Clock-rate Corollary to Newton's Law of Inertia

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Abstract

A key characteristic of Newton's First Law of Motion is that it satisfies the Law of Causality. It is pointed out that the same arguments that are the basis for the conclusion that the speed and direction of an object will not change in the absence of some unbalanced external force can be equally well applied to all other physical properties. For example, it must be expected that a clock in motion will maintain the same constant rate indefinitely under these conditions. It is thus clear that two such (inertial) clocks must have a constant ratio Q, which means that the time difference of any two events measured with these clocks must satisfy an analogous proportionality relation, i.e. $\Delta t' = \Delta t/Q$. The space-time mixing characteristic exhibited by the Lorentz transformation (LT) stands in contradiction to this proportionality, therefore demonstrating that the LT violates the Law of Causality. The experiments carried out with circumnavigating atomic clocks demonstrate that the same proportionality relationship for elapsed times is satisfied quite generally and is used to good effect in the operation of the Global Positioning System. A consequence of the elapsed-time proportionality is the absolute simultaneity of events occurring anywhere in the universe. Analogous proportional relationships, referred to as *uniform scaling*, exist for all physical

properties. Hence, there exists a Corollary to the Law of Inertia for properties that is applicable on a completely general basis.

Introduction. — The First Law of Kinetics, otherwise known as the Law of Inertia, states that an object in motion will continue at the same speed and direction (velocity) indefinitely in the absence of unbalanced external forces. In addition to clarifying the longstanding confusion about general questions concerning the existence of uniform motion, Newton emphasized the importance of subscribing to the ancient principle that no physical transformation occurs without some underlying cause (*Law of Causality*). Under these circumstances such an "inertial" object will move exclusively in a straight line without experiencing any amount of acceleration.

Inertial Clocks. —The question that will be analyzed in the present Letter is what one should expect for the physical properties of such a system consistent with the Law of Causality. In analogy to Newton's First Law, it seems perfectly straightforward to assume that an inertial clock will maintain the same constant rate indefinitely, for example, i.e. as long as it is not subjected to any unbalanced external force.

This conclusion has unavoidable consequences for Einstein's Special Theory of Relativity (STR) [1] and the Lorentz transformation (LT). The latter relates the values of elapsed times and distances traveled by objects which are not subject to any external forces including those of gravity. The key equation of interest is:

$$\Delta t' = \left(1 - v^2 c^{-2}\right)^{-0.5} \left(\Delta t - v c^{-2} \Delta x\right) = \gamma \eta^{-1} \Delta t \,. \quad (1)$$

In this equation $\Delta t'$and Δt are the measured time differences for two events that are obtained by two different observers moving with speed v relative to one another along the x axis with each one using an inertial clock which is stationary in his rest frame [Δx is the distance separating the two events along the x axis for one of the observers, c = 299792458 ms^{-1} is the speed of light in free space, $\gamma=(1-v^2c^{-2})^{-0.5}$ and $\eta = (1-vc^{-2}\Delta x/\Delta t]$. Einstein used the example of two lightning strikes on a moving train as it passes a stationary platform to illustrate the meaning of the LT variables in a particular case [2,3]. Accordingly, the observer on the train measures a time difference for the two strikes as Δt' with his stationary (inertial) clock while the platform observer measures the corresponding value Δt based on his stationary clock. The speed of the train relative to the platform is v and Δx is the distance between the two strikes measured by the platform observer.

As pointed out above, the rate of each clock in the above example, and quite generally for all applications of the LT, never changes. This means that the *ratio* of the rates of the two clocks is itself a constant. As a consequence, the respective elapsed times measured on the two clocks must always occur *in the same ratio* as the respective clock rates themselves, as expressed in the following equation [4]:

$$\Delta t' = \frac{\Delta t}{Q}, \qquad (2)$$

where Q is the ratio of the two clock rates. It is a positive definite quantity. If the clock in the "primed" rest frame (S') is slower than that in the other (S), then Q>1. Otherwise, Q<1.

Comparison of the above *proportionality* equation with eq. (1) of the LT shows unequivocally that they are hopelessly incompatible with one another. Moreover, it shows that the LT is totally *inconsistent* with the Law of Causality. As such, this comparison proves that the LT does *not* provide a valid description of the relationship between space and time coordinates in all natural processes.

This situation can be put in stark relief by carefully examining Einstein's example of two lightning strikes on a train discussed above. As has been shown, the inertial clock on the train runs at a rate which is always proportional to its counterpart on the station platform. If a time difference between the two strikes is measured to be Δt' based on the train clock, the corresponding value measured on the platform clock must be Δt'/Q. If the strikes are simultaneous on the train clock (Δt'=0), they must therefore also be simultaneous (Δt=0) on the platform clock. This is the only way that the Law of Causality can be satisfied in this example. This result is in complete agreement with Newton's conclusion of the absolute simultaneity of events occurring anywhere in the universe. At the same time, it rules out the possibility of remote non-simultaneity (RNS) espoused by Einstein [1], which is allowed by the above LT eq. (1) whenever both v and Δx are different from zero.

Real Clocks. —There are no truly inertial clocks in nature, so what happens in the real world in which clocks are constantly subjected to unbalanced external forces? An excellent example for studying this aspect of the problem is provided by the experiments with circumnavigating atomic clocks carried out by Hafele and Keating in 1970 [5]. They found that the rates of these clocks, after correcting for gravitational effects, are inversely proportional to their speed relative to the Earth's Center of Mass (ECM). The equation for the corresponding elapsed times Δt' and Δt for arbitrary portions of their trajectories is:

$$\Delta t' \gamma\left(v_{ECM}'\right) = \Delta t \gamma\left(v_{ECM}\right) \tag{3}$$

where v_{ECM}' and v_{ECM} are the respective speeds of the clocks relative to the ECM and $\gamma(v) = .(1-v^2c^{-2})^{-0.5}$. The value of Q in the above proportionality relation is then obtained as:

$$Q = \frac{\gamma\left(v_{ECM}'\right)}{\gamma\left(v_{ECM}\right)}. \tag{4}$$

A perfectly analogous relationship is obtained in the Mössbauer x-ray study employing a high-speed rotor [6-9], in which case the ECM is replaced by the rotor axis as the reference for determining the values of the speeds to be inserted in the γ (v) functions in eq. (4). As a consequence, eq. (3) has been referred to as the Universal Time-dilation Law (UTDL) [10,11].

The Global Positioning System (GPS) is perhaps the best example of how understanding the manner in which the rates of clocks change with acceleration can be used to great practical advantage. Prior to launch, the rates of atomic clocks on GPS satellites are adjusted so that they will run at the same rate in orbit as their counterparts left behind on the Earth's surface [12,13]. The amount of the adjustment is determined by application of the UTDL (gravitational effects are also included in this calculation). When a signal is sent from the satellite, its time of emission is registered on the adjusted satellite clock, while the corresponding time for reception of the signal is read from an earth-bound clock running at the same rate. The distance between the two positions of the signal can then be accurately determined by multiplying the time difference of these two events by c.

A point that is easily overlooked is that the adjustment of clock rates would be pointless were it not for the fact that the observers on the ground and satellite agree completely when the GPS signal is emitted [14]. If there were such a thing as RNS, as is predicted by the LT, there would be no reason to believe that this event occurs at exactly the same time for both of them. For this reason, it is not correct to claim that the operation of the GPS is consistent with STR and the above LT equation *in which space and time are mixed*. The corresponding proportionality relation [eq. (2)] connected with the UTDL of eq. (3) requires instead that the observers on the satellite and the ground always agree as to both the time of emission of each light signal from the satellite as well as that of its reception on the Earth's surface.

Other LT Contradictions. —There are many other examples which prove that the LT is physically invalid. The most egregious is perhaps Einstein's claim that two clocks can both be running slower than one another at the same time [1]. The circumnavigating atomic clock experiment [5] stands in direct conflict with this belief because it shows that the rate of a clock decreases with its speed relative to the ECM. As a consequence, there is no ambiguity as to which of two clocks runs slower. The same conclusion is supported by the results of the x-ray frequency measurements reported by Hay et al. [6]. Kündig [7] points out in his report that a clock located at the rim of the rotor runs slower than its counterpart in the laboratory. Sherwin notes [8] that this finding stands in contradiction to the LT prediction that each observer must find that it is the *other's* clock that is running slower. This would mean that the wavelengths of atomic lines emitted from a moving source always appear to be *red-shifted* to the observer. Instead, the experimental results show unequivocally that the frequency of the x-ray source is found to be *blue-shifted* relative to the detector when it is located at the rim of the rotor [6,7,9]

The LT also fails to predict the manner in which the lengths of moving objects vary with acceleration. It claims that there is FitzGerald-Lorentz length contraction (FLC), whereby the amount of the contraction varies with the orientation of the object relative to the observer [1].

In order for the speed of light to be independent of the rate of an accelerated clock, however, it is necessary that the distance traveled by the light pulse *increase* in exactly the same proportion as the clock slows down. Moreover, the effect must be independent of the orientation of the object. In other words, *isotropic length expansion accompanies time dilation*, in agreement with the proportionality relation for elapsed times which is consistent with the UTDL of eq. (3), not the type of anisotropic length contraction predicted by the FLC and LT [15].

Yet another example of the failure of the LT is its prediction of time reversal [16]. The ratio Q of the elapsed times $\Delta t'$ and Δt in eq. (2) that is consistent with the UTDL of eq.(3) must be positive to be consistent with experiment [4,5], but this is not true if the prediction is based on the LT eq. (1) in which space and time are mixed. Sommerfeld [17] based his faulty conclusion that light waves cannot travel faster than c on eq. (1), pointing out correctly that on this basis super-luminal waves would allow for time reversal to occur. This argument is contradicted by the fact that the index of refraction n can be less than unity, so that $c/n > c$. Subsequent experiments [18-19] indicated on the contrary that super-luminal speeds can in fact occur. More details about the failure of STR and the LT in particular may be found elsewhere [20].

Newton-Voigt Transformation. —The question therefore naturally arises whether there is an alternative space-time transformation which is consistent with the Newtonian Corollary for the Rates of Clocks and yet still satisfies both of Einstein's postulates: the Galilean Principle and the constancy of light in free space. The Newton-Voigt transformation (NVT) [21,22] given below achieves this goal, with $\eta=(1-vc^{-2}\Delta x/\Delta t)^{-1}$:

$$\Delta t' = \frac{\Delta t}{Q} \tag{2,5a}$$

$$\Delta x' = \left(\frac{\eta}{Q}\right)(\Delta x - v\Delta t) \tag{5b}$$

$$\Delta y' = \frac{\eta \Delta y}{\gamma Q} \tag{5c}$$

$$\Delta z' = \frac{\Delta z}{\gamma Q}. \tag{5d}$$

Note that the function η appears in the Relativistic Velocity Transformation (RVT [23]). Comparison with the four equations of the LT shows that the NVT relations can be obtained from the LT by multiplying each of them with $\eta/\gamma Q$. As a result, the three ratios of space to time are exactly the same in both sets of equations [24], thereby ensuring that the NVT is also consistent with the light-speed constancy postulate. Moreover, the inverse of the NVT set of equations can be obtained by the process of Galilean inversion in which exchanging the positions of the two observers can be simulated by interchanging the primed and unprimed quantities and reversing the sign of the relative speed v of the two observers (note that γ is unchanged by this operation). In the case of the NVT, Galilean inversion leads to the identity [21]: $\eta\eta' = \gamma^2 QQ'$. This relation can be satisfied by setting $Q'=1/Q$ in the inverse transformation. This is easily understandable when one identifies Q as a conversion factor between the two units of time; therefore Q' is the corresponding conversion factor in the reverse direction. The same identity can be used to prove that the RVT satisfies the Relativity Principle.

The true light-speed postulate. —It is easy to show, however, that Einstein's version of the light-speed postulate [1] is incorrect and that observers can measure light speeds c(v) in free space which are not equal to c. To this end, consider a light source that moves from a given origin O at a certain time at speed v along the x axis of the coordinate system. It emits a light pulse moving in the same direction at the same time it leaves the origin. After time ΔT has passed, the light source is separated from O by $v \Delta T$. During the same period, the light pulse has moved a distance of $c \Delta T$ relative to the source. This means that the *total distance* between the origin O and the light pulse is $v \Delta T + c \Delta T$ at this time. By definition, the speed of the light pulse relative to the origin is thus v+c, in contradiction to Einstein's postulate [25] and both the LT and NVT.

Moreover, if one insists that Einstein's postulate [1] is correct, then the speed of the light pulse relative to O is c and the distance separating it from O after time ΔT has passed is $c \Delta T$. Since the source itself is located at a distance of $v \Delta T$ relative to O at this time, it therefore follows by subtraction that the distance the light pulse has traveled in time ΔT is $c \Delta T - v \Delta T$, so its speed relative to its source is *not c*, as the postulate requires, rather it is c-v. This contradiction therefore proves that Einstein's postulate is invalid. This example shows that the correct speed of light in free space, or for that matter the speed of any object, can be obtained quite generally by *vector addition*, which is simply a different name for what is commonly referred to as the Galilean Velocity Transformation (GVT).

The phenomenon of the stellar aberration provides an example which clearly demonstrates the validity of the GVT. In 1727 Bradley ascribed the apparent movement of the positions of celestial objects at different times of the year to the speed v of the earth relative to the sun [26], and he used the classical theory of motion, i.e. the GVT, to quantify his position. Accordingly, the angle of aberration for light coming from the zenith is equal to $\tan^{-1}(v/c)$.

Einstein [1] assumed instead that the speed of light in free space is completely independent of the velocity of the emitting body, in accordance with his light-speed postulate.

On this basis he claimed that the above formula for the angle of aberration must be altered by multiplying it with γ (v). The value of v is too small to allow for an experimental test of Einstein's adjustment, but his version of the theory of stellar aberration has long been accepted by the astronomical community. Once it realized that Einstein's postulate is incorrect, however, it becomes clear that Bradley's formula is indeed the right one.

GVT or RVT?—The example of stellar aberration raises the issue of whether the GVT or the RVT should be used in a given case [26]. There is a clear dichotomous relationship between the two velocity transformations. If the speed of light is to be *measured in two different rest frames,* as in Bradley's example, the respective distances travelled by the light must be different and the relationship of the two velocities must be obtained by vector addition, i.e. with the GVT. The RVT only has validity for special cases in which the comparison is between measurements made by *a single observer for two different conditions.* In the case of the Fresnel-Foucault light damping experiment, the two conditions are for light moving in two different directions relative to the flow of a given medium through the apparatus. The GVT is not applicable in this situation and so the quantitative solution obtained by von Laue [27] correctly makes use of the RVT instead. The above condition for using the RVT is also satisfied by the phenomenon of Thomas spin precession [28], and thus the GVT also must be eschewed in this example [26].

Uniform Scaling—.The UTDL of eq. (3) for measured elapsed times of different observers of the same event has been shown to derive from the proportionality of eq. (2) between Δt' and Δt in the NVT. It is easy to see that as a consequence of the light-speed equality postulate, an analogous relation must exist for corresponding distances Δr' and Δr. First, it needs to be pointed out again that Einstein's version of the latter [1] is invalid and is therefore in need of replacement. This change is accomplished quite simply by realizing that all experimental information indicates that the speed of light *relative to its source* is always equal to c=299792458 ms^{-1}. This relationship is perfectly consistent with Maxwell's theory of electromagnetism as well as with the GVT. Since two observers measure the same value of the light speed in free space from different vantage points, it follows that any difference in the rates of their two clocks must be compensated for by an exactly proportional difference in the corresponding measured values $\Delta r'$ and Δr for the distance the light has traveled, that is:

$$\Delta r' = \frac{\Delta r}{Q} . \qquad (6)$$

A simple means of expressing the above relationships is to look upon Q as a *conversion factor* between the respective values of elapsed times and distances traveled by a given object as measured by the two observers. As already mentioned, the corresponding conversion factor Q' for the reverse relationship must be equal to the reciprocal of that in the forward direction.

Experiments with accelerated electrons by Bucherer [29] in 1909 found that their inertial masses m_I also increase with $\gamma(v)$, from which one must conclude that the corresponding scale factor for inertial mass is also Q. Formally, one can denote the scale factor for velocities as $Q^0=1$. In general, the corresponding scale factor for any other physical property can be determined on the basis of its composition in terms of the three fundamental quantities of time, distance and inertial mass. Accordingly, each factor must be expressed as an integral power of Q. For example, energy scales as Q because of the $E=mc^2$ relation, that is, the same as for inertial mass; angular momentum L scales as Q^2 since it has units of both inertial mass and distance. This general procedure has been referred to as *uniform scaling*. A much more detailed discussion of this point may be found in earlier work [30,31]. It is even possible to define scale factors for electromagnetic quantities [32,33]. There is also a comparable scheme for the *gravitational scaling* of physical quantities [34, 35].

It is important to see that the uniform scaling procedure is closely tied up with Galileo's RP. It is useful to amend this relationship as follows: *The laws of physics are the same in each inertial system but the units in which they are expressed can and do vary from one system to another.*

Generalized Corollary—The fact that the uniform scaling procedure applies to all physical properties makes clear that the corollary to Newton's Law of Inertia for the rates of clocks discussed at the outset can be extended to have a much broader range of application: The Law of Causality requires that an analogous relationship applies for distances, inertial masses, angular momentum etc. Hence, one can state without equivocation that *each physical property of an object in motion will maintain the same constant value indefinitely in the absence of any unbalanced external force.*

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