Uniform Scaling: Relativistic Energy-Momentum Relationships

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Abstract

A number of the most often cited results of relativity theory deal with the relationships between energy, momentum and inertial mass. The history of how Einstein and Planck came to these conclusions is reviewed. It is pointed out that considerations of how the speed of light is affected by the motion of the Earth played a determining role in these developments. After the Michelson-Morley null-interference result became available, Voigt introduced a new space-time transformation by amending the classical Galilean transformation so that the speed of light in free space has the same value of c regardless of the state of motion of both the light source and the observer. This led to the Lorentz transformation which has been the cornerstone of relativity theory for the past century. A thought experiment is presented which proves, however, that there are many situations for which the measured speed of light is NOT equal to c. Furthermore, it is pointed out that the rate of an inertial clock cannot change *spontaneously*, which result is perfectly compatible with Newton's First Law of Kinetics (Law of Inertia). This result contradicts the space-time mixing characteristic of the Lorentz transformation and leads to the conclusion that events which are spontaneous for one inertial frame will also be so for every

other one. The uniform scaling procedure is a generalization of this result for all other physical properties than elapsed times. Its application shows that the commonly accepted relationships between energy and momentum are only *special* cases in which it is assumed that the observer is stationary in the rest frame in which force has been applied to cause the object's acceleration.

Keywords: *Galileo's Relativity Principle, Voigt space-time transformation, Uniform Scaling, Light-speed constancy assumption, Hamilton-Voigt E-p transformation*

I. INTRODUCTION

The null-interference experiment carried out by Michelson and Morley [1] in 1887 led to the conclusion that the speed of light is independent of both the states of motion of the source and the observer. Upon hearing of this result, Voigt published a paper [2] in which he suggested for the first time that the classical (Galilean) space-time transformation could be amended by making an assumption that the speed of light in free space would have the same value c for observers in two different rest frames. His work led to the introduction of the Lorentz transformation by Larmor [3] and Lorentz [4] a few years later. This was followed in 1905 by Einstein's seminal paper [5] in which he ruled out the existence of an ether to explain the properties of light thus far observed to that point. In addition, he gave a new derivation of the Lorentz transformation.

Voigt's space-time transformation [2] also is consistent with the relativistic velocity transformation (RVT) first introduced by Einstein in his 1905 paper [5]. Two years later, the RVT received a significant verification from von Laue's derivation [6] of the Fresnel-Fizeau light-drag effect which was based on it. One of the main consequences of these theoretical developments was the conclusion that the classical Galilean transformation, which in reality is exactly the same as what is termed "vector addition in other more general contexts, is invalid since it is not consistent with the light-speed constancy assumption for observers in different rest frames. A simple thought experiment has been presented in recent work [7], however, that proves that the RVT is also not universally applicable. In reality, there is a *dichotomous* relation between the RVT and the classical velocity transformation.

In all the above cases, velocity/speed is defined as a ratio of distance moved by an object to the corresponding elapsed times during which the motion occurs. There is another possibility, however, namely through the use of the Hamilton Canonical Equations. Consideration of Newton's Second Law of Motion leads accordingly to the conclusion [8] that the speed of an accelerated object is equal to the derivative of energy E with respect to momentum p (v=dE/dp). On this basis it is possible to use the same logical argument used by Voigt [2] to obtain an E-p transformation instead. It therefore is also consistent with the light-speed constancy assumption underlying the RVT. It will be shown below that this E-p transformation leads to many of the most fundamental dynamical relations in relativity theory, including Einstein's [5] mass-energy equivalence equation, $E=mc^2$.

II. MYTHS IN RELATIVITY THEORY

For over a century mainstream physicists have promulgated the view that the classical (Galilean) space-time transformation is only accurate in the non-relativistic range and that it is merely the limiting form of the Lorentz transformation (LT) for low velocities. This attitude is clearly based on the assumption [2-5] of leading physicists of the late 19^{th} century that the speed of light has the same value for all observers independent of both their state of motion and that of the light source. This evolved into Einstein's light-speed constancy postulate (LSP) which he proposed in his 1905 paper [5]. The null-interference result of the Michelson-Morley experiment

[1] seemingly made this conclusion unavoidable, but there is another interpretation that also fits the facts, namely that the speed of light *relative to its source* is always equal to c in free space. This view is completely consistent with Maxwell's electromagnetic theory [9] as well as with the Michelson-Morey experiment.

Beyond this ambiguity there is a clear indication that the Galilean transformation is perfectly valid under certain circumstances. Consider, for example, the case of a light source moving with speed v away from the laboratory rest frame. According to the LSP, when a light pulse is emitted from the source *in the same direction*, it moves with speed c relative to both the source and the laboratory. Therefore, after time T has elapsed the distance travelled by the pulse is equal to cT both relative to the original rest frame and that of the moving source. Since the source is separated from the laboratory by a distance of vT at this time, however, it is unavoidable to conclude that this arrangement is impossible to achieve. Instead, it must be concluded that the light pulse is now separated from the original laboratory rest frame by a distance of vT + cT = (v+c) T. By definition this means that the laboratory observer measures the speed of light to be v+c, not c, in contradiction to the LSP. This result in turn is exactly the value of the light speed predicted by the Galilean transformation.

Nonetheless, it is still clear that the Galilean transformation cannot explain events such as the Fresnel-Fizeau light-drag and Thomas spin precession [10] effects. A way out of this impasse is simply to recognize that the ranges of applicability of the RVT and classical velocity transformations are mutually exclusive [7]. Examination of the above effects shows that only a single observer is involved. In all relevant cases, two events are considered under different circumstances. For example, the single laboratory observer in the Fizeau experiment measures the speed of light under the two distinct conditions, with and without flow of an external medium. On the contrary, whenever *two observers in different states of mo*tion measure the velocity of the same object, the velocities of the object, including a light pulse, are found to be different. In other words, before applying either of the two velocity transformations, it is necessary to decide which set of experimental conditions is involved. More discussion of this point is found in Ref. [7].

At the same time, it must be emphasized that the success of the RVT is in no way confirmation of the LT. On the contrary, the fact that the latter predicts remote non-simultaneity is a clear indication that it is inconsistent with physical reality [11]. The Law of Causality leads to the conclusion that an *inertial cloc*k, i.e. one that is moving at constant velocity, cannot change its rate spontaneously. This leads to the conclusion that elapsed times for a given event carried out by *two different inertial clocks* must always occur in the same ratio, i.e. $\Delta t' = \Delta t/Q$, where Q is the ratio of the two constant clock rates. If one of the measured time differences is zero ($\Delta t = 0$), it therefore follows that the other must be zero as well. This is to say that the event occurs *simultaneously* for the clocks in both rest frames. The LT is not consistent with this conclusion, therefore proving that it is inconsistent with the Law of Causality. There is a different space-time transformation (Newton-Voigt transformation [11]) which does not have this problem while still satisfying the LSP and the Galilean Relativity Principle (RP). It is consistent with absolute simultaneity and therefore also with the Law of Causality.

III. ENERGY-MOMENTUM TRANSFORMATION

The discussion in Sect. II makes clear that, although the LSP is not always valid, there are definite situations in which it must be used in the form of the RVT in place of the Galilean transformation in order to describe space-time relationships. It therefore is purposeful to combine the LSP with the Hamilton Canonical Equations in an analogous manner to that used by

Voigt [2] to derive his ground-breaking space-time transformation in 1887. The goal thereby is to describe relativistic dynamical relationships where the classical transformation is known to fail [12].

It is useful in this pursuit to examine the history of how the Hamilton Canonical Equations, specifically dE=vdp, were first introduced into relativity theory. Einstein [5] had formulated a theory of electromagnetism in which he equated the Lorentz electromagnetic force **F** to the product of mass and acceleration **a** = d**v**/dt of the electron. For this purpose Einstein found it necessary to define two different kinds of mass: longitudinal ($\mu\gamma^3$) and transverse ($\mu\gamma$), with γ (v) $=(1-v^2c^{-2})^{-0.5}$ and μ the rest mass of the electron. Planck was not satisfied with this approach and developed a different theory in which he introduced a general form of the momentum **p**= $\mu\gamma$**v**. He then used Newton's Second Law to define **F** as the time derivative of **p**, and this approach was soon accepted by Einstein [13]. Details of Planck's derivation [14] are given below for the three momentum components (x is the direction of the force **F**, so $dv_y/dt = dv_z/dt = 0$):

$$F_x = d/dt[\mu v_x(1-v^2c^{-2})^{-0.5}] = \mu a_x(1-v^2c^{-2})^{-0.5} + \mu v_x(1-v^2c^{-2})^{-1.5}c^{-2}(v_x dv_x + v_y dv_y + v_z dv_z)/dt=$$

$$\mu a_x(1-v^2c^{-2})^{-0.5}\,[1 + (1-v^2c^{-2})^{-1}v_x^2c^{-2}]= \mu a_x\,(1-v^2c^{-2})^{-1.5} = \mu a_x\gamma^3.$$

$$F_y = d/dt\,[\mu v_y(1-v^2c^{-2})^{-0.5}] = \mu a_y(1-v^2c^{-2})^{-0.5} + \mu v_y(1-v^2c^{-2})^{-1.5}c^{-2}(v_x dv_x + v_y dv_y + v_z dv_z)/dt=$$

$$\mu a_y\gamma.$$

$$F_z = \mu a_z\gamma.$$

The Hamilton Canonical Equations can then be used to obtain a general expression for the differential energy/work dE done as a result of the application of the force F in the x direction:

$$dE= F_x dx = dp_x\, dx/dt = v\, dp_x = \gamma^3\mu v dv.$$

Since $d\gamma= \gamma^3c^{-2}vdv$, it follows that

$$dE= \mu c^2 d\gamma = d(\gamma\mu c^2),$$

which upon integration gives the energy-mass equivalence relation:

$E= \gamma\mu c^2 = mc^2$.

The above results are based on the definition of relativistic mass as $m = \gamma\mu$. Thus, one can redefine Planck's general expression for momentum **p** as m**v**. At the time Planck made this suggestion, no experimental data were available to test the dependence of relativistic mass m on speed, but this changed in 1909 with Bucherer's study of accelerated electrons [15]. He found that indeed the mass of the electrons was proportional to γ (v), where v is the speed of the electrons relative to the laboratory from which they were accelerated.

It is interesting to see how Einstein originally came to the $E=mc^2$ relationship [5]. He based it on a thought experiment [16] in which two light waves of energy L/2 are emitted in opposite directions from a body. He used the formula he had developed for the transformation of a light beam to conclude that an observer moving with speed v relative to the rest frame of the body would measure the difference in energy $\Delta E'$ of the body to be $\gamma(v)$ times larger than the value of $\Delta E=L$ measured locally. Einstein then argued that $\Delta E'-\Delta E = L(\gamma-1)$ to be the kinetic energy of the body at rest. Therefore,

$\Delta E'-\Delta E = L(\gamma-1) \sim Lc^{-2}v^2/2 = mv^2/2$, and

$L/c^2 = m = E/c^2$.

It is at least a curiosity that Einstein used the non-relativistic value of the kinetic energy to arrive at this key result of relativity theory. As mentioned above, he changed his mind in favor of Planck's derivation in terms of Newton's Second Law and a new relativistic definition of mass [13].

Hamilton's Canonical Equations are a direct application of the Second Law, so the use of the LSP in arriving at an energy-momentum transformation is certainly suggested on this basis. In analogy to Voigt's original derivation [2] of a relativistic space-time transformation, one thus obtains the following E-p transformation in differential form [12,17]:

$$dE = \gamma(dE' + v dp_x')$$

$$dp_x = \gamma\,(dp_x' + vc^{-2}dE')$$

$$dp_y = dp_y'$$

$$dp_z = dp_z'.$$

One would like to go over to an analogous version with ordinary undifferentiated variables, but there is a problem with such a procedure. The corresponding space-time transformation can be obtained by straightforward integration of the differential quantities. This is possible because the speed v separating the two rest frames is constant. This is *not* the case with the Hamilton Canonical Equations, however, as the two rest frames in this case differ in their respective values of v. Nonetheless, there is merit in arriving at the hoped-for non-differential version simply by elimination of the d's in all the variables. The result is:

$$E = \gamma(E' + v p_x')$$

$$p_x = \gamma\,(p_x' + vc^{-2}E')$$

$$p_y = p_y'$$

$$p_z = p_z'.$$

The relativistic versions of the energy-momentum relationships are easily obtained on this basis if one considers a special case in which $p_x'=0$, i.e. the rest frame in which the accelerating force has been applied. This leads to $E=\gamma E'$ in the first equation and $p_x= vc^{-2}E = mv$ in the second, which is of course tantamount to the mass equivalence $E=mc^2$ relation. It also leads directly to Planck's momentum definition $\mathbf{p}=\gamma\mu\mathbf{v}$ if one defines the relativistic mass m to be equal to $\gamma\mu$, i.e.

γ times the rest mass μ ($E'=\mu c^2$). A key relationship in laboratory experiments is also easily obtained from the $E=\gamma E'$ equation. By squaring both sides, one finds:

$$E^2 = m^2c^4 = (1-v^2c^{-2})^{-1}E'^2 = (1-v^2c^{-2})^{-1}\,\mu^2c^4 \text{ and thus:}$$

$$E^2\,(1-v^2c^{-2}) = E^2 - E^2v^2c^{-2} = E^2 - m^2v^2c^2 =$$

$$E^2 - p^2c^2 = E'^2 = \mu^2c^4.$$

It should be noted, however, that all of the above standard results of relativity theory have been obtained on the basis of a *special case*, namely for $p_x'=0$, that is for the rest frame in which force has been applied to the particle. This is a standard situation in laboratory experiments, but the question remains as to how the Voigt adjustment of Hamilton's Canonical Equations functions for other cases in which both the object and the observer are moving relative to the above rest frame.

IV. UNIFORM SCALING GENERALIZATION

In the Hafele-Keating experiments with circumnavigating atomic clocks [18], it was found that the rate of each clock was inversely proportional to γ (v), where v is the speed of the clock relative to the Earth's center of mass (ECM). This experience indicates that the elapsed times for a given event satisfy the following relation for any two clocks [19]:

$$\gamma\,(v)\,\Delta t = \gamma\,(v')\Delta t',$$

where Δt and $\Delta t'$ are respectively the elapsed times for clocks moving with speed v and v' relative to the ECM. A perfectly analogous relation was found earlier in experiments measuring x-ray frequencies [20-22]. On this basis, the above equation can be looked upon as the Universal Time-dilation Law [23] or UTDL. It is also used in determining the change in the rates of atomic clocks prior to launch in the Global Positioning System [24].

The UTDL can be brought into a form that is suitable for space-time transformations, namely

$\Delta t' = \Delta t/Q$. Accordingly, the constant Q which appears in the NVT replacement [11] for the LT mentioned in Sect. II is defined as the ratio γ (v')/γ (v). If v'=0, as is the special case in laboratory experiments for the rest frame in which acceleration of the object to speed v is initiated, the relation between elapsed times becomes $\Delta t=\gamma$ (v) $\Delta t'$, Q = γ (v). This relation in turn is completely analogous to Einstein's 1905 result [5,16] for the energies of objects: $\Delta E=$
γ (v) $\Delta E'$.

Given the results for elapsed times, it is unavoidable to conclude that the latter relation is only valid for a special case in which v'=0, that is, when the observer is stationary in the same rest frame in which force has been applied to the object. Moreover, it seems clear that when this is not the case, the analogous relation for energies, i.e. $\Delta E=Q\ \Delta E'$, with the same value of Q as for elapsed times, is perfectly accurate. Further consideration leads to the conclusion that the same proportionality relation holds for inertial masses. Since the speed of light is assumed to have the same constant value of c for all observers (at the same gravitational potential), it also follows that the $E=mc^2$ relation retains its validity for nay value of Q. This in turn leads to the conclusion that that the same proportionality for energy, inertial mass and elapsed times holds for distances as well. In other words, *length expansion accompanies time dilat*ion, not the type of asymmetric length contraction that is predicted by the LT.

The above proportionality relations are primary examples of the uniform scaling procedure. More details may be found elsewhere [25-28]. The basic idea is that there is a constant Q which can be used in an unequivocal manner to deduce the ratios of values of any physical property for any pair of rest frames. The *conversion factor* between the values of a given property is always equal to an integral factor of Q. For example, the conversion factors for length, time and inertial mass are each equal to Q itself. Knowledge of the composition of any other physical property in terms of these three fundamental quantities is sufficient to compute the corresponding factors for that property. For example, the factor for speed/velocity is unity, i.e. Q^0, because it is defined as the ratio of distance travelled to elapsed time. This value is obviously consistent with the constancy of the speed of light relative to the source from which it is emitted. The factors for momentum and angular momentum (also Planck's constant h) are Q and Q^2, respectively. It is also possible to define unique conversion factors for all electromagnetic quantities [29,30].

Perhaps the simplest means of understanding the underlying basis for the uniform scaling procedure is to imagine that each rest frame has its own unique set of units. Consistent with Galileo's Relativity Principle, these units appear to be exactly the same for the *in situ* observer in each rest frame. An observer in a different rest frame can determine that his units are different than those in the other rest frame, for example, by comparing his value for the elapsed time of a given event with that of his colleague for the same event. This is in fact the most straightforward procedure for determining the value of Q relating the two rest frames, as discussed above. In essence, there is a law which is analogous to the UTDL for each physical property. On this basis, it is possible to make an addendum to the RP, namely that although the laws of physics are the same in every rest frame, the units on which they are based differ from one rest frame to another [31].

V. CONCLUSION

Well-known formulas such as $\mathbf{p}=\gamma(v)\mu\mathbf{v}$ and $E=\gamma(v)\mu c^2$ are special cases in which the observer is in the same rest frame as that in which force is applied to the objects of measurement. It needs to be recognized that the value of any property depends on the unit in which it is expressed. There is clearly an inverse relationship between the value and the unit in any measurement. It stands to reason that when the observer himself is accelerated, his unit of energy/mass *increases.* As a result, his measured values of the energy/mass of an object *decrease.*

The experiments with circumnavigating atomic clocks carried out by Hafele and Keating [18] are very instructive in this regard. Their results show that there is an inverse relationship between elapsed times measured on a given clock and the speed of the clock relative to the ECM. This leads to a general formula referred to as the Universal Time-dilation Law (UTDL). Accordingly, the ratio of two γ values constitutes a *conversion factor* between the values of the elapsed times measured by two clocks *for any object.* This conversion factor is denoted by $Q = \gamma(v')/\gamma(v)$, where v' is the speed of the object and v is the speed of the observer relative to the *relevant rest frame* (such as the ECM in the HK experiments [18]). The same factor applies for other properties, including energy, inertial mass and momentum. As a result, the aforementioned formulas for momentum and energy are generalized to $\mathbf{p}=Q\mu\mathbf{v}$ and $E= Q\mu c^2$ when both the object and the observer are moving relative to the laboratory in which force has been applied, or in the HK case, relative to the ECM.

The above formalism is rendered essential for moving objects by virtue of the assumption of complete objectivity in the measuring process. Uniform scaling is the antithesis of Einstein's version of relativity [5]. To believe in his theory one must accept as fact that two clocks in relative motion *can both be running slower than the other*. The same situation is claimed to hold for inertial masses, energies and lengths of objects, even though there has never been a single experimental observation that is consistent with Einstein's view. It could be expected, for example, that the frequencies of light are always red-shifted when the source is moving relative to the observer. Such an effect has never been observed despite numerous attempts to confirm it. By contrast, the uniform scaling procedure assumes that if Clock A is found to run *more slowly* than Clock B, then it is an absolute certainty that Clock B is running *faster* than Clock A at the same time.

Einstein's misunderstanding of the measuring process is directly tied to his belief in the correctness of the Lorentz transformation [3-5]. The alternative NVT [11] subscribes to a fundamentally different view of timing relationships, namely if Clock A runs *Q times slower* than Clock B, then Clock B runs *Q times faster*, the same as one finds in everyday comparisons of ordinary household clocks. Indeed, the conversion factor in the "reverse" direction is always the reciprocal of the forward one, as can be seen from the above definition of Q. Reversing the vantage points of the two observers means that the corresponding factor (Q') is $\gamma(v)/\gamma(v') = 1/Q$. The same relationships between conversion factors hold for any physical property. Conversion factors have no place in Einstein's version of relativity theory for the simple reason that it is totally ambiguous as to which clock rate is faster.

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