

# **Description of Virtual Photons and Pion Clouds**

## **in the XBPS Model of Elementary Particles**

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### **Abstract**

The description of muons and pions and other light elementary particles in the Exponentially-damped Breit-Pauli- Schrödinger (XBPS) model is reviewed. An analogy is made between the configuration interaction method of electronic structure and the use of virtual photons in the highly accurate theory of quantum electrodynamics. This leads to a discussion of the pion cloud found to surround protons in high-energy scattering experiments. The possibility that the mass of the bare proton is significantly greater than the proton actually observed experimentally is consistent with calculations carried out with the XBPS method. The model is supported by the failure of attempts to prove that the proton is subject to spontaneous decay. The formation of a pion cloud from massless  $e^+e^-$  and  $\nu\bar{\nu}$  binaries is illustrated. The nomenclature developed in previous work to describe light elementary particles in terms of integral numbers of protons, electrons, neutrinos and their anti-particles is extended to apply to hyperons as well. Finally, the quark-lepton theory of elementary particles and the XBPS model are compared in some detail.

*Keywords: XBPS model of elementary particles, hyperons, pion cloud, proton-antiproton interaction, quarks and leptons*

## I. INTRODUCTION

In previous work [1] it has been shown how calculations based on the Exponentially-damped Breit-Pauli Schrödinger (XBPS) model can be used to give a description of small metastable particles such as muons, pions and Kaons in terms of their decay products. The latter consist entirely of electrons, neutrinos and their anti-particles. The goal of the present work is to extend this analysis to the description of hyperons, which have protons as decay products. One of the first subjects to be dealt with is the role that virtual particles play in the theory of quantum electrodynamics. This raises the question of how such effects can be dealt with in the XBPS model [2-4]. A central aspect in that discussion is the pion cloud that appears to surround protons based on high-energy scattering experiments. The results of XBPS calculations which simulate the interaction of protons with photons allow for a possible explanation for the existence of the pion cloud in terms of the interaction of pions with protons. They also shine light on the interaction of protons with anti-protons, which lead to the production of 1836 times more energy (1.873 GeV) than in the decay of positronium ( $e^+e^-$ ).

In addition the nomenclature developed in Ref. [1] can be extended to the description of the hyperons. Their composition is assumed in each case to consist of exclusively protons, electrons and neutrinos along with their respective anti-particles. The XBPS model describes the reactions of hyperons in terms of strictly balanced equations through appropriate addition of  $e^+e^-$ ,  $\nu\bar{\nu}$  and  $p^+p^-$  mass-less binaries. Finally, a comparison between the XBPS model and the highly popular quark model for the description of elementary particles will also be undertaken.

## II. VIRTUAL PARTICLES IN THE XBPS MODEL

One of the most surprising aspects in the development of theoretical physics since the introduction of the Schrödinger [5] and Dirac [6] equations was the necessity of assuming that there are virtual photons in the neighborhood of atoms and molecules which need to be considered in the form of radiative corrections in computations for “isolated” systems [7-9]. In introducing these effects it is customary to go to some length to argue that real photons are not involved because the assumed entities violate energy and momentum conservation laws. It can be pointed out that such behavior is not inconsistent with the fundamental laws of physics, however, because Heisenberg’s uncertainty principle [10] allows such minor deviations from them.

Because of the indisputable accuracy produced in quantum electrodynamics calculations making use of radiative corrections, the assumption of virtual photons has become a fixture in

theoretical physics. The idea received further impetus from Yukawa [11] in 1936 when he suggested that virtual particles with a mass of roughly  $100 \text{ MeV}/c^2$  might perform a similar function in the transmission of nuclear forces. As pointed out in Ref. [1], this revolutionary hypothesis was ultimately followed by the discovery of the pions in cosmic radiation. Attempts to formulate a theory of nuclear binding along the lines of quantum electrodynamics by substituting pions for photons have generally been accepted to be quantitatively unreliable [12], but this fact has been ascribed to difficulties in treating such interactions at a suitably high computational level. In more recent times the W or intermediate boson was sought and found [13] as the analogue of the photon and pion in the weak interaction, lending further weight to this theoretical development.

It is well to mention that one of the main arguments in favor of the exchange of virtual particles is taken from general relativity theory [14], according to which it is assumed that no interactions at a distance exist. While forces such as gravity can be described quite well in terms of a formula involving the distance between two interacting bodies, it is argued that this circumstance does not prove that the actual mechanism from which the force arises can be explained solely on the basis of the two objects in question. Newton is often criticized for having assumed that the gravitational interaction is instantaneous, but in 1692 he wrote [15] that it was an “absurdity” to think that bodies act upon one another through a “vacuum, without the mediation of anything else.” The long period of searching for an “aether” which transmits the electromagnetic force was in line with this view as well. The concept of virtual photons and virtual pions tries to get around such difficulties by assuming a) there is a vacuum, but b) particles can be created and annihilated anywhere at any time in it, and are thus always available for transmitting the observed forces. Once the creation-annihilation hypothesis is questioned [3,4], however, it is necessary to assume that massless binaries such as  $e^+e^-$ ,  $\nu\bar{\nu}$  and  $p^+p^-$  are always available in high concentration everywhere in the physical universe. Adoption of this alternative formulation thus suggests that the concept of virtual particles might no longer be required, or at least that the distinction between them and physically existent photons may be primarily of a heuristic nature.

To see how the computational techniques which are based on the assumption of virtual particles can be recast in a physical model in which neither a vacuum nor anything but real particles is accepted, it is helpful to consider the configuration interaction technique described in Sect. 3.2 of Ref. [4] in a somewhat different light. Let us take the CI wavefunction for the helium

ground state, for example. To a good approximation it consists of a single configuration in which the  $1s$  orbital is doubly occupied. To obtain better agreement with experiment, however, it is necessary to mix in additional configurations, for example, the  $2p^2$  species. The energy of the latter configuration is much higher than that of  $1s^2$ , and yet quantum mechanics allows the two functions to form a linear combination which gives a *better* description of the helium ground state *than either of them alone*. One is simply employing a different language when this process is referred to as involving a “virtual”  $2p^2$  state. In this way it is possible to incorporate into the present model the idea of virtual entities whose interactions violate conservation laws in a manner consistent with the provisions of the uncertainty principle. In order to go a step beyond this and bring in *virtual particles*, it is only necessary to have configurations containing spin orbitals corresponding to several particle types within the same system.

When a helium atom interacts with virtual photons according to the present model, it is necessary to deal with at least four particles, the two helium electrons and the  $e^+e^-$  complex which has been identified with the photon in Ref. [3-4]. The unperturbed system contains a massless  $0^-$  photon state, while the radiative corrections correspond to a configuration of much higher energy in which the electron and positron are no longer bound together. Such effects are relatively small and it is thus reasonable to expect that their treatment in low-order perturbation theory is adequate to obtain the desired accuracy. The matrix element involved is provided by the methods of quantum electrodynamics [16], thereby circumventing the problem of working with explicit wavefunctions for photons which take account of the internal structure assumed for them in the present model. Even if exact solutions for the various  $e^+e^-$  states can be obtained from the XBPS model, it can still be anticipated that such an explicit CI treatment of the He atom plus photon would be much more difficult to carry through with the required accuracy than in the standard perturbational approach. A similar situation has already been met in Ref. [17] where the possibility of computing line-widths in terms of complex energy eigenvalues [18], in principle the preferred technique, was compared to the simpler approach involving the Fermi golden rule [19]. When very small imaginary parts of the energy eigenvalue are involved, better accuracy can invariably be obtained in the perturbational treatment because the terms omitted in such a procedure are negligibly small. One needs to carry out the computations with much more effort to approach the same level of accuracy when a strictly variational treatment is employed.

In this sense the possible advantages of assuming that real photons are involved in the quantum electrodynamics theory are only to be found on a conceptual level. The computational

techniques of the existing theory are well-known to be capable of great accuracy and, except perhaps in rare instances not yet encountered, are in no need of further improvement. The situation is qualitatively different for nuclear interactions, however. The main reason for this distinction appears to be less a matter of the relative strengths of photon and pion interactions, however, than the fact that the operators used to describe the two types of interactions in a quantum mechanical treatment are known far more accurately for the predominantly Coulomb forces governing atomic and molecular systems. One has an excellent starting point in the Dirac equation [6] or related two-component methods [20] from which to apply the necessary radiative corrections. The main effects are still to be found in the electrons and nuclei of the atoms and molecules of interest in quantum electrodynamics treatments, even though the most interesting results often emerge only after the photon interactions are included.

The main thesis of the XBPS model is that exponentially damped momentum-dependent operators of relatively short range are required to obtain a similarly useful starting point for the calculation of nuclear binding. By attributing a non-zero value to the charge-to-rest-mass ratio of the neutrino [2], it has been possible to describe the neutron and deuteron as collections of protons, electrons and antineutrinos interacting primarily through such short-range interactions, and thereby offsetting the large kinetic energies required for the confinement of light fermions within nuclear dimensions. By analogy to the quantum electrodynamics description of atomic systems, it can then be expected that the inclusion of pions into the theoretical treatment is required to obtain a truly comprehensive representation of the corresponding observed characteristics of nuclear interactions. The difference between the present model and earlier proposals to describe nuclear binding is that it foresees only a relatively minor role for pions, which might well be amenable to a perturbative treatment similarly as for photons in processes dominated by the electromagnetic interaction. The emphasis in the XBPS model is more on the nature of the quantum mechanical representation of the short-range interaction itself in terms of the exponentially damped Breit-Pauli terms such as spin-orbit, orbit-orbit and spin-spin coupling. This position in no way contradicts the concept of certain particles being responsible for the transmission of forces, but it clearly places less emphasis upon the idea than is normally done in quantum field theory.

A more significant difference between the XBPS model and other suggested approaches is clearly that it denies the need for assuming *virtual* particles to describe any aspect of the corresponding experimental phenomena. Instead of a vacuum filled with nothing, it is assumed

that free space as we perceive it is in reality permeated with large numbers of massless binary systems which are always available to participate in interactions with other (massive) particles [3,4]. These  $e^+e^-$ ,  $\nu\bar{\nu}$  and  $p^+p^-$  systems are not observable directly as long as they remain in their respective massless states, but they exist just as surely in this condition as do particles with non-zero relativistic masses. To describe the interaction of an atomic or nuclear system with such entities, it is necessary to expand the number of particles treated explicitly in the configuration interaction calculations, at least in principle. The resulting wavefunctions have contributions from configurations which contain excited states of such binary systems. As a result, their effects become observable in practical experiments, the most well-known, of which are the magnetic moment determinations for electrons and protons and the measurement of the Lamb shift [16]. A small admixture of such excited binary states in the wavefunction means that the measured properties of the overall system are directly affected by them.

Such a CI treatment, by construction, requires a set of configurations which share a common occupation of spin orbitals corresponding to definite numbers of the same type of particles, which is to say, perfect elemental balance is a strict condition for treating a physical interaction in this manner. This does not exclude the introduction of external fields into the governing Hamiltonian, but in a broader sense it leaves open the possibility that any such additional effect ultimately has its origin in the existence of other particles in the neighborhood of the system in question. It is a “particles only” approach, often deemed to be *anathema* to the quantum field theory. There is no need to see a contradiction in these two views of physical theory, however. As usual it is the creation-annihilation hypothesis [2,3], in this case in the form of a vacuum model of free space, which only makes them seem mutually incompatible. If massless particle-antiparticle binaries exist, as the XBPS model’s calculations indicate they do, then they must exist everywhere in great numbers at all times according to the laws of statistical mechanics [3]. This possibility then allows a ready explanation of how action-at-a-distance effects are transmitted throughout space, and especially why it is necessary to assume fields external to the observed atomic and nuclear systems in order to obtain a suitably accurate description of their interactions.

### III. INTERACTIONS OF PROTONS WITH PIONS IN THE XBPS MODEL

The most explicit evidence for the interaction of pions with nuclear matter is gleaned from elastic scattering of high-energy electrons off protons and neutrons [21-23]. Values for the proton

form factors vary with momentum transfer in a manner corresponding to a charge distribution which varies exponentially with the distance away from the proton's center of mass. This result can be satisfactorily interpreted in terms of a model in which the proton is surrounded by a cloud of pions. When an electron approaches a proton at sufficiently high energy, it thus encounters not only the bare nucleon but also the less massive pions. As a result, it appears that the proton itself is not a point charge, unlike the electron and positron, for example. The calculations discussed in Refs. [24-27] combined with the arguments of Ref. [1] regarding the composition of pions in the XBPS model allow for a straightforward interpretation of the elastic scattering data, however, which puts matters in a different light and does not require the assumption of virtual particles in arriving at its conclusions.

As shown in Fig. 1, the XBPS model envisions a bare proton being continuously surrounded by numerous  $e^+e^-$  and  $\nu\bar{\nu}$  massless binaries whose influence must be taken into account in explicit calculations in order to properly represent the physical situation. A minimum number of these binaries are required to explain the observed effects. The lowest-energy configuration is a product of the proton's spin orbital and the two-particle  $0^-$  wavefunctions for one  $e^+e^-$  and two  $\nu\bar{\nu}$  binaries in their respective massless states. The proton is able to approach the lighter particles relatively closely as a result of the form of the exponential damping factor in the XBPS Hamiltonian (see Table I and the discussion in Ref. [24]). As a result, a *higher-energy* configuration is also needed to obtain a satisfactory description of the system. This corresponds to a neutron and a positive pion, which are formed by decomposing an  $e^+e^-$  binary to form a  $p^+e^-$  complex plus a positron. These species interact further with the  $\nu\bar{\nu}$  binaries, with the antineutrino from one of them joining the proton-electron system to form a neutron, while the corresponding neutrino plus an additional  $\nu\bar{\nu}$  pair combine with the positron to form  $\pi^+$  (11.2 composition vector, see Table II of Ref. [1]). This (seven-particle) excited configuration mixes with the lower-energy proton-plus-massless-binary species (and other configurations of intermediate occupation) to form the final eigenfunction. According to the usual statistical interpretation of such wavefunctions, it follows that the system as a whole spends a certain fraction of its time in the excited neutron-plus- $\pi^+$  configuration, which circumstance leads to the deflection of high-energy electrons away from the proton itself. Since  $\pi^+$  has a positive electric charge and a relatively small rest mass, it can be expected that the same configuration-mixing effect tends to raise the combined system's magnetic moment relative to what would be obtained if the  $e^+e^-$  and

$\nu\bar{\nu}$  binaries always remained in their massless states, i.e. a residual bare proton.

The above effect is far greater than in the case of an electron interacting with photons, with the proton magnetic moment being 2.5 times larger than would be expected for a bare (Dirac) proton, compared to an analogous deviation of only 0.05% in the case of the electron. The point to be stressed in the present context, however, is that there is really no necessity to talk about virtual pions in describing this effect once allowance is made for the existence of massless  $e^+e^-$  and  $\nu\bar{\nu}$  binaries in the neighborhood of the proton. Ultimately, it is the large mass of the proton which allows it to interact more strongly with such entities than do lighter particles such as electrons and neutrinos.

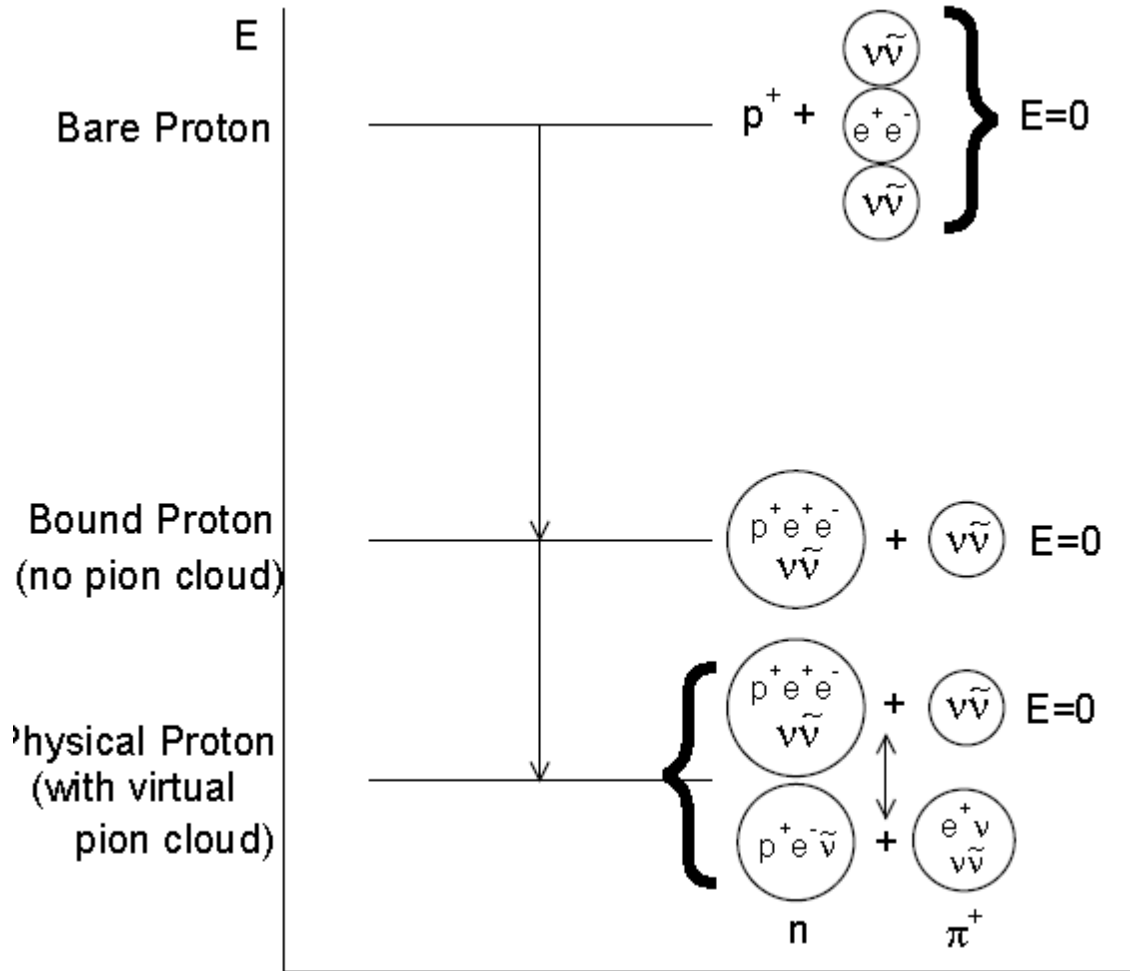


Fig. 1. Schematic energy diagram showing the relative stabilities of the bare proton (upper line), proton-photon-photon complex (middle line) and physical proton with a virtual pion cloud, as foreseen in the XBPS model. Calculations indicate that the rest mass of the (physically unobserved) proton is several electronic mass units (1.0-1.5



MeV/c<sup>2</sup>) larger than the value of 1836  $m_{oe}$  measured for the proton in its naturally occurring environment. The  $n\pi^+$  virtual state is interpreted as a configuration interaction component of the actual (bound) proton wavefunction.

To give further consideration to this point, a series of calculations has been carried out for the tri-atomic  $p^+e^+e^-$  system. In the 4s,2p basis employed for the  $p^+e^-\bar{\nu}$  system in Ref. [25], a full CI energy of -186373.864 hartree results, which represents a proton binding energy of 148783 hartree to the  $e^+e^-$  ground state obtained at the same level of treatment. While this quantity is likely to become significantly smaller as the quality of the basis set is improved, it still indicates that there is a substantial interaction between the proton and such a particle-antiparticle binary. This result is clearly consistent with the arguments given in Ref. [25] regarding the mechanism of proton binding to the  $e^-\bar{\nu}$  complex, especially when one employs a  $q/m_o$  value of 1.0 a.u. for the antineutrino in the  $p^+e^-\bar{\nu}$  calculations (Table 1 of Ref. [25]).

Recognition of this point raises another question, however, namely whether such a large binding energy between the proton and the  $e^+e^-$  binary system is compatible with the theory of nuclear binding which has been discussed in Ref. [2]. A calculation for the  $p^{+2}e^+e^-$  system for the same basis (an optimum scale factor of 0.10 is found in both cases, as compared to the value of 0.16 obtained above) yields a total energy of -305503.616 hartree, which corresponds to a binding energy for the second proton of 119221 hartree, 29562 hartree smaller than for the first. Since, according to the present model, there is always an abundance of massless  $e^+e^-$  species with which to interact, this result indicates that the protons are reluctant to group together around a single binary of this type. This is in sharp contrast to the experience with the  $e^-\bar{\nu}$  complex when computed at the same level of treatment. There the first proton is not bound at all, existing in a  $p^+e^-\bar{\nu}$  resonance state, whereas the second combines with it to form the strongly bound deuteron.

The results of the above calculations nonetheless suggest that the energy of the bare proton would actually be significantly greater if it were not always surrounded by  $e^+e^-$  (and  $\nu\bar{\nu}$ ) systems, which according to the present model corresponds more closely to its physical state actually observed in experiments (Fig. 1). Increasing the rest mass of the proton would have almost no effect on the calculated total energies because they are referenced to the energy of each system's component particles separated to infinity. On this basis one can expect that the mass of the bare proton might be greater than that measured in the laboratory by as much as 2-3 electronic mass units (1.0-1.5 MeV/c<sup>2</sup>). Since the polarization effects which cause this stabilization are charge-dependent, it can be expected that their influence on the neutron's rest mass *vis-a-vis* that in its

isolated (bare) state is much smaller. Such a development would also be at least qualitatively consistent with the premises of the isospin theory [28-30], in which the proton and neutron are assumed to be of equal mass in the absence of the electromagnetic interaction. Recognition of these effects would tend to indicate that the  $|q/m_0|$  value which must be assumed for the neutrinos in order to compute a neutron ( $p^+e^-\bar{\nu}$ ) rest mass in agreement with the measured value might need to be increased slightly if such environmental influences were properly taken into account in the theoretical treatment. It also can be noted that the above calculations are consistent with the well-known observation that the interactions of protons and other hadrons with photons are not correctly described by quantum electrodynamics [31].

A more crucial question arises from this general discussion, however, namely do the electron scattering data for nucleons *imply that the proton is not a point charge*? The answer is not at all clear once it is realized that the extension in the charge distribution observed in the electron elastic scattering from protons may be tied up in large part with the fact that *the proton is not an isolated system under the governing experimental conditions*. It could be that the exponential nature of the charge distribution derives from the combination of *both* the proton and the  $e^+e^-$  and  $\nu\bar{\nu}$  species to which it is tightly bound according to the present calculations (the pion cloud also results from this interaction in this view; see Fig. 1). To investigate this point further it is clearly necessary to consider the effects of probes in the scattering experiments which have significantly smaller de Broglie radii than the electrons employed in the above study [21,23,32].

When inelastic scattering processes are investigated with incident electron energies in the 10 GeV range [33], the nature of the cross sections is different than above and continuum states are observed whose charge distributions are no longer exponentially decreasing with the distance away from the proton's center. In essence the results indicate that point charge scatterers are now involved. There have been essentially two interpretations of this phenomenon, identifying the point particles with some internal structure of the proton [34,35] or with the bare nucleon itself [36]. In the present model it seems tempting to focus on the idea that as the energy of the electrons is taken up by the proton they come into excited states which are no longer able to bind pions (or alternatively the  $e^+e^-$  and  $\nu\bar{\nu}$  binary systems) even a small percentage of the time. A 10 GeV electron energy corresponds to a proton kinetic energy of 2-3.5 GeV in its own inertial system, which is comparable to what is computed for the  $p^+p^-$  binary in its massless state (Sect. 3.6 of Ref. [4]). Under these conditions the interparticle distances are so small that even the advantage that the proton normally enjoys by virtue of the form of the exponential damping

factors in the XBPS Hamiltonian in the present model is unable to offset the enormous centrifugal (kinetic energy) effects impeding against binding. The proton is therefore no longer attracted by neighboring  $e^+e^-$  and  $\nu\bar{\nu}$  species and behaves as an isolated system. The fact that the curvature in the measured cross sections for inelastic electron scattering steadily decreases with the energy of the continuum states is at least consistent with such an interpretation. We shall have occasion to return to this question later in this study when the subject of the quark model of elementary particles is taken up in Sect. VII).

#### IV. PROTON-ANTIPROTON INTERACTION

The reaction of a proton with an antiproton can be described in a perfectly analogous manner to that of an electron and positron in the XBPS model by virtue of the scaling properties of the Hamiltonian employed (Sect. 2.4 of Ref. [4]). According to this treatment, the energy released is  $m_{op}/m_{oe}$  times greater in the  $p^+p^-$  interaction than in the  $e^+e^-$  case, and its mean interparticle distance is smaller by the same ratio. It might be expected that when  $p^+$  and  $p^-$  bind together, the result is a pair of photons with correspondingly greater energy than for  $e^+e^-$ , but the most commonly occurring process observed experimentally involves instead the production of a number of pions. It is therefore important to consider this process in somewhat more detail in order to find a possible explanation for this phenomenon.

Because the  $p^+p^-$  massless binary system is much less likely to interact with its surroundings by virtue of the relatively small radius, it seems reasonable to assume that the energy given off as a result of its formation is borne by other particles in the neighborhood. Initially the proton and antiproton can be taken as being at rest in their center of mass, and it has been argued in Sect. 2.1 of Ref. [3] that the product of their interaction must also remain localized in the same region of space. In view of the zero rest mass of the  $p^+p^-$  binary, this situation cannot hold if it takes up any energy itself, in which case it must move with the speed of light out of the area. The same argument has been used to explain the quantized nature of photon interactions, particularly the well-known fact that they tend to give up all rather than only part of their energy when absorbed by atomic or molecular systems. The enormous amount of energy released in the proton-antiproton interaction must be taken up by something, however, and because of the conservation of momentum requirement, the most likely recipients in the present model are two photons, initially also possessing zero energy and therefore likely at rest in the neighborhood of the reaction.

In this respect the  $e^+e^-$  and  $p^+p^-$  particle-antiparticle reactions initially proceed in a very similar manner, but the amount of energy released in the latter process is sufficient to cause neighboring  $e^+e^-$  species to decompose into their elements, unlike the situation when a positron and electron react with one another. As shown in Fig. 2, a relatively uncomplicated scenario for the observed pion production can be imagined once two pairs of electrons and positrons are set free as a result of the proton-antiproton interaction. The conservation laws essentially require that the two  $e^+e^-$  species push off one another (see Fig. 4 of Ref. [3]), so that initially one  $e^+$  and  $e^-$  remain near the original center of mass, whereas the other two particles go off in opposite directions at high speed. The next step then involves the attachment of  $\nu\bar{\nu}$  binaries to each of the fragment systems: one to  $e^-$  to form a transient  $\mu^-$  (see Table 2 of Ref. [1]), one to  $e^+$  to form  $\mu^+$ , and finally one to the remaining  $e^+e^-$  pair to form a  $\pi^0$  species. At such high energies the muons offer an attractive target to another  $\nu\bar{\nu}$  binary, thereby producing the respective charged pions  $\pi^+$  ( $\nu$ ) and  $\pi^-$  ( $\bar{\nu}$ ). Alternatively, only two  $\nu\bar{\nu}$  species might be involved, with the unused  $\nu$  and  $\bar{\nu}$  particles taking up some of the released energy as well.

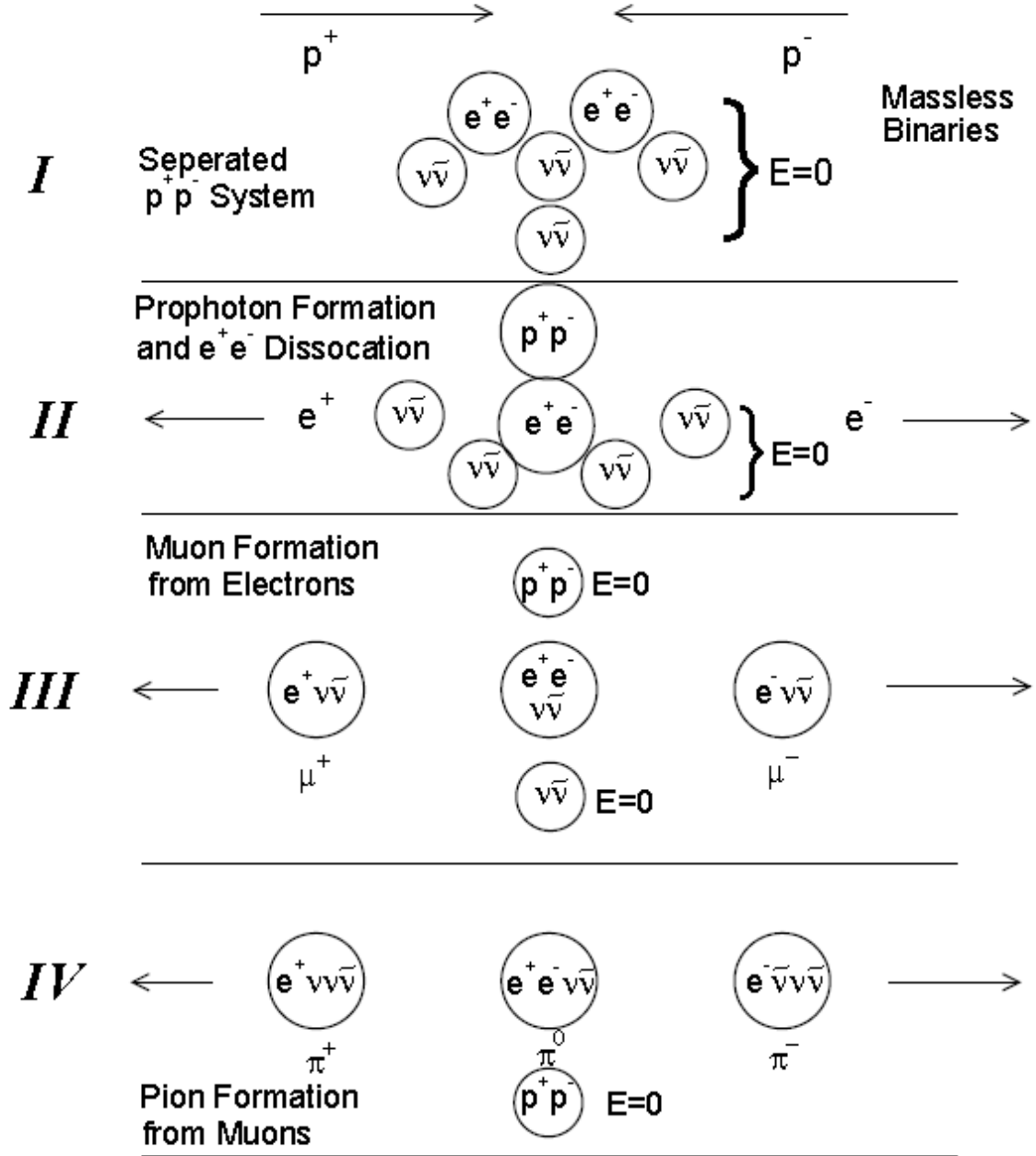


Fig. 2. Schematic diagram showing four stages of the proton-antiproton fusion process producing a system of three pions and the massless prophoton ( $p^+p^-$ ) system. In the first stage the proton and antiproton are shown to collide with one another in the presence of massless  $e^+e^-$  and  $\nu\bar{\nu}$  systems (I). Their fusion leads to the production of 1.88 GeV of energy which is taken up initially by two massless  $e^+e^-$  species, leading to their dissociation in the second stage (II). The latter particles further combine with  $\nu\bar{\nu}$  species to form the  $e^+\nu\bar{\nu}$  ( $\mu^+$ ),  $e^+e^-\nu\bar{\nu}$  ( $\pi^0$ ) and  $e^-\nu\bar{\nu}$  ( $\mu^-$ ) intermediate products in the third step (III), while in the final stage the muons react further to form the corresponding charged pion systems (IV). The prophoton product is assumed to remain in the translationless state upon its formation and thus escapes experimental detection.

In accord with the notation used in Table 2 of Ref. [1], the reacting system involved is not 100.100, i.e.  $p^+p^-$ , but rather 124.124, which yields as products a  $\pi^+$  (11.2), a  $\pi^-$  (2.11), a  $\pi^0$  (11.11) and the 100.100  $p^+p^-$  massless binary. The mechanism described above leads to the production of only three pions, but it is obvious how alternative processes might occur which would produce still larger numbers of such mesons. Given the amount of energy produced by the  $p^+p^-$  interaction and the relatively small barriers to decomposition of the massless  $e^+e^-$  and  $\nu\bar{\nu}$  binary systems, it is not surprising that a variety of such multiple-pion products is observed.

## V. COMPOSITION OF HYPERONS IN THE XBPS MODEL

The next elementary particles, in order of increasing rest mass, are the various meta-stable baryons, or hyperons, as they are also frequently called. The suggested elemental composition of these particles is given in Table 1, along with a survey of their respective decay processes [37,38]. There is a well-known baryon conservation law governing reactions of such systems, and this is obviously consistent with the main thesis of the present work, namely that all truly elementary (stable) particles are in continuous existence, regardless of the nature of the interactions to which they are subjected. These elements are the proton, electron and neutrino and their antiparticles, and since each of the hyperons decays into a collection of such particles of which one is always the proton (or antiproton), it is clear that each of them must contain such a heavy element if the creation-annihilation hypothesis is to be avoided. The simplest meta-stable baryon is the neutron in this view, with a 1.110 composition (Table 1), whereas the proton is one of the elements (100).

The next lightest hyperon is the  $\Lambda$  particle, which decays into the major sets of products  $p\pi^-$  and  $n\pi^0$ . The latter differ by a single  $e^+e^-$  binary according to the assignments of Table 1. The decay energy is of the order of the pion value, amounting to 177 MeV. The most likely composition (2.111) would thus seem to be one proton plus two antineutrinos, one electron and one neutrino, i.e. the components of  $p^+ + \pi^-$ . Alternatively, one might take it to be an excited state of the neutron itself which upon decay attracts a  $\nu\bar{\nu}$  binary to its emitted  $e^-$  and  $\bar{\nu}$  particles to form a negative pion. In view of the observation that a proton is surrounded by a pion cloud even when it is in its ground state, the former assignment is slightly preferred and given explicitly in Table 1. With this choice only a single  $e^+e^-$  massless binary must be added to  $\Lambda$  to produce its  $n\pi^0$  decay products. The other groups of fragments listed occur far less frequently and are easily

correlated with the above structure for  $\Lambda$  when allowance is made for the usual addition or loss of particle-antiparticle binary systems. The un-symmetric nature of the  $\Lambda$  composition vector indicates that a distinct antiparticle exists with charge-conjugated composition, and this is observed as well.

Table 1. Classification of baryons by means of composition vectors. For notation see Table 2 of Ref. [1].

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
proton	P	938.2592±0.0052	100	stable	-	-
neutron	n	939.5527±0.0052 ( $\tau=918s$ )	1.110	$p^+e^-\nu$	1.0000	0
lambda	$\Lambda$	1115.59±0.05 ( $\tau=2.521 \times 10^{-10}s$ )	2.111	$p^+\pi^-$ $n\pi^0$ $p^+e^-\bar{\nu}$ $p^+\mu^-\bar{\nu}$ $p^+\pi^-\gamma$	0.642 0.358 $8.13 \times 10^{-4}$ $1.57 \times 10^{-4}$ $8.5 \times 10^{-4}$	0 10 $\bar{1}$ 0 10
sigma plus	$\Sigma^+$	1189.41±0.07 ( $\tau=8.00 \times 10^{-11}s$ )	11.111	$p^+\pi^0$ $n\pi^+$ $p^+\gamma$ $n^+\pi^+\gamma$ $\Lambda e^+\nu$ $n\mu^+\nu$ $ne^+\nu$ $p^+e^+e^-$	0.516 0.484 $1.24 \times 10^{-3}$ $1.31 \times 10^{-4}$ $2.02 \times 10^{-5}$ $< 2.4 \times 10^{-5}$ $< 1.0 \times 10^{-5}$ $< 7.0 \times 10^{-6}$	0 1 $\bar{1}$ 11 1 1 0 $\bar{1}$
neutral sigma	$\Sigma^0$	1192.48±0.10 ( $\tau<-1.0 \times 10^{-14}s$ )	2.111	$\Lambda\gamma$ $\Lambda e^+e^-$	1.000 $5.45 \times 10^{-5}$	10 10
sigma minus	$\Sigma^-$	1197.34±0.07 ( $\tau = 1.484 \times 10^{-10}s$ )	2.120	$n\pi^-$ $ne^-\bar{\nu}$	1.000 $1.10 \times 10^{-3}$	1 0

neutral xi	$\Xi^0$	1314.9±0.6 ( $\tau=2.98 \times 10^{-10}\text{s}$ )	12.121	$n\mu^-\bar{\nu}$	$4.5 \times 10^{-4}$	1
				$\Lambda e^-\bar{\nu}$	$6.0 \times 10^{-5}$	1
				$n\pi^-\gamma$	$1.0 \times 10^{-4}$	11
				$\Lambda\pi^0$	1.000	1
				$p^+\pi^-$	$<0.9 \times 10^{-3}$	$\bar{1}0$
				$p^+e^-\bar{\nu}$	$<1.3 \times 10^{-3}$	$\bar{1}\bar{1}$
				$\Sigma^+e^-\bar{\nu}$	$<1.5 \times 10^{-3}$	0
				$\Sigma^-e^+\nu$	$<1.5 \times 10^{-3}$	0
				$\Sigma^+\mu^-\bar{\nu}$	$<1.5 \times 10^{-3}$	1
				$\Sigma^-\mu^+\nu$	$<1.5 \times 10^{-3}$	1
				$p^+\mu^-\bar{\nu}$	$<1.3 \times 10^{-3}$	$\bar{1}0$

TABLE 1 continued

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
xi minus	$\Xi^-$	1321.29±0.14 ( $\tau = 1.672 \times 10^{-10}\text{s}$ )	3.121	$\Lambda\pi^-$	1.000	1
				$\Lambda e^-\bar{\nu}$	$7.0 \times 10^{-4}$	0
				$\Sigma^0 e^-\bar{\nu}$	$<5.0 \times 10^{-4}$	0
				$\Lambda\mu^-\bar{\nu}$	$<1.3 \times 10^{-3}$	1
				$\Sigma^0\mu^-\bar{\nu}$	$<5.0 \times 10^{-3}$	1
				$n\pi^-$	$<1.1 \times 10^{-3}$	0
				$ne^-\bar{\nu}$	$<0.01$	$\bar{1}$
omega minus	$\Omega^-$	1672.5±0.5 ( $\tau = 1.3 \times 10^{-10}\text{s}$ )	13.131	$\Xi^0\pi^-$	-	1
				$\Xi^0\pi^-$	-	1
				-	-	2

The next three hyperons have nearly the same rest masses and are grouped together as  $\Sigma^+$ ,  $\Sigma^0$  and  $\Sigma^-$ , in order of increasing energy. Again two major decay modes are noted for  $\Sigma^+$ , namely  $p\pi^0$  and  $n\pi^+$ . With the usual *caveat* an assignment of 11.111 for its composition is made in Table 1,



which is identical to that of its  $p\pi^0$  products. A single  $v\bar{v}$  binary must then be added to obtain the  $n\pi^+$  fragments, observed in 48.4% of  $\Sigma^+$  decays. Otherwise there are six minor decay modes such as  $p\gamma$  and  $n\pi^+\gamma$ , which are easily understandable in the usual way. The composition of  $\Sigma^+$  implies that an antiparticle also exists, as observed.

The neutral sigma particle  $\Sigma^0$  also has an antiparticle. Its decay products consist almost exclusively of a  $\Lambda$  species plus a photon or a separated  $e^+e^-$  pair. This suggests something akin to a conventional radiative emission is involved in the decay, so the  $\Sigma^0$  structure is probably best assumed to be the same as that of the  $\Lambda$  particle. The assumption is that the 77 MeV higher rest energy of  $\Sigma^0$  relative to  $\Lambda$  is simply a consequence of the mass-energy equivalence relation for two different states of the same system.

The heaviest of the sigma particles is  $\Sigma^-$ , again with its own antiparticle. It also has only a single major decay mode, producing  $n\pi^-$ . As such it is probably best to assume that the original system contains two electrons, thereby clearly distinguishing its composition from that of  $\Sigma^+$ . Since the other two  $\Sigma$  species are assigned a composition of five particles, it is reasonable to delete a  $v\bar{v}$  pair from the  $n\pi^-$  fragments in arriving at the  $\Sigma^-$  structure, so that a like number of constituents is assumed for all members of this hyperon family. A restructuring of the bonding between the  $\Sigma^-$  and  $v\bar{v}$  species can then lead to  $\Lambda e^-\bar{v}$  products (Table 1). As usual the possibility must be left open that each of these assignments may need to be altered by the addition or subtraction of  $e^+e^-$  and/or  $v\bar{v}$  binary systems.

The next two hyperons in order of increasing rest mass comprise the  $\Xi$  family,  $\Xi^0$  and  $\Xi^-$ . Both particles have essentially unique decay products,  $\Lambda\pi^0$  and  $\Lambda\pi^-$  respectively. Their difference in rest mass compared to the  $\Sigma$  family members is about 130 MeV/ $c^2$ , with  $\Xi^0$  being more stable than  $\Xi^-$  by 6.4 MeV. This suggests a composition of several more particles than for the  $\Sigma$  species. Since  $\Xi^0$  is more stable it seems reasonable to assume that it benefits more from the Coulomb interaction. Taking the number of constituent particles to be seven in each case leads to assignments of 12.121 for  $\Xi^0$  and 3.121 for  $\Xi^-$ . Whether the combination of one proton, two electrons and a positron ( $\Xi^0$ ) reaps more benefit from the Coulomb interaction than just two electrons and a proton remains somewhat of an open question in this assignment, but at least the assumption that one can estimate such effects by algebraically adding the products of each pair of

charges does speak in favor of it. As usual each of the minor decay products of both particles can be obtained with the help of integral numbers of particle-antiparticle binaries (Table 1) .

The last particle classified as a hyperon is the  $\Omega^-$ , with a rest mass of  $1672.5 \text{ MeV}/c^2$ , some  $340 \text{ MeV}/c^2$  greater than for the  $\Xi$  particles. It is also a fermion, but it is found to have a quartet state, in contrast to all the other hyperons. The compositions of two of its known decay products,  $\Xi^0\pi^-$  and  $\Xi^-\pi^0$ , is the same according to Table 2 of Ref. [1] and the present Table 1, namely 14.132. The other possibility ( $\Lambda K^-$ ) has an additional  $\nu\bar{\nu}$  pair. The assignment given in Table 1 of 13.131 assumes that  $\Omega^-$  has two more constituent elements than the  $\Xi$  family members and that  $\nu\bar{\nu}$  massless binaries are always involved in its decays.

Before closing this discussion, however, it should be noted that there are a large number of other resonances found as a result of scattering pions off protons. They are often referred to as excited nucleon states. Their decay products are generally of the same order. The main experimental distinction between these two groups of systems lies in the magnitude of their lifetimes or linewidths. The  $N^*$  and  $\Delta$  resonances all have widths of ca.  $100 \text{ MeV}$ , corresponding to lifetimes which are shorter by a factor of  $10^{12}$ - $10^{13}$  compared to those of the hyperons. Especially in the present context, it seems pertinent to mention that a distinction on this basis is somewhat artificial. Designating something an elementary particle because it only decays with a lifetime of  $10^{-10} \text{ s}$ , while refraining from the same terminology for less stable systems yielding a very similar product spectrum, is in itself an indication that the whole classification system is somewhat inconsistent. The present view is that hyperons and the above broad resonances are all meta-stable combinations of protons and lighter stable fermions. They are distinguished from one another mainly on the basis of the number and type of such particles they contain, as well as the manner in which they are bound together by short-range interactions.

## VI. REACTIONS OF ELEMENTARY PARTICLES

Throughout the present study we have been discussing reactions of particles generated in high-energy collisions, but the emphasis has been placed on one-particle decays. In this section we will briefly sample other types of reactions occurring for such systems as a further illustration of the utility of the assignments given in Table 2 of Ref. [1] and Table 1. We can start by considering the collision of two protons:  $p^+ + p^+ \rightarrow d^+ + e^+$ . As usual the equation as conventionally written is not balanced. On the left-hand side the composition vector is 200, i.e.

two protons, while on the right it is 11.210. Balance is obtained by adding an  $e^+e^-$  and a  $\nu\bar{\nu}$  binary to the initial system and by taking account of the fact that positron emission in such a process is inevitably accompanied by the release of a like number of antineutrinos. The result is 11.211 on both sides. Nothing is created or destroyed in this view. It only appears that nothing beside the two protons is present at the start of the reaction. Instead there are always an infinite number of massless particle-antiparticle binaries available to participate in the reaction in a non-trivial way provided sufficient energy is available to cause their decomposition or attachment *en masse* to a neighboring system.

It only makes sense to include the binary systems explicitly in the reaction's equation when they have a different relationship to their surroundings before and after the process has occurred. The situation is thus wholly similar to that encountered when a chemical reaction takes place in a water solvent. We could write down  $10^{23}$  water molecules on both sides of a typical equation, but this generally serves no useful purpose and clearly conceals the essential submicroscopic nature of the observed process. It occasionally happens, however, that a true picture of such a reaction only emerges after careful study, with the result that it is realized that one or more of the water molecules does play an essential role in the process, remaining for example as part of the product molecules generated by it.

The massless binaries perform both the functions of reaction partner and catalyst in elementary particle reactions, depending on the specific case. One major difference between water and massless binaries, however, is that the latter cannot be observed in their state of lowest energy, which has been found in the present work to have  $0^-$  symmetry. This characteristic makes a unique assignment of elemental composition to each participant in a given reaction effectively impossible. That need not be an important deficiency in the utility of the theoretical model, however, because the main interest is in the identity and probability of a certain reaction's occurrence. This aspect is not unduly affected by the miscounting of one or another of the binary systems, as long as the same error is made on both sides of the equation. These are the rules of the game which should be kept in mind in considering other reactions below.

Let us now consider a more complicated process in which a negative kaon collides with a high-energy proton. According to Table 2 of Ref. [1] the elemental composition of the initial particles is 13.122. The products of this reaction are  $K^0$ ,  $K^+$  and  $\Omega^-$ , respectively 13.13, 22.13 and 13.131, which gives a total of 48.157. The difference between the two totals is 35.35, *i.e.* three  $e^+e^-$  and five  $\nu\bar{\nu}$  species, which must be added to the reactants to ensure elemental balance.

These are distributed as follows: one  $e^+e^-$  and three  $\nu\bar{\nu}$ s are needed to form  $K^0$ , an electron is added to  $p^+ + K^-$  and a neutrino is lost to give  $\Omega^-$ , while the remaining particles go into the formation of the positive kaon (including the positron counterpart of the  $\Omega^-$  electron and the neutrino lost from  $K^-$ ). As usual the assignment is not unique. One would particularly like to know if the  $K^-$  elements are used exclusively in the formation of  $\Omega^-$  or if they are distributed among two or more of the products. Only accurate calculations can presumably help to remove such uncertainties. The reaction is highly endothermic, which is consistent with the relatively large number of particle-antiparticle systems which become included in the process.

As another example let us consider the  $p + \pi^-$  reaction which gives as products  $n\pi^+\pi^-$ . From the point of view of merely balancing the equation it is necessary to compare a proton with a neutron-plus- $\pi^+$  combination. The difference is again an integral number of binaries (12), which must be supplied on the reactant side of the corresponding equation. One can imagine the process starting with the decomposition of the  $e^+e^-$  binary. The electron combines with the proton and a  $\bar{\nu}$  to form a neutron, while the  $e^+$  and  $\nu$  counterparts result in the positive pion with the addition of the second  $\nu\bar{\nu}$  species. In actuality it might well be that the initial  $\pi^-$  species is decomposed in the course of the reaction and simply reformed, perhaps with different constituent particles, at the conclusion of the process.

Another set of products observed in  $p\pi^-$  collisions is  $\Lambda K^0$ . We begin with a 2.111 composition and end with  $2.111 + 12.12 = 14.123$ . The  $K^0$  species is its own antiparticle and thus can be formed directly from only binary systems, provided the minimum of 496.7 MeV kinetic energy is provided at the center of mass in the original collision. The  $\nu_e {}^{37}\text{Cl}$  to  $e^- {}^{37}\text{Ar}$  process can be similarly described by suppressing the common  ${}^{36}\text{Cl}$  nucleus on both sides, which changes it to the typical  $\beta$ -decay reaction  $\nu_e n \rightarrow e^- p^+$ . This equation is balanced by adding a massless  $\nu\bar{\nu}$  species to the products which is assumed to go undetected.

The  $\nu n$  reaction, which is crucial in the arguments supporting a second type of neutrino, as discussed in Sect. III of Ref. [1], is already balanced for  $\mu^- p^+$  formation. Only a suitable amount of kinetic energy is additionally required to make the latter process possible. In its absence a massless  $\nu\bar{\nu}$  species can be formed, giving the appearance that only an electron and proton are emitted in the process. This is the only process which can occur unless the sum of the kinetic energy of the  $\nu + n$  system in its center of mass and the decay energy of the neutron exceeds the 105 MeV rest energy difference between the muon and the electron.

As an example of a more complicated reaction, let us consider the result of a deuteron colliding with a  $K^+$  particle. One of the more unusual possibilities is the formation [39] of the products  $\bar{\Omega}^+ \Lambda p^+ \pi^+ \pi^-$ . One starts with  $d^+$  (1.210) and  $K^+$  (22.13), or a total composition vector of 23.223. The  $\Omega^-$  assignment in Table 1 is 13.131, which implies 131.113 for  $p^+ \bar{\Omega}^+$ . Adding a  $p^+ p^-$  binary to the reactant side gives 123.323, which leaves a deficiency of a proton and anti-proton on the product side. These are covered by the  $\Lambda$  particles of 2.111 composition (Table 1). Adding these as products of the reaction gives a result of 135.335, which provides a potential elemental balance through the addition of one  $p^+ p^-$ , one  $e^+ e^-$  and two  $\nu \bar{\nu}$  species as reactants. The  $\pi^+ \pi^-$  pair is unchanged by charge conjugation and thus can also be explained in terms of integral numbers of particle-antiparticle binaries (one  $e^+ e^-$  and three  $\nu \bar{\nu}$ ) becoming involved as products of this reaction.

In summary, the present analysis in terms of composition vectors of protons, electrons, neutrinos and their respective antiparticles at least provides a handy bookkeeping device for checking if a series of reaction products is feasible. In the model the occurrence of the given reaction depends on a number of factors, especially whether sufficient energy is available. The actual reaction probabilities can then be computed in principle, and in this way give a quantitative description of the various processes without the need of further assumptions. Traditionally one has tended to rationalize whether such reactions are allowed or forbidden by defining a series of quantities such as isospin [28-30], hypercharge [40,41] and muon quantum number [42] which have no counterpart in atomic and molecular calculations. In the XBPS model it can at least be imagined that these quantities need not be considered explicitly in calculations which are capable of providing a suitably quantitative description of such processes.

## VII. THE QUARK MODEL AND ITS RELATION TO THE XBPS TREATMENT

The prevailing theory of elementary particles is deeply bound up with concepts of symmetry, involving a number of quantities specifically introduced to aid in unraveling the tangled web that the experimental data for such systems has produced. As mentioned in Sect. 5 of Ref. [2], the basic assumption in many theories which have been put forward is that all matter is ultimately formed from a series of building blocks, but the models differ in their specific choices for these quantities. Between 1949 and 1956, Fermi, Yang [43] and Sakata [44] developed one of the first such variations on this theme. They suggested a set of elements consisting of the proton,

neutron and lambda particles, thereby setting a precedent for allowing unstable particles to serve as elements. One of the most fundamental concepts employed in this approach was the provision that a boson can be constructed from an even number of such fermionic elements. Some of the new particles which could be expected from this model have never been found in experiment, however, which fact has generally brought it into disuse.

The suggestion of quarks as building blocks brought with it the new assumption, at least relative to the Fermi- Yang-Sakata model, that the building blocks need not be found among the list of observed particles. In 1964 when the latter model was first introduced by Gell-Mann [45] and Zweig [46], only three quarks and three antiquarks were needed to explain the structure of all known particles. In the meantime this number has been expanded to six each [47], and the resulting improved theory has won great respect among physicists for the detail with which it is able to describe observed relationships among this otherwise heterogeneous group of systems. Symmetry is the watchword in this theory and the more it has been refined and tested, the more it has come to be accepted as reality.

The present calculations are based on a model which does not require the existence of quarks or any other particles as elements except the proton, electron, neutrino and their antiparticles. It also does not make use of quantum numbers other than those known in the field of atomic physics at the time of the introduction of the Schrödinger [5] and Dirac [6] equations. On this basis it can be argued that the present model is relatively free of unproven assumptions, which is after all one of the most important criteria to be satisfied by any physical theory. Emphasis is placed to a large extent on the identification of a Hamiltonian which when employed in standard quantum mechanical procedures leads to a suitably accurate description of observed phenomena, particularly for such quantities as the rest masses and lifetimes of elementary particles.

As mentioned earlier, the fact that the XBPS model might ultimately be improved to the point of delivering a high degree of quantitative reliability does not necessarily clash with the precepts of the quark theory. It is conceivable that the elements assumed in the present model, particularly the proton, are composed of still less massive building blocks such as quarks or even more fundamental particles. Even if experiments continue to be *unable to provide definitive, positive, direct evidence* that quarks exist, such as isolating and identifying a particle with a charge of  $\pm 1/3 e$  or  $\pm 2/3 e$ , it can still be true that such entities nevertheless exist with essentially the same properties as are required for them in the theory. Similarly the fact that one or the other

of its key predictions cannot be verified, such as for example that the proton decay with an extremely long but nonetheless finite lifetime [48,49], in no way constitutes a contradiction of the model as a whole. In short, the possibility that two fundamentally different theories might explain the same set of observations inevitably leaves one with an uneasy feeling, but does not in itself amount to indisputable evidence that at least one of them is seriously flawed.

Yet one knows from experience with simpler problems in the field of mathematics that multiple solutions of different characteristics are more the rule than the exception. The goal of finding a set of elements or building blocks from which to synthesize all known material particles is similar to the familiar exercise in the theory of finite-dimensional linear spaces of determining a set of basis functions to span a given linear manifold. It is impossible to find a suitable choice of functions for this purpose whose number is less than the dimension of the corresponding space, but there is no difficulty in finding a solution *involving more than this minimal number*. It is easier to construct a polynomial fit to a given set of experimental results which satisfies a given least-squares criterion when the ratio between the number of terms in the expansion and that of available data points is relatively large. In general, one is inclined to favor solutions which rely on a minimal number of elements (basis functions or free parameters) to accomplish the desired purpose.

The quark model, in its most modern form holds that there are a total of twelve fermions of non-integral electronic charge plus a number of leptons and their antiparticles which nature uses as building blocks. The XBPS model by contrast hypothesizes that only six elements are needed, the proton, electron, neutrino and their respective antiparticles. It also uses only two intrinsic properties of these particles in its formulation, namely their electronic charge and rest mass (or the ratio of these quantities in the case of the massless neutrinos). *Other quantities such as angular momentum quantum numbers and parities arise naturally out of the solutions of differential equations*. No direct use is made of quantities such as isospin, muon quantum number and hypercharge. It is proposed instead that quasi-degenerate groups of particles such as the pions arise naturally from the theory when one solves a Schrödinger equation in which the true elemental compositions of these systems appear explicitly. It is clear then that the present model at least attempts to represent the structure of elementary particles with a far less elaborate system of hypotheses than does the quark theory. It is equally clear that the success of the XBPS model ultimately lies in its ability to obtain reliable approximate solutions to the Schrödinger equations

arising from it, without greatly increasing the number of free parameters and/or *ad hoc* assumptions needed to attain this objective.

If the latter goal can be achieved in a systematic manner, it will constitute strong evidence supporting the major assumptions which have been made in previous work [2,3], such as that of the ubiquitous presence of massless particle-antiparticle binary systems or the non-zero charge-to-rest-mass ratios of the neutrinos. In this sense there is a clear parallel between the present model and that of the quark theory. Both contain assumptions which of their very nature cannot be contradicted by experiment. In the latter case there is *no way to prove that quarks do not exist or that protons do not have a finite lifetime*. On the other hand, as already mentioned, these assumptions *might be verifiable*, at least in principle. It is not difficult to imagine experimental investigations which could produce such results, but if quarks really do not exist the wasted effort could be enormous. Similarly, the basic assumptions underlying the XBPS model appear to be impossible to refute by experimental means. How does one *prove that two particles pass out of existence*, for example, or how can one be *certain that a neutrino does not have a given charge-to-rest-mass ratio* when theoretical calculations indicate that conventional magnetic fields would be incapable of deflecting it regardless of what value it might have [2,24] ? The likelihood is thus that none of the above assumptions in either theoretical model will ever be definitively proven to be either true or false. Instead the theories must be judged on their respective abilities to make detailed predictions regarding other types of experiments, especially of those which will first become possible in the coming years.

Before concluding this discussion it is well to consider several further points, one of an experimental and two others of a theoretical nature. The deep inelastic scattering experiments [33] mentioned in Sect. III are often taken as an indication that the proton does have internal components which are point charges, just as the corresponding elastic form factors [21-23] are used to justify the position that the proton has a finite radius. If the assumptions of the XBPS model are correct, a quite different interpretation can be given to the same results. Accordingly, the proton, just as the electron (and neutrino), could be regarded as a point entity. The elastic scattering data [21-23] could simply be a consequence of the admixture of neutron-pion character (or binding with  $e^+e^-$  and  $\nu\bar{\nu}$  binary systems; see Fig. 1) in the wavefunction of the proton in its normal environment, whereas the higher-energy data [33] for the corresponding continuum might be a reflection of the free proton's isolated state. In the XBPS model, the charge/probability distributions of all these particles have finite dimensions, but the terms in the corresponding



Hamiltonian are constructed entirely under the assumption that each of them is a point particle. From this point of view an experimental proof that the proton has a finite lifetime would be inconsistent with the present model because it would be incongruous to hold that a point-charge system could be meta-stable.

On the theoretical side it is interesting to compare the way isospin degeneracy is defined in the description of elementary particles as opposed to its original usage in the field of nuclear physics [28-30]. The classic example common to both fields is the nucleon doublet consisting of the proton and neutron, i.e. with  $I = \frac{1}{2}$ . Each of these particles has its own antiparticle, which together form a *different* isospin doublet. The pi mesons on the other hand are said to form a single isospin triplet, consisting of the  $\pi^+$ ,  $\pi^0$  and  $\pi^-$  particles, even though the two charged members bear the same particle-antiparticle relationship as the proton and antiproton in the first example. Similarly the  $K^+$  and  $K^-$  particles are paired in the same isospin doublet. According to this prescription the nucleons actually form an isospin quartet, but as already mentioned the standard theory assigns them to two different doublets instead. In so doing, however, one in effect asserts that there is an “accidental” degeneracy between the particles and antiparticles of the respective nucleon doublets. The question remains, however, why the particle-antiparticle relationship of the charged pions should be described as a true degeneracy in the theory while that of the proton and antiproton is given a less fundamental characterization.

The history of quantum mechanics has led to the belief that accidental degeneracies are either not real, as turned out to be the case in the  $s_{1/2} - p_{1/2}$  example for hydrogenic atoms [49], or that some “hidden” symmetry can be found in the Hamiltonian which leads to the realization that a higher-order group is actually involved [50]. In the case of the proton-antiproton pair it can be claimed that the former attitude is probably justified because charge conjugation is not thought to be a true symmetry operation [51], but this would imply that respective particles and antiparticles actually do not have the same total energy (rest mass). There is no evidence to support such a view, although one can always claim that this situation may one day change. Even if this eventuality should come about, however, one can wonder why there is no interaction capable of mixing the two “basis functions” anyway, similarly as happens in the famous example of ortho-para hydrogen conversion [52].

The work of Gell-Mann and Pais [40] on the neutral kaons (see Sect. III of Ref. [1]), which do comprise an isospin doublet according to the theory, did pursue the possibility of such hybrids of degenerate particles ( $K^0$  and  $\bar{K}^0$ ) being formed, a phenomenon referred to as hypercharge

oscillations. Their conclusion was that a basis of hybrid kaons which would be separately symmetric and anti-symmetric with respect to the charge conjugation operation  $C$  should possess distinct decay lifetimes. The observation of three-pion decays several years later [53] gave strong support to this interpretation. As mentioned in Sect. IV, however, further experiments [54] indicated that the long-lived kaon ( $K_L^0$ ; see Table II of Ref. [1]) exhibits both two- and three-pion decays, which led to the abandonment of the hypothesis of hybrid particles which are eigenfunctions of  $CP$ , but not the concept of hypercharge oscillations itself.

The question that seems pertinent in the present general discussion, however, is why such “linear-combination” particles *do not occur for multiplet partners possessing different electronic charge*, such as for the three pions or the charged kaon pair. Such hybrid particles would necessarily have non-integral electric charges and consequently there is no evidence whatsoever that they exist. While it is always possible to come up with an argument based on one or more conservation laws to rationalize such a finding, it should at least be acknowledged that a far more straightforward explanation can be found by simply rejecting the idea in the first place that elementary particles can be treated as basis functions for irreducible representations of some mathematical group. Even in the case of the neutral kaons, it is still possible to interpret the experimental observations in terms of two particles (or two different states of the same particle) with well-defined lifetimes ( $K_S^0$  and  $K_L^0$ ). Their lack of electronic charge simply makes it impossible to rule out that oscillations actually do not occur.

Especially when attention is focused on charged systems, it is seen that elementary particles do not generally behave like conventional angular momentum basis functions. For example, the spatial properties of the latter can be altered in a continuous manner simply by systematically changing the direction of a perturbing magnetic field. In this sense the analogy between angular momentum and isospin is not at all as closely drawn as generally assumed. *Where else in physical applications of group theory does a single basis for a degenerate irreducible representation play such an exclusive role as in this model for the structure of elementary particles?*

By contrast, the XBPS model seeks to deal with the fact that two different particles of the same rest mass exhibit *distinct* properties, which are *fixed* characteristics of each system, by employing the following two basic assumptions: a) the true building blocks of nature are the proton, electron, neutrino and their respective antiparticles, each of which is an immutable element and b) the governing Hamiltonian commutes with the charge-conjugation operation, so that any given composite particle must have an antiparticle (in some cases identical with itself)

with exactly the same rest mass. The possibility of encountering a single particle which is a *hybrid* of two or more other particles is therefore excluded in the XBPS model, whereas it is clearly implied by any theory which treats these objects as basis functions for a particular group representation.

The other theoretical point mentioned above has more to do with the internal consistency of the XBPS model. Especially in view of the desirability of keeping the number of free parameters to a minimum in constructing theoretical models, it can be regarded as a positive aspect of the XBPS methodology that it appears capable of accomplishing the goal of obtaining maximum binding energies for particle-antiparticle pairs of exactly  $2mc^2$  with the aid of only a single parameter, the exponential damping factor  $A$ . The scaling arguments of Refs. [2-3] make clear, however, that this result is only valid for particles with electronic charges of  $\pm e$  or zero. In that sense the properties of the various quark particles (non-integral charge) are incompatible with the present assumptions of the XBPS model. The impasse can only be averted by allowing two additional damping constants, one for particles with  $\pm 1/3 e$  and one for those with  $\pm 2/3 e$ . Such an additional complication in the underlying theoretical framework might be acceptable on general grounds, but it would be a step backward and presumably should only be taken if more is accomplished thereby than just ensuring that particular values of quark-antiquark binding energies are obtained.

In summary, the quark model and that presented in the present study differ in a number of important respects, although their ultimate goals are quite similar. The XBPS model employs notably fewer assumptions and seeks to compute rest masses of elementary particles in an *ab initio* fashion. The quark model derives its plausibility from its capacity to identify and rationalize an orderly grouping of meta-stable particles observed in high-energy experiments. In so doing it claims, for example, that a charged  $\pi$  meson is a diatomic system composed of one type of quark and the antiparticle of another type. When the resultant particle decays a short time later, however, the fragments observed are ultimately electrons and neutrinos, i.e. leptons in the terminology of present-day physics, which do not appear to have a quark composition themselves. This model therefore relies firmly on the creation-annihilation hypothesis whose validity has been questioned in the present study, and which at the very least cannot be proven experimentally. The XBPS model by contrast asserts that the order perceived in the quark theory has its origin in the structure of a single Hamiltonian operator which governs the interactions of three well-established stable particles and their antiparticles. All other substances are thought to

be compounds of such elements in various degrees of excitation whose composition can be inferred to at least within an integral number of  $p^+p^-$ ,  $e^+e^-$  and  $\nu\bar{\nu}$  binary systems on the basis of the identity of their various decay fragments.

## VIII. CONCLUSION

In addition to its explanation for the origin of the strong and weak interactions of nuclear systems, the XBPS model allows for a re-interpretation of virtual processes in quantum electrodynamics and related theories. Accordingly, when one speaks of virtual photons affecting the properties of a given system, one can describe the interactions in terms of the original atom or molecule combined with one or more  $e^+e^-$  and  $\nu\bar{\nu}$  massless binaries (phantons). In the lowest-order representation only the massless photon is involved, but an excited configuration in which the electron and positron are separated makes a contribution to the overall wavefunction in higher order. An analogy is made to CI treatments of electronic structure, in which it is routine to mix in configurations of relatively high energy, which one can refer to as virtual states, in order to obtain a more accurate description of the physical system.

The ubiquitous presence of real photons in the universe, as assumed in the XBPS model, thus provides a straightforward explanation for the fact that perfect agreement with experimental data for an apparently isolated system *cannot* be obtained without making some provision for interactions external to it, i.e. virtual processes. In this view the results of quantum electrodynamics can be obtained, at least in principle, by including real  $e^+e^-$  binaries explicitly in the theoretical treatment, in which case variational methods would be applicable. Since the internal structure of the photon has little to do with the electromagnetic interactions responsible for such effects, it is reasonable to exclude such details in the theoretical treatment employed. Instead, low-order perturbation theory based on matrix elements between the  $E = 0$  and  $E \neq 0$   $e^+e^-$  states which are derived on the basis of certain assumptions about the nature of the photon field can be employed with high accuracy, as prescribed in the standard quantum-electrodynamics approach.

The same type of argumentation applies to virtual pions in nuclear interactions, even though a first-order perturbation theory treatment is unrealistic in this instance. To make the analogy more concrete, it is necessary to ascribe a definite elemental composition to the pions (Sect. II of Ref. [1]). Since it is known that  $\pi^+$  decays into  $\mu^+$  and  $\nu$ , and  $\mu^+$  itself into  $e^+$ ,  $\nu$  and  $\bar{\nu}$ , the particle balance hypothesis suggests that the charged pions have a tetra-atomic composition,

i.e. they consist of a single electron and three neutrinos. The (virtual) pion cloud in nuclear structure theory can thus come about as the result of an interaction of a proton with three phantons, namely one photon and two ( $\nu \bar{\nu}$ ) photrinos (Sect. III). An excited configuration in which the above particles are redistributed to form a neutron ( $p^+ e^- \bar{\nu}$ ) and a positive pion ( $e^+ \nu \bar{\nu} \nu$ ) thus makes a significant contribution to the overall wavefunction of the system in a CI expansion of which the leading term is the proton-phantons complex.

The form of the exponential damping factor in the XBPS Hamiltonian offers a clear mechanism by which such configuration mixing can occur, since it allows the proton to interact strongly with the lighter particles through the spin-same-orbit and Darwin terms, for which only the large  $q/m_0$  values of the electron and antineutrino serve as coupling constants. Because its orbital is effectively un-damped in such interactions, the proton is relatively free to maximize this effect through contraction of its charge distribution, being hampered in this respect by only the accompanying kinetic energy enhancement. The XBPS calculations indicate that the binding energy of the bare proton to a single  $e^+ e^-$  species is on the order of 1.0 MeV or more. On this basis it can be argued that the actual mass of the bare proton is larger by several electronic mass units than that measured for the proton in its natural environment. Further calculations indicate that the binding energy of a second proton to such an  $e^+ e^-$  unit is significantly less than the first, which helps to explain why nuclei form themselves around electron-antineutrino pairs instead of such particle-antiparticle binary systems.

The finding that the bare proton is strongly bound to the  $e^+ e^-$  (and  $\nu \bar{\nu}$  but not  $p^+ p^-$ ) phantons in the XBPS calculations suggests a simple explanation for the existence of the “pion cloud.” This has been indicated by elastic electron scattering experiments in the GeV range, which show a definite extension of the proton’s charge distribution. Consistent with this interpretation is the fact that higher-energy inelastic scattering resonance continua of protons in the 2-4 GeV range exhibit point-charge characteristics. According to the de Broglie relation the interparticle distances involved in the elastic scattering processes correspond to a range in which the proton can interact strongly with neighboring photons and photrinos, whereas those associated with the inelastic scattering processes are too small to produce similar bonding effects, with the result that the properties of the bare (Dirac) proton are observed under these conditions. Ultimately, it is simply the large mass of the proton and its consequently small  $q/m_0$  ratio which in this view is responsible for its perception as other than a point-charge particle in low-energy scattering experiments.

The proton-antiproton (annihilation) interaction can be understood on this basis as well (Sect. IV). Just as in positronium decay, it is assumed in the XBPS model that (at least) two (massless) photons take up the energy given off in this process. Because the magnitude of the energy expended is so much greater in this case, however, the  $e^+$  and  $e^-$  components of the interacting photons are set free. One positron and one electron form the basis for charged pion formation, combining with a photrino and a  $\nu$  and  $\bar{\nu}$ , respectively, to form  $\pi^+$  and  $\pi^-$ . Another  $(e^+, e^-)$  pair can also combine with a photrino to give a neutral pion. In this interpretation the proton and antiproton do not lose their existence in the process, but instead form a massless  $p^+p^-$  prophoton at rest in the original center-of-mass coordinate system. The fact that the  $E = 0$  form is apparently always reached in the process is consistent with the observation noted in Sect. 2.3 of Ref. [3], namely that only in this state can a system of zero rest mass travel with less than the speed of light. Intuitively, such a result seems essential in order to have the required transition from the initially stationary  $p^+$  and  $p^-$  systems to occur with any finite probability. The same argument also explains why all the energy of a photon is given up in the photoelectric effect and other absorption processes, even though the energy and momentum conservation laws do not in themselves require this result (see Sect. 2.3 of Ref. [3]). At the same time, the greater activity of pions as compared to muons in the prophoton formation process itself is attributed to the unsaturated structure of the latter, which can be thought of as heavy electrons with  $\nu \bar{\nu}$  adjuncts. At scattering energies below 100 MeV, this distinction between muons and pions is no longer critical, which explains why the muons are stable with respect to pion formation under such conditions, as is observed in the study of cosmic rays.

In addition to providing a theory of elementary particles which is governed by a strict elemental balance of protons, electrons, neutrinos and their antiparticles, the XBPS model also gives insight into other processes which have aroused the interest of physicists over the past several decades. The longitudinal polarization of electrons and neutrinos in  $\beta$  decay is noted to follow a simple rule [51]. Accordingly, the direction of each of their momenta is invariably found to be (predominantly) parallel to that of its magnetic moment. In the case of neutrinos this conclusion is based on the signs of their respective charge-to-rest-mass ratios needed to obtain the observed binding energies of the neutron and its antiparticle. Heavier systems such as the proton and muon do not follow the above rule, but this behavior is easily understood on the basis of the conservation of energy and linear momentum laws. The latter would always have to be

violated in such cases in order to allow conformance with the above correlation found between the respective directions of the magnetic moments and momenta of lighter particles.

With these observations in mind, it can be noted that the concept of a neutron or muon as a “molecule” composed of certain elements, rather than as a single fundamental particle itself, opens up a new possibility for explaining the observed polarization effects in the decay of these systems. Each of the constituent particles in such a decaying system can be expected to be subject to its own distinct field (due to damped Breit-Pauli interactions in the XBPS model), and this circumstance could lead to correlations between the momenta and spins of such species similar to those observed. If the decay particles are simply created at the time of the decomposition of a single particle, no comparable assumption is warranted in the absence of external fields, which is the basis of the argument against parity conservation in such experiments. Thus the creation-and-annihilation hypothesis for material particles can be seen as a key underlying assumption in the long-accepted interpretation of the longitudinal polarization phenomenon accompanying these decay processes.

Once the creation-and-annihilation hypothesis is seen to not be essential in explaining other classes of observations in modern physics, however, the possibility arises that it also might not be essential in this area either. This observation in turn suggests *that parity might well be conserved in all physical processes after all*. Instead, the above correlation found between momentum and magnetic moment directions is seen to be consistent with the expected effects of the short-range interactions of the Breit-Pauli Hamiltonian, which would tend to force the spins of each of the decay particles into alignment with the respective partial fields acting on them. If the field gradients are always positive at the location of the particles *immediately after the decay process* begins, the latter would not only rotate so that their magnetic moments (or their equivalent in the case of the short-range neutrino interactions) become parallel to their respective fields, they would also be accelerated in that direction as well (Stern-Gerlach effect). Such an *inertial* effect for the incipient fields arising at the time of  $\beta$  decay would thus explain the longitudinal polarization phenomenon without requiring that the Hamiltonian employed not commute with the parity operation. In addition, it can be noted that most (if not all) experimental attempts to provide evidence for parity violations in other contexts are inconclusive because they employ external perturbations which themselves do not commute with this operation. Typical examples are use of an electric field to induce a dipole moment in an atom or of a radiation field to demonstrate optical rotation. Since each of these fields is *ungerade*, it is impossible to know if

the effect observed stems from some inherent property of the free-space Hamiltonian or from the effects of the applied perturbation itself.

Recognition of the above uncertainties in the parity non-conservation argument suggests [51] that one should re-examine the theoretical arguments which led Yang and Lee to propose the idea in the first place. Their solution to the  $\tau$ - $\Theta$  puzzle involves a clear assumption about the parity of the pion, namely that it is negative. One usually attempts to justify this choice on the basis of experiments in which low-energy negative pions are scattered from deuterons, but it is well-known that no *definitive* conclusion based on experiment alone can be reached regarding the parity of any of the participants in this reaction. Instead, one ultimately uses the fact that a choice of negative parity for the pion would allow an assignment of positive parity to both the proton and neutron, which at best is only made plausible by an argument based on isospin theory. Nor can it be argued that *only* by assuming a negative pion parity can any of the experiments be satisfactorily interpreted which are thought to provide evidence for parity violations in nature. In fact, the situation is quite the opposite, namely if the parity of the pion is positive, there are no difficulties whatsoever in understanding why a given system might decay into both an even and odd number of them, as is observed for both charged and neutral kaons (and other systems).

With this background in mind, it is interesting to note that the calculations of the XBPS model indicate that the parity of each pion is *positive*, while it is that of the neutron in the relevant  $d\pi^-$  scattering experiments which is negative. This finding ultimately rests on the fact that the phantom ground state is computed to have  $0^-$  symmetry in the present model, and the same result is obtained for the analogous  $e^- \bar{\nu}$  complex which is thought to be a constituent of the neutron. Finally, it is important to note that if parity really is conserved in all natural processes, as these considerations strongly suggest, then there is also no reason to doubt that both the charge-conjugation and the time-reversal operations are perfectly conservative, since previous claims to the contrary have been based on the same types of experimental evidence as discussed above.

Another puzzling characteristic of the behavior of neutrinos may also be connected with the existence of massless phantom systems, namely the unexpectedly low cross sections observed for the reactions of solar neutrinos arriving at the earth's surface. Under laboratory conditions the reaction producing the neutrinos is assumed to involve a massless  $\nu \bar{\nu}$  system according to the present model. The antineutrino is used to form a neutron, while the neutrino is released with high energy. Because of the  $10^6$ - $10^7$  °K temperature in the sun's core, a different distribution of photrino translational states is expected there than on the earth, in close analogy to what one



knows for the blackbody radiation spectra of photons under the same two sets of conditions. If a significant number of dissociated  $\nu$  and  $\bar{\nu}$  species are present in equilibrium with their bound  $\nu\bar{\nu}$  counterparts in the solar environment, it seems at least conceivable that a rather large fraction of the key fusion processes known to occur there would be induced *by free antineutrinos*. In such reactions no neutrinos would be emitted. This possibility would destroy the simple relationship which has been used in predicting the outcome of the solar neutrino measurements made at the earth's surface.

The key unknown quantity in this proposal is clearly the height of the energy barrier for photrino dissociation, but the expected higher reactivity of free antineutrinos *vis-a-vis* bound  $\nu\bar{\nu}$  species could also be a significant factor in reducing the fraction of neutrino-producing fusion processes occurring on the sun as compared to what is observed at ordinary temperatures. This example provides a clear illustration of how the phantom hypothesis differs from the creation-annihilation concept. The present model assumes a well-defined structure for the  $\nu\bar{\nu}$  system normally which is required to cause the fusion reactions of interest to be balanced, but this leaves open another possibility, namely that such a composite particle may behave differently under one set of experimental conditions than another. By contrast, to remain consistent with the creation-annihilation hypothesis, it must be assumed that the environment can have no effect of this nature, because in this view the neutrino and antineutrino are thought to be non-existent prior to the reaction's occurrence.

In summary, the present model asserts that a consistent theory of elementary particle interactions can be built upon the premise that all matter ultimately is composed of electrons, protons, neutrinos and their antiparticles. Such a model is characterized by very simple conservation laws, the most basic of which requires that the same elements are present before, during and after every physical process. Each of the above elements is assumed to be perfectly stable and to possess a point charge distribution. There is no unambiguous experimental finding which stands in contradiction to this view. A model based on the existence of only point particles as the constituents of all matter is the ultimate concretization of the theory of the Greek philosopher Democritus which holds that the only physical realities are "Atoms and Voids", and most importantly in the present context, that the former are absolutely indestructible. A point particle has no volume and thus can never be divided into still more elemental parts. To the elemental balance principle is added the energy, linear and angular momentum conservation laws. Parity is assumed to commute with the corresponding Hamiltonian, similarly as the

charge conjugation and time reversal operations.

On this basis, it is suggested that there may be no need to assume that particles exist which are not composed of the above elements. The present model also does not make use of a number of theoretical quantities invented in the quantum age, such as isospin, hypercharge and the muon quantum number. Such a development in no way precludes the utility and relevance of these properties. Rather, it questions whether they are *essential* to the goal of understanding all processes involving elementary particles. In a similar vein, while the quark model in its various forms has been quite successful in predicting the existence of hitherto unknown elementary particles, as well as elucidating the symmetry relationships which exist among such systems as a whole, it can still be argued that such explanations may not be *unique*. Furthermore, there is a fundamental relationship connected with treating particles as basis functions for an irreducible representation of a group which does seem to be lacking on the basis of experimental observations. The classical applications of group theory in quantum mechanics, such as the description of angular momentum, allow for equivalent irreducible representations for which different basis sets are employed (for example, real and complex  $l^2$  eigenfunctions). One knows, for example, that the choice of such angular momentum basis functions can be systematically varied by changing the direction of an applied magnetic field for atomic Zeeman effect experiments. To demonstrate the analogous relationship in elementary particle physics, it is necessary to find different *linear combinations* of degenerate particles, as it were, for a particular irreducible representation. When the original basis functions are of opposite charge, as is the case for pions and charged kaons, for example, this would mean isolating a particle with non-integral electronic charge, something which has never been found in any experiment to date. While it is easy for a theoretician to construct arguments which rationalize the failure to discover such hybrid charged particles in nature, it also can be argued that the real reason for this development is that the true elements which nature has provided us are completely immutable, regardless of any external force which might be applied to them.

In the absence of the positive identification of particles with fractional electronic charges, one should at least ask the question as to whether a theory which only requires the existence of known stable particles as the sole building blocks of nature may not ultimately be sufficient to explain all measureable phenomena. The present model thus envisions the universe as consisting exclusively of point particles joined together by forces which are not greatly different than those envisioned in the Dirac equation. In particular, the same Hamiltonian is used to represent the

electromagnetic and weak interactions, and additional proton-proton interactions have been investigated, suggested to a large extent by the success of the nuclear shell model, which give promise of ultimately incorporating a quantitative description of the strong interaction in the present model as well. The fact that the creation and annihilation of material particles can never be unambiguously verified by purely experimental means should serve as a strong impulse toward the further development of such an alternative theoretical model, one that avoids making the above assumption and instead requires a strict elemental balance in all naturally occurring processes.

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