pulses is 2c, not c as the RVT predicts. On the contrary, in the example of a Type B experiment involving collisions of elementary particles, it is a single observer in the laboratory who measures the velocity of a photon or other emitted particle *before and after* a decay process occurs; the relationship between the two values is successfully described using the RVT but not the GVT. It is a curious fact of history that the RVT can be derived from the GVT by simply introducing a free parameter in the GT space-time equations. This is not a true derivation in the traditional meaning of this term, since there is no physical justification for altering the GT in this way.

In this context, it is interesting to see that the results of the Michelson-Morley experiment depend on the motion of light waves in different directions *at the same time.* There is only a single observer in this case, so there is no possibility to apply the GVT to arrive at the null-interference effect observed. Yet, there is no velocity v to insert in the RVT to obtain this result either. One can resolve the matter by adopting a replacement for Einstein's LSP (which is shown to be invalid based on application of the distance reframing procedure; see the discussion in Section VI) as follows: the speed of light in free space relative to its source is always equal to c. This includes looking upon the mirrors in which the light waves are reflected in the experiment as light sources. The latter postulate makes a clear distinction between sound and light waves.

Finally, it is pointed out that the Law of Causality leads to the conclusion that the relationship between the measurements of the elapsed times (Δt and Δt') for the same event obtained by two observers in different rest frames must differ by the same ratio Q as their respective clock rates; i.e. Δt'=Δt/Q. This equation is referred to Newtonian Simultaneity. It precludes the possibility of remote non-simultaneity (RNS) expected on the basis of the LT. This proportionality equation can be combined with the RVT to obtain the LT replacement given in eqs. (13a-d) and is referred to as the Newton-Voigt Transformation (NVT). It also paves the way for the Uniform Scaling Method which applies to all physical properties and guarantees that the false LT prediction of the FLC is avoided. In its place the NVT predicts that the unit of distance changes in direct proportion to the unit of time upon acceleration of the observer.

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