Inclusion of Breit-Pauli Short-range Interactions in the Quantum Mechanical Theory of Sub-atomic Systems

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Abstract

Following the introduction of the Schrödinger and Dirac quantum mechanical equations for the description of hydrogenic atoms, there was a longstanding attempt to develop a similar approach which is applicable to the treatment of nuclear reactions. There was consensus that interactions of shorter range than the Coulomb force are involved, but there was also the strongly held view that the corresponding Hamiltonian that satisfies this requirement must be invariant to a Lorentz transformation. The latter conclusion is questioned on the basis of a simple example involving the interaction of an electron in an electromagnetic field. It leads to a contradiction according to which observers in different rest frames must disagree on whether the path of the electron is curved or linear. A solution to this dilemma is provided by the assumption of a different definition of the velocity parameter v in the Lorentz Force law, specifically that it is taken to be the speed of the electron relative to the origin of the electromagnetic field rather than to each observer. Moreover, the requirement that the Hamiltonian operator be invariant to a Lorentz transformation is shown to be satisfied by merely *employing the same form* for the operator in each rest frame. A problem with the limiting behaviour of the Breit-Pauli Approximation short-range terms such as spin-orbit and spin-spin coupling is shown to be eliminated by multiplying them with a momentum-dependent exponential factor similar to that advocated much earlier by

Yukawa in connection with his theory of elementary particle interactions. The relation between the internal motion of the particles and that of the center of mass of the system is also discussed. The evaluation of properties in rest frames moving with high velocity relative to the center of mass of the system under consideration can be undertaken with the aid of the Uniform Scaling procedure rather than by explicit computation of expectation values based on solutions in which the effects of internal and translational motion are intertwined.

Keywords: Schrödinger Equation, Dirac Equation, Breit-Pauli Approximation, Uniform Scaling, Exponential Damping, XBPS Model

I. INTRODUCTION

The main tool in the XBPS theoretical model [1,2] is a Schrödinger equation whose Hamiltonian contains a relativistic free-particle term for the kinetic energy in addition to potential terms which are short-range in nature. Specifically, the latter are exponentially-damped, momentum-dependent operators, which in the limit of velocities much smaller than that of light, approach those used in the conventional Breit-Pauli approximation [3-5] to the Dirac equation [6,7]. Since the XBPS formalism is applied to describe interactions such as nuclear binding and elementary particle processes, it is evident that it must adhere to the strictures imposed by relativity theory for phenomena involving high relative velocities. Theoretical aspects regarding this point will be considered in the present work, with particular emphasis on the treatment of translational motion employed therein. This general topic will also be seen to have relevance in another important area of quantum mechanical theory, namely the particle-wave duality interpretation of physical processes.

II. RELATIONSHIP BETWEEN THE SCHRÖDINGER EQUATION AND MULTI-COMPONENT RELATIVITIC FORMULATIONS

The Dirac equation [7] is the cornerstone of a four-component theory for one-electron atoms. Just as the Schrödinger equation [8], it can be looked upon as an eigenvalue equation, i.e. $H_D \Psi = E \Psi$, for which the corresponding Hamiltonian consists of a 4 x 4 matrix of operators and

the eigenvectors are corresponding four-dimensional vectors called spinors. For a time-independent H_D the same type of separation of time and spin-spatial variables can be obtained as for the Schrödinger equation. In this case E becomes a four-dimensional constant matrix and the spinors which serve as solutions to the corresponding time-independent Dirac equation are functions of spatial and spin coordinates only. Dirac was led to this equation by relativistic considerations [9], especially the desire to accurately describe atomic fine structure which is known to arise from *the motion of the electrons* and thus could not be satisfactorily represented by means of the original electrostatic Schrödinger equation. Since the goal of the present study has been the quantitative description of systems involving higher relative velocities than those of electrons in atomic systems, it is natural to ask whether the inherently relativistic nature of the interactions involved is compatible with a theoretical treatment in which the multi-component spinor wavefunctions analogous to those of the Dirac equation are *not* employed. To answer this question, it is important to examine the nature of the Dirac four-component theory in some detail.

As already noted, the differential equation in this formulation employs fourth-order operator matrices. In the time-independent version, the energy is represented by a constant matrix, whereas H_D itself contains non-vanishing elements of several types, including some in off-diagonal positions. As with all equations involving matrices, it is possible to apply a unitary transformation to each of the objects appearing in it to obtain an equivalent differential equation. The constant E matrix is unaffected thereby, but the spinor eigenfunctions as well as the Hamiltonian matrix will generally have a different appearance after such a transformation. An interesting possibility suggests itself as a result, namely that a particular unitary transformation U might succeed in producing a diagonal form H_D' for the Dirac Hamiltonian. The transformed Dirac four-component equation thus takes on the following appearance:

$$\begin{pmatrix} H_1 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & H_3 & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix} \begin{pmatrix} \Psi_1' \\ \Psi_2' \\ \Psi_3' \\ \Psi_4' \end{pmatrix} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & 0 & E \end{pmatrix} \begin{pmatrix} \Psi_1' \\ \Psi_2' \\ \Psi_3' \\ \Psi_4' \end{pmatrix}$$

where $U\Psi = \Psi'$ gives the relationship between the original and transformed spinor eigenfunctions.

Foldy and Wouthuysen [10] adopted such an approach and succeeded in bringing the free-particle Dirac Hamiltonian to diagonal form by using

$$U = [(\alpha\ p)\beta + m_o + E_p]\ [2E_p\ (m_o + E_p)]^{-1/2}$$

In this definition α_i and ß are the four-dimensional Dirac matrices [9], and E_p and m_o are constant matrices for $(p^2 + m_o^2)^{1/2}$ and the electronic rest mass respectively (in a system of units in which c = 1). The corresponding diagonal elements in the free-particle H_D' are then $H_1 = H_2 = -H_3 = -H_4 = E_p$, which is the Einstein relativistic energy for a system of rest mass m_o and momentum **p**. The same one-electron term also appears in the XBPS Hamiltonian (see Table I of Ref. [2]). When the Coulomb potential $\varphi = -Z/r$ is introduced, the situation becomes more complicated, but Foldy and Wouthuysen suggested a series of contact transformations which in principle can achieve the desired diagonal form of H_D' in this case as well. By carrying out a finite number of such transformations it is possible to obtain an approximation to the Dirac equation which is closely related to that used in the Breit-Pauli formalism [11].

The main interest in the FW transformations in the present context is less in explicit mathematical details, but rather the theoretical implications of employing such formal diagonalization procedures in the first place. Once such a unitary transformation is applied, *the Dirac equation reduces to a series of four ordinary Schrödinger equations* involving H_1, H_2 H_3 and H_4, respectively, namely:

$$H_1\Psi_1' = E\Psi_1'$$
$$H_2\Psi_2' = E\Psi_2'$$
$$H_3\Psi_3' = E\Psi_3'$$
$$H_4\Psi_4' = E\Psi_4'$$

The simplest way of proceeding is thus to solve each of these Schrödinger equations separately, obtaining four complete sets of eigenfunctions for each component. Under the circumstances it would no longer be necessary to employ spinor eigenfuntions with more than a single non-zero component. For example, if Ψ_1' is an eigenfunction of H_1 then the spinor $(\Psi_1', 0, 0, 0)$ is an

eigenfunction of H_D' as well, with the same energy eigenvalue as in the ordinary Schrödinger equation, $H_1\Psi_1' = E\Psi_1'$. If the individual eigenfunctions for each component Hamiltonian form an orthonormal set, as is always possible, such a choice of spinor wave-functions with only a single non-zero component would be orthonormal as well. A reverse transformation then allows one to generate spinor eigenfunctions for the original H_D operator from each of the above simpler spinor functions with a single non-zero component: $\Psi = U^{-1}\Psi$. There is thus clearly a one-to-one correspondence between the two sets of spinor eigenfunctions with identical eigenvalues.

As a result there are essentially (exactly in the case of free particles) four times as many spinor eigenfunctions of the Dirac equation as there are simple eigenfunctions of any one of the above Schrödinger equations involving H_1 to H_4, as has been pointed out earlier by Bethe and Salpeter [12]. The question that arises from this observation is whether the additional solutions of the Dirac equation are important physically. *In the case of the free-particle Hamiltonian, the answer is clearly no,* since the four diagonal Hamiltonian operators are identical except for sign. For each eigenfunction of H_1 there is an identical one for H_2, H_3 and H_4. The corresponding eigenvalues are equal for H_1 and H_2, and only of opposite sign for H_3 and H_4. It is common to think of the pairs of equal eigenvalues as corresponding to different spin directions of the free particle, but such a distinction is not essential. Instead one can have a complete set of solutions for both spin directions (assuming the system involved is a fermion) for each of these operators. This is the way the XBPS Hamiltonian is treated, for example, with all degenerate components being generated by the same operator. Under the circumstances, it is a matter of semantics whether one refers to such a treatment as a one-component or a two-component method. At least for the free particle, half of the solutions of the Dirac equation are simply redundant, serving no useful purpose from a physical point of view. Moreover, the negative-energy solutions don't really tell us anything new either. In summary, in the free-particle example, solution of a single Schrödinger equation tells us all we need to know.

When a potential is added to the Dirac Hamiltonian there is no longer such a simple relationship between the positive- and negative-energy solutions. This is because the potential has the same sign in all four diagonal terms of H_D whereas the kinetic energy terms for the first two components continue to differ in sign relative to the last two. Nonetheless, there is still a clear mathematical relationship between the two sets of spinor eigenfunctions which allows corresponding positive- and negative-energy solutions to be converted into one another by simply

changing the signs of E, p and the electronic charge e and interchanging the first two components with the last two [12,13] relative to the original solution. The new function thus corresponds to a system in which the electron of the original problem is replaced by its antiparticle. Dirac originally used this relationship to formulate his hole theory of the electron, which ultimately led to his prediction of the existence of the positron [6]. Furthermore, since the added potential only appears in the diagonal positions of H_D, it seems likely that the interesting simplifying feature of the transformed free-particle Dirac equation, namely $H_1 = H_2$ and $H_3 = H_4$ in the diagonal H_D' matrix, is also a characteristic of the corresponding hydrogenic version. In other words, all the physically meaningful positive-energy solutions of the Dirac equation with a central-field potential can also be obtained from a single Schrödinger equation.

The same type of analysis can be carried over to multi-component treatments of many-particle systems, such as the Bethe-Salpeter equation for helium-like atoms [14]. In this case, one can generalize the Dirac approach to define corresponding differential equations whose solutions are spinors of dimension 4^N, where N is the number of particles. Such matrices can also be diagonalized, at least in principle, and the resulting H_i diagonal elements must have eigenfunction sets which can be used to produce the 4^N-component spinor solutions of the original system of coupled equations. If one continues to allow each of these Hamiltonians H_i to act on the entire spin space spanned by the particles, it again seems possible (and even likely) that all physically meaningful results for the corresponding positive energy states can be obtained by solving a single many-particle Schrödinger equation. Furthermore, as long as the original multi-component set of equations is invariant to a Lorentz transformation [14,15], so must the corresponding diagonal (uncoupled) set as well.

In summary, because of the form of the Dirac equation and its generalizations, it appears to always be possible to obtain a completely equivalent version which *decouples* the various components. This being the case, there is no reason to rule out the possibility that a corresponding Schrödinger equation might exist whose solution contains all the physically meaningful information about a given system which can be obtained from quantum mechanical theory. Since an analogous multi-component treatment for nuclei and elementary particles is not known, it therefore seems reasonable to search for a concrete form of the Schrödinger equation which is capable of attaining the same objective.

Indeed, once such a form is known, it is always possible to construct a multi-component version by writing down a diagonal matrix containing Hamiltonian operators corresponding to different combinations of particles and antiparticles and subjecting it to an arbitrary unitary transformation. The latter complication would serve no useful purpose, however. From this point of view the advantage of the original Dirac multi-component formulation is clearly that it provides a quite satisfactory description of the desired atomic interactions which is superior to any Schrödinger equation yet discovered. It brings with it a large number of redundant solutions, however, which either apply to a different physical system than that of primary interest, or are simply repetitions of other useful solutions. Under the circumstances, when dealing with interactions for which no appropriate multi-component formulation is known, such as in the present set of applications, there is no compelling reason to expect *that a Schrödinger-type approach may not offer the most straightforward means of achieving* a suitably accurate description of the phenomena of interest.

III. THE ROLE OF TRANSLATION IN THE MULTI-COMPONENT DIRAC THEORY

One of the mathematical characteristics of the multi-component Dirac equation [7] which sets it apart from its non-relativistic counterparts is that it can be brought into an explicitly covariant form [15]. The variables contained in it represent exclusively internal degrees of freedom, however. Implicit in this formulation is therefore the complete separation of internal motion from that of the center of mass. Such an arrangement for the theory seems intuitively obvious, but as pointed out in Sect. 3 of Ref. [2], a mathematically sound procedure for effecting the desired coordinate separation is only given explicitly for a non-relativistic treatment of the motion of several particles. Nonetheless, the failure to obtain a clean separation of internal and center-of-mass coordinates in the XBPS Hamiltonian leads to some hitherto unexpected effects for the description of short-range interactions, particularly a relationship between the strength of the various Breit-Pauli terms (Table I of Ref. [2]) and the magnitude of the translational energy.

The question that therefore needs to be examined more carefully is whether the separation of internal and center-of-mass coordinates in the Dirac equation is consistent with experimental observations. In this context, it is well to note that there is strong theoretical evidence [16,17] that the distance between two objects is measured to be different in two inertial systems moving with non-zero relative velocity to one another. Experiment has demonstrated that the rate of an atomic

clock is slowed as it increases its speed relative to a certain rest frame. For example, in the Hafele-Keating study of circumnavigating atomic clocks [18], the Earth's Center of Mass (ECM) satisfies this requirement. Therefore, since the speed of light is the same in every rest frame (after taking account of gravitational effects), it follows that distances expand as the rates of clocks slow down [19].

As an example, consider the computation of the mean distance between the electron and proton in the hydrogenic ground state. In quantum mechanics this quantity is given as the expectation value $<\Psi|(r_e-r_p)|\Psi>$ for the corresponding eigenfunction Ψ. The distance between these two particles is not fixed but the above consideration should be relevant in this determination, that is, it seems reasonable to expect that a different result should be obtained if the center of mass of an H atom is at rest with respect to the observer or if it is moving relative to him at high speed. If the total wavefunction is always written as a product of two factors only one of which depends on the internal coordinates, however, as is the consequence of separating out the center-of-mass motion, it is clear that the expectation value in question cannot depend on the translational state of the system. The internal part of the wavefunction is completely independent of the motion of the center of mass. If, on the other hand, the two types of motion become increasingly intertwined as the translational energy becomes large compared to the system's rest energy m_oc^2, one can at least see a mechanism in which the distance scaling effects [16,17,19] might evolve in a natural way from the computations.

One might argue that it is not really necessary to obtain different results for internal properties of systems as a function of their translational state. Instead, *one can compute the property for the system at rest* and use the time and distance relationships of the corrected relativity theory [16,17] (uniform scaling method) to obtain the corresponding results when the system is moving relative to the observer. The situation is similar to that encountered in the computation of the energy lost when positronium decays, namely $2m_{oe}c^2$, in accordance with the classical mass-energy equivalence relation. It seems fair to say that such interpretations may be overly simplistic, however, and more importantly, that they may hinder the development of a more internally consistent formulation of mechanical theory as a whole.

In the case of property computations corresponding to states of high translational energy, this line of reasoning suggests that the separation of internal and center-of-mass coordinates is an approximation whose .validity is lost at velocities close to the speed of light. An attempt to recast

the XBPS Hamiltonian of Table I of Ref. [2] in terms of center-of-mass and relative coordinates leads to cross terms involving odd powers of both types of conjugate momenta. Although matrix elements for such terms cannot mix configurations of different translational momentum k, they can connect species of different angular momentum quantum number l.

Since the differentiations of the center-of-mass momentum operator always lead to factors of |k| in the corresponding matrix element expressions, it is clear that no such mixing can occur for zero translational momentum. By the same token, however, it seems reasonable to expect that the magnitude of such interactions increases steadily as the speed of the center of mass increases. This would mean in effect *that different internal configurations are used to describe states of high translational energy than are required for the corresponding system when it is at rest with respect to the observer.* Since the translational kinetic energy operator commutes with any Hamiltonian which is a function of the inter-particle distances only, none of this prevents the total eigenfunctions from having a well-defined translational energy. This effect is seen to arise naturally in the XBPS formulation from the fact that the cross terms involving both types of coordinates have only vanishing matrix elements between functions of different k.

IV. LORENTZ INVARIANCE CONDITION FOR QUANTUM MECHANICAL TREATMENTS: APPLICATION OF THE BOHR CORRESPONDENCE PRINCIPLE

The above discussion also raises a question about the way in which the Lorentz invariance condition of Einstein's special theory of relativity [20] is applied in quantum mechanics. In the classical theory, the discussion of relativistic effects is invariably tied up with the translation of various systems. By contrast, the Lorentz invariance property of the Dirac equation [15] for a particle moving in a central field refers to a non-translating system, i.e. only internal coordinates are involved. In classical expositions of special relativity theory [20], however, the treatment of two interacting systems is generally given in terms of the original Cartesian coordinates of each particle. For example, in discussing the problem of two charged particles moving with the same velocity, Sard [21] proceeds as follows. In the inertial system of the charged particles themselves, the equation of motion is:

$$\mu_A \frac{d^2 y_A^t}{d\, t^{t\,2}} = \frac{e_A e_B}{(y_A^t - y_B^t)^2}$$

and $$\mu_B \frac{d^2 y_B^t}{d\, t^{t\,2}} = \frac{-e_A e_B}{(y_A^t - y_B^t)^2}$$

where the two particles are denoted by the indices A and B, the direction y is perpendicular to their momenta, and y^t refers to a coordinate measured in the moving particles' own inertial system S^t. These equations express the fact that from the point of view of the particles themselves there are no magnetic forces (see the discussion in Sect. VI of Ref. [22]).

The standard relativistic treatment of electromagnetic interactions is based on the premise that the components of the electric **E** and magnetic **B** field vectors transform according to the following equations [23] (c is the speed of light in free space, 299792458 ms^{-1}):

$$E_x' = E_x \qquad B_x' = B_x$$
$$E_y' = \gamma (E_y - vc^{-1}B_z) \qquad B_y' = \gamma (B_y + vc^{-1}E_z)$$
$$E_z' = \gamma (E_z + vc^{-1}B_y) \qquad B_z' = \gamma (B_z - vc^{-1}E_y).$$

Einstein derived this set of relations [20] by assuming that Maxwell's equations must be invariant to a Lorentz transformation of spatial and time coordinates between different rest frames. It was further assumed that the components of the electromagnetic force **F** on charged particles e are given in terms of the above field components by the Lorentz Force equation:

$$\mathbf{F} = e\,(\mathbf{E} + c^{-1}\mathbf{v} \times \mathbf{B}).$$

In this equation it has been generally assumed that **v** is the velocity of charged particles relative to the observer, a point which will prove worthy of further discussion subsequently [24].

There is ample evidence [25] that the Lorentz Force satisfies the equation of motion expected from Newton's Second Law, namely:

$$e\,(\mathbf{E} + c^{-1}\mathbf{v} \times \mathbf{B}) = d/dt\,(\gamma\mu\mathbf{v}),$$

i.e. the force **F** equals the time rate of change for the relativistic momentum $\mathbf{p} = \gamma\mu\mathbf{v}$, with $\gamma = (1\text{-}v^2c^{-2})^{-0.5}$ and μ is the rest mass of the particle/electron. Nonetheless, as will be seen from the following concrete example which makes use of this equation, there is still an uncertainty in the

definition of **v** therein when the observer is located in a different rest frame than that of the laboratory.

Consider the effects of an electromagnetic field with only the two components, E_x and B_y, acting on an electron, as has been done in Ref. [25]. From the point of view of an observer located at the origin of the field, the electron will initially move along the x axis. This is because the Lorentz force **F** only depends on E_x at the instant the field is applied since the value of v=0 negates any effect from the corresponding magnetic field component B_y. This situation changes as time goes by and the electron is accelerated to non-zero speeds. The $\mathbf{v} \times \mathbf{B}$ term gradually produces a force component in the z direction, causing the electron to veer away from its initial path. Depending on the relative strengths of the constant values of E_x and B_y, the amount of deflection can be quite significant over time. This situation is easily reproduced in the laboratory and there is no doubt that it is consistent with the Lorentz Force law.

Next consider the same example from the perspective of an observer *co-moving* with the electron. Since the speed v of the electron relative to the observer is zero at all times, it follows according to Einstein's transformation law that the *magnetic field has no effect.* As a result one expects that, from the perspective of this observer, *the electron continues indefinitely along a straight line parallel to the x axis.* This predicted trajectory is therefore clearly distinguishable from that discussed first from the vantage point of the laboratory observer.

This behaviour raises the question of whether it is reasonable to expect that the electron would appear to follow a different path for the two observers. No one has ever ridden along with an accelerated electron or other charged particle to verify that the predicted straight-line trajectory would actually be found by such an observer. Since the curved path expected from the laboratory

perspective is routinely observed, however, it would therefore seem on the contrary *that the straight-line result is pure fiction,* an artefact of a physically unrealistic theory.

Does this example prove that Galileo's RP does not apply to electromagnetic interactions? Clearly not [24]. The reason is because there is another quite straightforward way to satisfy both Maxwell's equations and the RP at the same time, namely to insist that all observers, regardless of their state of motion, *see exactly the same results of any given interaction*. In particular, the hypothetical observer co-moving with the accelerated electron must record the same curved trajectory as is viewed from the laboratory perspective.

The measured values for the parameters of the electron's path may still differ for the two observers, however. This is *because the units in which they express their respective measured values may not be the same* [16,17.19]. We know, for example, from the time-dilation experiments [18] that the clocks they employ to measure elapsed times can run at different rates. This fact does not change the above conclusion about the trajectory of the electron in the above example, however. There is no reason to doubt that all observers should agree that a curved path is followed as a consequence of the interaction of crossed electric and magnetic fields.

There is a detail that needs to be considered in both Maxwell's equations and the Lorentz Force law which is crucial for deciding how to apply the RP to electromagnetic interactions. It is the interpretation of the velocity that appears in both expressions. At some point in history, physicists came to the consensus that **v** is the velocity of the electrons or other charged particles *relative to the observer* in any given interaction. This decision has quite important consequences vis-a-vis the measurement process in general. It means that the results of any measurement are thought to depend on the perspective of the observer. *Measurement is subjective*, in other words.

There is a clear alternative interpretation [24] of the velocities which appear in Maxwell's equations and the Lorentz Force law, however, one which eliminates the need to assume that observers can disagree on the trajectories of particles affected by these interactions. It is simply necessary to assume that the variable **v** in these equations is the velocity of the electron *relative to the rest frame where the electromagnetic field originates*. This is a quantity which all observers can agree upon at least in principle. Just changing the unit in which velocity measurements are expressed can have no effect on the measured trajectory of the particle. In particular, an observer co-moving with the electron in the example of the previous section can therefore use Maxwell's equations and the Lorentz Force law to conclude that the path being followed is exactly the same as reported by his counterpart located at the origin of the electromagnetic field, except perhaps for a difference in the sets of physical units in which each expresses his results.

The above interpretation allows for a much less restrictive interpretation of the RP. It is not necessary that the form of the physical law describing this or any other interaction be invariant to a particular space-time transformation in an arbitrarily chosen rest frame. In the case of electromagnetic interactions, it is only necessary that the same laws, in this case Maxwell's equations and the Lorentz Force law, apply in any rest frame *where the electron currently exists*. Its velocity **v** relative to the origin of the interaction uniquely determines the magnitudes of the electric and magnetic fields as well as the corresponding force acting on it. By contrast, the velocity of the electron relative to the observer himself plays no direct role in determining such quantities, thereby removing any element of subjectivity from the process. All observers, regardless of their own state of motion, must agree on the results of the interaction, except that they will generally not agree on the numerical values of their measurements because of differences in their respective choice of physical units.

In quantum mechanics the equation of motion is of a different form, however, namely one in which *the wavefunction of a system is sought rather than a fixed trajectory.* If the corresponding Hamiltonian is given in terms of the coordinates of each particle rather than assuming a separation into internal and center-of-mass coordinates, similarly as in the example from classical relativity theory discussed above, the following possibility suggests itself. The laws of nature must be the same in all inertial systems according to the principle of relativity [20.26,27]. *In the case* of a *quantum mechanical formulation, this condition can be fulfilled if the Hamiltonian operator simply has the same form for all observers.* The coordinates on which this operator depends necessarily vary from one inertial system to another in describing a given system.

If two observers moving relative to one another start out with the same Schrödinger or Dirac equation $(H - E)\,\Psi = 0$, how then can they come to different conclusions about the properties of a system in their respective inertial systems? The answer lies in the fact that *there is more than one solution to the above quantum mechanical differential equation.* The computation of the properties of a given system thus requires *not only the solution of the corresponding equation, but also the correct identification of one of the resulting eigenfunctions* as that describing the state of the system actually observed. For example, the hydrogen atom ground state in the conventional language actually corresponds to an infinite number of different translational states. If two observers *disagree* on the magnitude of the translational energy of a given system, they will simply *choose different solutions* of *the same differential equation* to describe it. This possibility distinguishes quantum mechanics from its *c1assical* counterpart, for which *a definite answer is expected* for each set of initial conditions.

What these considerations show is that the principle of relativity can be incorporated into a quantum mechanical theory by simply requiring that the Hamiltonian H(q.p,t) of any system always be expressed in terms of the coordinates of a given observer. Applying the Lorentz transformation to H(q,p,t)-E(t) simply gives back the same expression in terms of the coordinates of a different inertial system, i.e. H(q',p',t') - E(t'). Under the circumstances *it is not really necessary that the two quantities be exactly equal,* i.e. that H(q,p,t)-E(t) = H(q',p',t')-E(t'), in order to satisfy the principle of relativity. Rather it is sufficient that the *form* of H-E be the *same* for all observers, so that each of them can generate the *same* set of solutions to the corresponding quantum mechanical equations of motion from which the pertinent translational states

corresponding to different perceptions of the same physical system can be properly selected. If the Hamiltonian is incorrect for any reason, there will inevitably be disagreement between prediction and observation on this basis, so this possibility still excludes the use of distinctly non-relativistic terms such as $p^2/2m$ in representing the kinetic energy. On the other hand, it suggests that failure to find a covariant form for H-E, i.e. one that satisfies the condition $H(q,p,t) - E(t) = H(q',p',t') - E(t')$ upon application of a Lorentz transformation, *does not constitute proof that the corresponding quantum mechanical formulation is defective.*

Since the present discussion involves a comparison of the theory of quantum mechanics with its classical counterpart, it is relevant to consider the implications of the Bohr correspondence principle [28] in this regard. In accordance with this prescription, it must be expected that the quantum mechanical formulation reverts to the classical one in limiting situations in which the less general theory has established validity. One difficulty with applying this principle to a relativistic quantum mechanical theory is that two separate limiting processes are involved, as commonly achieved by setting Planck's constant h to zero and the speed of light in free space c to infinity. The minimal condition which must be fulfilled with regard to the theory of special relativity is that the classical limit of the quantum mechanical equations of motion can be cast in covariant form.

For this purpose one typically employs the relativistic four-vectors [29] (**p**,iE/c and (**A**, iφ). The Breit-Pauli terms [5] are closely related to the classical magnetic interactions containing the vector potential A, as has been discussed in detail by Slater [30], so the ingredients for such a covariant limiting form are present when a Hamiltonian containing such interactions is employed. Because of the linearity of the Lorentz transformation, it is clearly essential that the r^{-3} dependence of the Breit-Pauli terms (see Table I of Ref. [2]) tend toward r^{-1} as a result of the limiting process. Since a constant velocity is always involved in such a transformation, however, it is not difficult to imagine how this occurs. Under these conditions the angular momentum $\mathbf{l} = \mathbf{r} \times \mathbf{p}$ itself varies as the first power of r, and since the spin-independent Breit-Pauli terms are second-order in **1** (or $\mathbf{r} \cdot \mathbf{p}$ in the case of the Darwin term), the necessary changeover from r^{-3} to r^{-1} variation is ensured. The corresponding spin-dependent terms can be plausibly ignored since they have no classical analogue and can thus be assumed to vanish in the pertinent limiting process.

The fact that the Breit-Pauli terms can be derived as an approximation to the Dirac equation [3-5], which itself is invariant to a Lorentz transformation, is perhaps the best indication that such

a limiting relationship can be satisfied by a differential equation containing such momentum-dependent short-range interactions. The essential point remains, however, that it is not necessary that the relativistic quantum mechanical theory be Lorentz-invariant. As discussed above, the principle of relativity is automatically satisfied by employing the same functional form in all inertial systems for the (H-E) operator appearing in the pertinent Schrödinger equation.

V. THE ROLE OF SHORT-RANGE INTERACTIONS IN HIGH-ENERGY PROCESSES

Ever since the first scattering experiments on nuclei, it has been recognized that forces of shorter range than those of the Coulomb effect and gravitation are at work in nature. Yukawa [31] was the first to suggest that the interactions in question were exponential in character, but he assumed a potential which varies as r^{-1} as the internuclear distance approaches zero. In the XBPS model the short-range interactions vary as r^{-3} over a large range of interparticle distance, until the momentum-dependent damping factors begin to have a dominant influence on the theoretical description (Table I of Ref. [2]). Because the Lorentz transformation is linear in spatial and time coordinates, it is difficult to imagine an equation of motion involving a potential which is invariant to such a change in variables. By not assuming the usual separation in terms of internal and center-of-mass coordinates, it has been possible to employ exponentially-damped Breit-Pauli r^{-3} potentials in the XBPS model to obtain a realistic description of nuclear binding [1,2,22]. The combination of short-range momentum-dependent potentials plus an interdependence of internal and translational motion allows a simple explanation for the fact that particles and antiparticles can interact strongly with another. In the inertial system in which the center of mass is at rest, $\mathbf{p}_1 = -\mathbf{p}_2$, from which it follows that particles of equal rest mass must move with the same speed relative to their midpoint. This circumstance allows maximum benefit to be taken of attractive short-range interactions which are momentum-dependent, whereas a much weaker effect is expected for two particles with greatly different rest masses.

It would appear then that not only is it consistent with relativity theory to employ such short-range interactions to describe high-energy processes, but even that it is actually essential that this be done in order to obtain satisfactory agreement with experimental observations. The arguments of the last section show how the results of the corresponding Schrödinger equation can be interpreted in a manner consistent with the principle of relativity, even though a covariant form for its Hamiltonian appears to be excluded. Furthermore, by *not* factoring out the center-of-mass

motion, it is possible to explain at least in principle how the properties of physical systems can vary with their translational state, as relativity theory [16,17,19] indicates they must. None of this precludes the possibility of defining a Schrödinger or Dirac equation which is completely independent of center-of-mass coordinates.

Formally, one must transform to the usual system of internal and center-of-mass coordinates and retain only the terms of the Hamiltonian which are independent of translational variables. Solution of the corresponding differential equation would then only produce information about states of zero translational energy. It is doubtful that such a simplified operator would be Lorentz-invariant, but its application would have an advantage relative to Hamiltonians of the XBPS type employed in the present work, in which the center-of-mass and internal types of motion are intertwined.

A related question to that discussed above is whether it is essential to employ a separate time coordinate for each particle in defining a proper equation of motion. If one believes in the prediction of remote non-simultaneity of the Lorentz transformation, it could be argued that such an arrangement is essential [20]. It has been shown of the basis of the Law of Causality [17], however, that this view in not correct. The results of the Hafele-Keating study [18] with circumnavigating clocks are in complete agreement with this assessment. Furthermore, this same principle is assumed in the operation of the Global Positioning navigation system [32,33].

The situation is even clearer if one assumes, as is done in the XBPS model, that all the particles of a given system are referenced to the position of an observer *who is always located at the origin of the coordinate system* in which the computations are carried out. In that case, it is essential to employ a single temporal variable in describing the physical interactions. There is no need for more than a single clock with which to measure the time of events occurring in the same inertial system. This choice is also consistent with assuming that each solution of the corresponding differential equation corresponds to a single total energy. As a result it is possible to describe the time-dependence of the XBPS eigenfunctions in the usual way as $\exp(-iEt/\hbar)$, where E is the total energy of the corresponding state. Since the particles move independently of one another, it is nonetheless necessary to employ a separate set of spatial and spin coordinates for each of them, but the former correspond to different length measurements in the same inertial system, thus requiring the use of only a single unit of distance (meter stick) to determine their values experimentally.

VI. CONCLUSIONS

There are several characteristics of the Dirac equation which are commonly misunderstood. For example, it is not true that it is fundamentally different than the Schrödinger because of its four-component nature. The fact is that since the Hamiltonian matrix of operators is Hermitian, it is possible to find a unitary transformation to bring it into diagonal form, thereby producing four independent Schödinger-like equations. This possibility was first used by Foldy and Wouthuysen to define their relativistic equation for the description of the electronic structure of atoms. . The simplest way of proceeding is then to solve each of these equations, thereby obtaining four sets of eigenfunctions. It is therefore always possible in principle to completely *decouple* the original components represented in the Dirac equation. It is exactly this argumentation which is used in developing the XBPS model which employs short-range interactions to describe subatomic particles. In other words, what one has in this model is a Schödinger equation which employs short-range momentum dependent operators that do not appear in the original version used to describe the hydrogen atom.

The variables in the Dirac equation for hydrogenic atoms contain only interparticle distances, so implicit in this formulation is the complete separation of internal motion from that of the system's center of mass. When more than two particles are involved, the situation becomes more complicated, however, because there is no way to completely separate out the translational motion; this is only possible in the non-relativistic treatment. As a result, it is unavoidable that the two types of motion become intertwined as the translational energy of the system increases. In effect, this means that different internal configurations must be used to describe states of high translational energy than is the case for the corresponding system when it is at rest relative to the observer. In an exact treatment, one can restrict attention exclusively to states with zero translational energy, but when only finite numbers of basis functions are employed, as is necessarily the case in the XBPS model, one must identify states with a minimum of translational energy for each internal state and compute properties with the corresponding wavefunctions on this basis.

Over the past century, it has been concluded that the Hamiltonian operator in relativistic calculations must be invariant to a Lorentz transformation. It has been shown in a simple example, however, that defining v in the Lorentz Force law as the value of the electron speed relative to each observer himself, leads to a contradiction. According to that definition, one must

conclude that different observers will disagree on whether the path of the electron is curved or not. If instead, the electron speed is always taken *relative to the rest frame in which the electromagnetic field originates,* the above contradiction is removed. Different observers will always agree on the nature of the electron's path, but they will disagree on the values of most properties because they use different units in which to express their respective values. The uniform scaling method allows one to account for these differences on a completely quantitative basis.

The restrictions of the Relativity Principle can be achieved by requiring that the Hamiltonian of any system always be *expressed in the coordinates of each observer.* Applying the Lorentz transformation (or any other that relates the two rest frames) results in identically the same operator in any other inertial frame. Each observer can then generate the same set of solutions in his own coordinate system. This line of argumentation thus supports the view of the XBPS model that the Hamiltonian operator can be chosen solely on the basis of its ability to obtain satisfactory agreement with experimental findings.

The above considerations allow one to employ operators in the Hamiltonian which are of shorter range (r^{-3} instead of only r^{-1}) than those in the original Schrödinger and Dirac equations. Problems with the limiting values of the computed energies are avoided by multiplying the Breit-Pauli operators with Yukawa-type momentum dependent exponential factors. Agreement with experimental energy values has thus been obtained with the aid of a single free parameter [1,2,22]. Because the calculations are formally carried out from the vantage point of a single observer, it is essential to employ only a single time variable corresponding to his stationary clock/. There is no need for more than one such variable in the calculations, although a separate distance variable must be used for each of the particles in the system under consideration.

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