# Rebuttal of Fermi's Denial of Nuclear Electrons

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**Abstract**

The discovery of the neutron by Chadwick in 1932 is discussed in detail. Pauli pointed out that the profile for neutron decay indicates unequivocally that a third particle, in addition to the proton and electron, is involved, which has since been referred to as the antineutrino $\bar{\nu}$. Fermi then argued that the electron could not have been present in the neutron prior to decay. He based his conclusion on the assumption that the laws of physics must be in accord with the Lorentz transformation, which Einstein used as the cornerstone of his Special Theory of Relativity (STR) that he introduced in 1905. On this basis, it should be impossible for a potential to exist which is capable of binding an electron to a proton in such a small space (500 Mev would be required according to Fermi's calculation). The present work assesses this claim on the basis of recent theoretical developments which make use of the exponentially damped Breit-Pauli-Schrödinger (XPBS) equation. Calculations of this type have been successful is showing that the binding energy of an electron to a positron might be exactly equal to the energy equivalent of an electron and positron ($2m_{oe}c^2$). To this end it is assumed that the charge-to-mass ratio for $\bar{\nu}$ is non-zero. It is pointed out that this eventuality would still be consistent with the known extreme penetrability of these particles through matter, as demonstrated by experiments carried out by Reines and Cowan. By assuming a value for this ratio in the 0.5-0.7 a.u. range, it proves possible to obtain a total energy of the neutron relative to its separated particles which is consistent with experimental data (+0.7825 MeV). The corresponding $p^+e^-$ separation is $3.62\alpha^2$=10.2 fermi=$10.2\text{x}10^{-15}$ m, which is well within Fermi's expected range for this quantity.

*Keywords: Neutron Composition, Neutrino Penetrability, Creation-Annihilation Hypothesis, Exponentially-Damped Breit-Pauli-Schrödinger equation (XBPS)*

## INTRODUCTION

In preceding work [1,2], a model for describing processes involving the interaction of electromagnetic radiation with matter has been examined whose main ingredient is the assumption of an $e^+e^-$ "molecular" structure of the photon. Accordingly, the hypothesis of disintegrating matter is replaced with the assumption of indestructible particles, including electrons and positrons, which in a specific state of binding can lose all their mass, thereby defying experimental observation but still retaining their own existence.

The latter distinction seems subtle enough when formulated in this manner, but there is a potentially critical difference. Put simply, if particles can be created from pure energy, there is nothing fundamentally excluding the possibility that much larger objects can also be formed by this mechanism. It has been argued,[3] for example, that whole universes could be created from the "energy equivalent to just a few pounds of matter." Later in the same article it is pointed out that the key element in the author's theory 'is that quantum physics permits the spontaneous creation of something from nothing." If on the other hand, matter can never be created or destroyed by any mechanism, but instead only disappears from view under certain well-defined circumstances, the possibilities are much less fantastic. A much more sober view of the universe emerges, and consequently it behooves us to find out which of the above two hypotheses corresponds to the true facts.

It is thus important to see that the creation-annihilation concept does have definite relevance beyond the subject of the interaction of photons with other particles. In the preceding work[1] it was noted. for example, that the formation of protons and antiprotons from photon collisions demonstrates that $e^+e^-$ systems cannot be the only mass-less particle-antiparticle binaries which must be assumed if one hopes to do without the latter hypothesis . Once one has grasped the possibility that the creation-annihilation assumption may not actually be needed to explain relatively low-energy phenomena, there is thus a challenge to follow through on such argumentation in the high-energy regime as well.

One is reminded that considerable impetus was given to the creation-annihilation concept at the time when it first became possible to carry out high-energy experiments in the laboratory. It is therefore to be expected that as the available energy from accelerators and related devices increases, the number of phenomena requiring similar explanations will tend to multiply, as has indeed been the case. New particles with non-zero rest mass have been identified over the past

century, and these have provided fertile breeding ground for new theories. In the present chapter the task of surveying such high-energy experiments will be taken up, with particular emphasis on the question of how the assumption of the creation-and-annihilation of matter plays a role in the theories which are used to describe them.

## II. THE NEUTRON

After the results of Rutherford's scattering experiments were understood,[4] one had the model of atoms containing a relatively massive nucleus of positive charge being orbited by a number of electrons. The simplest atom is hydrogen with a single proton and electron. The heavier nuclei have rest masses which are nearly integer multiples of that of the proton, but charges which are always a smaller (exact) multiple of the electronic charge. On this basis it was first assumed that the nucleus itself generally contains both protons and electrons. The discovery of the neutron in 1932 by Chadwick[5] caused a shift in this position, however. The neutron was found to have a slightly larger mass than the proton and to have no electric charge. It is meta-stable with a half-life of 1000 s, decaying into a proton and electron (or "β-ray" in the earlier terminology). Other unstable nuclei were also known to undergo β decay.

Several objections to the assumption of a proton-electron constitution for the nucleus quickly arose on the basis of these findings. The most important was based on the fact that the energy spectrum of the emitted electron is not mono-energetic, as would have to be expected from the laws of conservation of energy and momentum when a single particle decomposes into two fragments. One popular interpretation for this observation was supported by Bohr.[6] He suggested that the usual conservation laws might no longer be valid at high energies (0.8 MeV is released upon the neutron's decay). Pauli took a different view,[7] however, which has since been generally accepted, namely that the continuous spectrum observed indicates that at least one more particle must be involved in beta decay. Accordingly, the energy lost in the process can be divided between several emitted particles in a continuous distribution without violating the above conservation laws. The name "neutrino" was subsequently coined by Fermi[8] for this new particle.

Other evidence was also found for the neutrino hypothesis, however. If a neutron were to consist of only a proton and an electron, it should possess integral spin and exhibit boson statistics, so that a nucleus such as $^{14}N$ (containing seven protons and seven neutrons) would

behave as a fermion with half-integral nuclear spin. Since the opposite behavior is observed, it was suggested by Pauli that the neutrino (if indeed it is a single particle) is also a fermion with half-integral spin, and that its presence in the neutron thus explains this aspect of the nuclear puzzle as well. This argument eventually carried the day and the third neutron decay product was later renamed the anti-neutrino $\bar{\nu}$.

What needs to be emphasized in the present context is that in the search for a quantitative theory of beta decay, confidence in the straightforward idea of the neutron being composed of a proton, electron and antineutrino was eventually lost. This development occurred primarily because of the realization that the relatively small radius of the neutron implies that an electron bound within it would have to possess an enormous amount of kinetic energy. A figure of 100-500 MeV could be computed for this quantity based on the de Broglie relation, which connects the magnitude of this radius (wavelength) with the electron's momentum. Since the known electromagnetic and gravitational forces are far too weak to explain how the electron could enjoy a net attraction inside the nucleus under these conditions, it was clear that an impasse had been reached. In addition, the fact that the magnetic moment of the neutron was measured[9] to be much smaller than that of the electron seemed to be totally inconsistent with such a composition.

The problem was ultimately circumvented by Fermi with his suggestion that the electron might be annihilated in the presence of a strong nuclear force. It is interesting to consider Fermi's original remarks on this point, as given in translated form by Wentzel.[10] "The simplest way for the construction of a theory which permits a quantitative discussion of the phenomena involving nuclear electrons seems then to examine the hypothesis that the electrons do not exist as such in the nucleus before the ß emission occurs, but that they, so to say, acquire their existence at the very moment when they are emitted; in the same manner as a quantum of light, emitted by an atom in a quantum jump, can in no way be considered as pre-existing in the atom prior to the emission process. In this theory, then, the total number of the electrons and of the neutrinos (like the total number of light quanta in the theory of radiation) will not necessarily be constant, since there might be processes of creation or destruction of those light particles."

In at least one sense this was a significant departure from the original hypothesis used to interpret positronium decay, because up to that point charged particles were only thought to undergo creation or annihilation pair-wise with their respective antiparticles. If one continues to doubt, as in the preceding chapters, that electrons and positrons are really destroyed in

positronium decay, however, then it is only consistent to question the very similar hypothesis made by Fermi in explaining the β decay phenomenon. Yet if one insists on the continuous existence of the electron in an alternative theory, one must face up squarely to the need for finding a potential which is strong enough to overcome the undeniably high kinetic energy the electron would possess in the small volume occupied by a nuc1eus. The problem is more severe than this, however, because in taking this approach one also must provide a suitable explanation for the role of the antineutrino itself in β decay. Before considering these points further, however, a few additional details of the experimental observations should be carefully considered.

A whole series of nuclear reactions could be found which are closely related to one another. Written in the form of reactions these are:

| | | |
|---|---|---|
| β decay: | $n \rightarrow p^+ + e^- + \bar{\nu}$ | |
| β+ decay | $p^+ \rightarrow n + e^+ + \nu$ | |
| electron capture: | $e^- + p^+ \rightarrow n + \nu$ | II. 1 |
| $\bar{\nu}$ absorption: | $\bar{\nu} + p^+ \rightarrow n + e^+$ | |
| ν absorption: | $\nu + n \rightarrow p^+ + e^-$ | |

The notation is schematic, with $p^+$ and n generally denoting constituents of heavier nuc1ei actually present.

As always is the case, the classical chemist's balanced equation becomes a casualty of the assumption that matter such as electrons or neutrinos can be created or destroyed. None of the above five reactive equations is balanced in the traditional sense. In the first, an electron and antineutrino are created, while in the second, both $(e^-,e^+)$ and $(\nu, \bar{\nu})$ pairs are created, with subsequent annihilation of $e^-$ and $\bar{\nu}$ in the process of neutron formation. In the third, a $(\nu, \bar{\nu})$ pair is created and then the neutrino plus a captured electron from an associated atom are destroyed. Finally, an $(e^-,e^+)$ pair is created in the fourth reaction, followed by the annihilation of $e^-$ and $\bar{\nu}$ in the neutron formation, while in the last case the neutron decomposition produces the

same two particles, only to have the antineutrino destroyed along with the neutrino which induces the reaction.

The hypothesis of mass-less particle-antiparticle binary systems is only partially successful in restoring particle balance to these equations, but strict adherence to the original definition of elements advocated by Boyle and others before him completes the process. In particular, when we surrender the idea that material particles can be created or destroyed in such interactions, we are forced to conclude that *the neutron is not an element at all.* Rather, it is a compound of its known decay products, i.e. $p^+$, $e^-$ and $\bar{\nu}$, akin to a tri-atomic molecule in the usual chemical notation. The fact that the inertial mass of the neutron is greater than that of its separated constituents is perhaps surprising, but according to the Einstein mass-energy equivalence principle of his Special Theory of Relativity (STR), this is what one must expect from the fact that the dissociation energy of the neutron is negative (-0.8 MeV). There is considerable precedent for such a situation, namely in the description of excimer complexes, which has had a great impact in the field of laser research. Excimers are bound (meta-stable) molecular states for collections of atoms with a ground state that is characterized by a repulsive potential curve and is therefore unbound. The fact that the neutron undergoes spontaneous decay is perfectly consistent with this analogy. Only the energies involved are far greater than those released in excimer lasers. In the language of scattering theory, the neutron is thus a resonance with a relatively long lifetime. The nature of the forces which hold the neutron together in this meta-stable state constitutes a major unanswered question in this model, but in this respect the situation is wholly similar to that for the $e^+e^-$ tight-binding state which has been associated with the photon in Ref. 1.

For the sake of clarity the conventional use of the term “elementary particle” for the neutron and related systems will continue to be made below.[11] Nevertheless, it is well to keep in mind that the definitions of atoms and elements that Democritus and Boyle espoused exclude the neutron from this classification because of its instability. With these preparatory remarks, we can rewrite the five β decay reactions given in eq. (II.1) in completely balanced form:

i´) $p^+e^-\nu(n)\rightarrow p^+ + e^- + \bar{\nu}$

ii´) $e^+e^- + \nu\,\bar{\nu} + p^+ \rightarrow p^+e^-\,\bar{\nu}\,(n) + e^+ + \nu$

iii´) $\nu\,\bar{\nu} + e^- + p^+ \rightarrow p^+e^-\,\bar{\nu}\,(n) + \nu$ II.1´

iv´) $e^+e^- + \bar{\nu} + p^+ \rightarrow p^+e^-\,\bar{\nu}\,(n) + e^+$

v') $\nu + p^+e^-\,\bar{\nu}\,(n) \rightarrow p^+ + e^- + \nu\,\bar{\nu}$

In this way the number of distinct particle-antiparticle binaries has been kept to three in number, namely $e^+e^-$ , $\nu\,\bar{\nu}$ and $p^+p^-$. The need for $n\bar{n}$ pairs is thereby avoided by virtue of the tri-atomic composition ascribed to a single neutron itself. Thus neutron-antineutron production involves the interaction of each of the three elemental binary systems, i.e.

$$p^+p^- + e^+e^- + v\bar{v} \rightarrow (p^+ + e^- + \bar{v}) + (p^- + e^+ + v) \rightarrow n + \bar{n} \qquad \textit{II.2}$$

The economy of the assumptions in the above model is especially desirable when one attempts to construct a quantitative theory which is capable of generating wave-functions and properties for such mass-less particle combinations. The elemental balance in the above equations also has the advantage of making the additional assumption of charge conservation unnecessary. Since charge is an intrinsic property of each particle, never to be altered in the course of physical transformations, its conservation is always ensured thereby. In addition, we must continue to assume that energy and each of the components of the net linear and angular momentum are conserved in such processes, consistent with STR and the mass-energy equivalence principle. More generally, it can be said that there is an unwritten rule in the physical sciences according to which the number of theoretical assumptions should be held to an absolute minimum. In practical terms, an irrefutable contradiction based on experimental evidence should be established before adding any new postulate to the theoretical framework in whatever form. In this sense we can take a lesson from the conservation laws of classical physics, according to which a truly remarkable variety of observations is understandable on the basis of only three fundamental principles.[12]

## III. THE NEUTRINO

The neutrino is associated with many puzzles that are still unresolved over 80 years after its discovery. It apparently bears no electric charge and thus is unaffected by electric fields. It also seems to be essentially mass-less, although this conclusion itself has been the subject of much controversy. During his lifetime Pauli compained[13] that the upper limit for its rest mass was set too high based on what was already know then. For a long time it was something of a bookkeeping device, merely accounting for energy otherwise missing in the β decay spectrum. In 1956 the first successful experiment involving neutrinos as reactant species was performed by Reines and Cowans[14] and this achievement allowed considerably more confidence that the particle was more than a theoretical construction. In essence, Reines and Cowan[14] carried out the first antineutrino absorption reaction [eq. (iv)] in the laboratory. Extreme sensitivity was required. The actual mode of detection was the appearance of a photon signaling the reaction of the positron product with an electron, but the time between positive events was typically 30 hours. As a result it was clear that antineutrinos are extremely penetrating, with a chance of one part in $10^{12}$ of being captured while passing through the earth along a diametric ray. The almost complete absence of ionization products observed during the experiment also led to the conclusion that the particle has virtually no magnetic moment, but this raised difficult questions about what forces actually govern the neutrino's behavior. The lack of either electric charge or a magnetic moment also left uncertain the exact relationship between $\nu$ and $\bar{\nu}$ themselves, since previously every other particle could be distinguished from its antiparticle on the basis of these properties.

An answer to this question was provided by Lee and Yang[15] when they concluded that parity might not be conserved in beta decay. A decisive experiment[16] suggested by their work was carried out shortly thereafter by Wu *et al*. on the longitudinal polarization of decay electrons emitted by the $^{60}$Co nucleus in the presence of a strong magnetic field at low temperature, using a method proposed earlier by Rose and Gorter.[17] As $^{60}$Co decays into $^{60}$Ni the nuclear spin changes by one unit from I=5 to 4, and is referred to as a Gamov-Teller transition,[18] as opposed to the other possibility (Fermi transition) with $\Delta$I=0. Because only the lowest-energy nuclear spin component is occupied in the presence of a strong magnetic field at low temperature, it was clear that the emitted electron must be ejected with its spin parallel to that of the $^{60}$Ni nucleus. Under these conditions, a preferred direction for the departing electrons could be detected, namely

opposite to that of the nuclear spin. One can summarize this result by saying that the electron behaves predominantly as a left-handed screw in β decay. In addition, subsequent studies of nuclear recoil indicated that the antineutrinos turn exclusively as right-handed screws[19] in the same experiment.

Out of these investigations came a new theory of the neutrino proposed independently by Lee and Yang[20] ,Landau[21] and Salam,[22] according to which the neutrino and antineutrino could be distinguished on the basis of their respective left- and right-handedness, or alternatively their negative and positive helicities (defined as $I \cdot p/|p|$). It was referred to as the two-component theory of the neutrino because it postulated that certain spin-momentum orientations are forbidden to this particle. The idea was not entirely new to theoretical physics as Weyl had proposed something very similar[23] nearly 30 years earlier from strictly mathematical considerations based on STR. Since the parity P was not conserved in this theory, Pauli initially criticized it.[24] The observed asymmetry in the $^{60}$Co experiments of Wu *et a1*. led him to reverse his position,[25] however, although he went to some length at the time to express his amazement at this development.

Charge conjugation is also violated according to the two-component neutrino theory, but Yang noted that the product CP should still be conserved,[26] i.e. by simultaneously inverting the coordinate system and changing all particles into their antiparticles, so that a left-handed $\nu$ becomes an (observable) right-handed $\bar{\nu}$. A key assumption in this theory was that the neutrinos have zero rest masses, a possibility, which, as already mentioned, has thus far not been refuted by experiment. The puzzle was later further complicated, however, by new experiments,[27] also involving neutrinos, which provided strong evidence that even CP is violated in β decay.
Since a theorem by Schwinger,[28] Luders[29] and Pauli[30] proved that the product of CP and the time reversal operation T should be conserved under the most general of circumstances, this observation seemed to indicate that neither T itself nor any two of these operators together are conserved in such processes.

The interpretation of the polarization phenomena discussed above has received support from a wide variety of other experimental observations, including studies on free neutron decay[31] and on muons.[32-33] In the first instance, it was found that the proton is predominantly left-handed, which ultimately led to the convention of referring to the particle emitted in neutron decay as the antineutrino $\bar{\nu}$ rather than the neutrino $\nu$, as originally suggested by Fermi. In this way all

antiparticles present in nuclei (and anti-nuclei) are right-handed, while their counterparts in charge conjugation are left-handed. Another important distinction between $\nu$ and $\bar{\nu}$ was demonstrated by the fact that antineutrinos do not undergo the capture reaction of eq.(II.1.v) upon interaction with neutrons.[34] Attempts to observe a double beta decay in which no antineutrinos are emitted[35] have also never been successful. In the present context it should be noted that this result is also consistent with the belief in a definite composition for the neutron needed to achieve particle balance in the five beta decay reactions listed in the preceding section.

Another puzzle connected with neutrinos was discovered in 1964 by Bacahl and Davis[36] as a result of their investigation of nuclear fusion processes on the sun. The fusion reaction is closely related to eq. (II.1.iii), with neutrinos being set free in the process. Detailed knowledge of the solar reaction profile enables a relatively precise calculation of the magnitude of the associated neutrino flux reaching the earth, but the above experiments have invariably indicated a large discrepancy in the computed value. Less than half the expected neutrinos are ever observed. In addition there is evidence that the neutrinos involved in the $\beta$ decay of muons are not the same[37] as those associated with neutrons and heavy nuclei. One distinguishes then between electron ($\nu_e$) and muon ($\nu_\mu$) neutrinos, and there is also theoretical evidence[38] that a third type (or flavor) also exists, namely the tau neutrino ($\nu_\tau$).

Yet the greatest puzzle of all in this connection seems to be why such charge-less, extremely penetrating particles are involved in almost every high-energy reaction ever observed. Given this situation, the question of whether neutrinos can be created or annihilated seems a secondary issue, but one that nonetheless permeates any relevant theoretical discussion of the nature of these particles, beginning with the Fermi theory of $\beta$ decay put forward over a half-century ago.[8]

## IV. NUCLEAR STRUCTURE AND THE STRONG INTERACTION

The theory of nuc1ear structure relies firmly on the assumed elemental nature of the neutron.[39] Any nucleus can be described as containing a definite number of protons and neutrons, thereby providing a means of discussing nuclear reactions with balanced equations along the lines foreseen by Boyle in his general theory of chemical transformations. The fact that the neutron undergoes spontaneous emission complicates this simple picture, however, suggesting an alternative interpretation according to which the real elements are the electron and neutrino, in

addition to the proton. The proton itself has been theorized[40] to decay as well, but careful experiments[41] have never found any positive evidence for this expectation. By any reckoning, the proton is an extremely stable substance, with a lifetime far longer than that of the neutron.

The question of the nature of the forces which hold nuclei together is still largely open. One refers to them collectively as the strong interaction, a term which above all takes cognizance of the fact that the forces in question have properties which are far different from either the electromagnetic or gravitational type. In the time of Rutherford, three nuclear decay modes were well known, involving $\alpha$-, ß- and $\gamma$-rays respectively. In the meantime one can add at least two more decay products to this list, namely neutrons and the small nuclei which result from fission of their heavy counterparts. It is interesting to note that the creation-annihilation hypothesis is only invoked to explain two of these five possibilities, the $\beta$ decay discussed in the preceding two sections, as well as the gamma or photon emission process. In the other three cases the decay products have much larger rest masses, and thus are thought to be present in the original nuclei prior to decomposition as well as subsequent to it.

As noted previously, the justification for this distinction in the case of electron ($\beta$) emission is the belief[8,10] that no potential could be sufficiently attractive to outweigh the large kinetic energy such a light particle would have in the confines of the nuclear volume. Yet the fact that the forces binding nuclei together are a matter of some uncertainty themselves lends a degree of tentativeness to this conclusion. On the other hand, if we assume that neither the electron nor anything else can be annihilated under any circumstances, we are left with no choice but to look for a potential that is capable of binding an electron so strongly. In Refs. 1-2, a similar line of reasoning has led to the conclusion that the electron and positron form a much more tightly bound system than anything inferred from a solution of the electrostatic Schrödinger equation, which corresponds to a binding energy of 1.02 MeV. For comparison purposes, it is interesting to note that nuclei are bound together on the average by roughly 8 MeV/nucleon, a number which is somewhat larger but of the same order of magnitude as that required in the interpretation of a tightly bound electron-positron system. Moreover, it is over 200 times smaller than the $p^+p^-$ binding energy which must be assumed on the same basis, namely 1.85 GeV.

The possibility may thus exist that in searching for a potential which binds particles with their antiparticles together with such force as to cause a total loss of mass relative to the reactant species, one is approaching the same goal as has long been sought in the context of nuclear

interactions. Merely saying that the energies given off in the particle-antiparticle interactions can be computed on the basis of STR and the mass-energy equivalence relation does not after all conform to the standards physicists have adhered to in dealing with other types of elemental processes. In almost every other conceivable situation, a detailed description of the forces involved in causing systems to be either attracted or repelled by one another has become an important goal of subsequent research, and this line of approach has been rewarded with genuine scientific progress on numerous occasions in the past.

With this motivation, it is instructive to review what is known about the strong interaction from experimental studies of nuclear processes and what progress has been made in interpreting these observations theoretically. To begin with, one has a good idea from scattering experiments what the associated potential must look like. One distinguishes three types of fundamental processes, namely pp, nn and pn scattering. Especially when Coulomb repulsion effects are subtracted for proton pairs, it is found that the potential that causes nuclear binding is essentially the same in all three cases. On this basis the nuclear force is assumed to be charge-independent and, by inference, quite distinct from the electromagnetic force. Some evidence of at least a secondary role for electromagnetism is suggested by the existence of nuclear magnetic and quadrupole moments, not to mention the positive electronic charge of all nuclei, but it is felt that such effects can be dealt with separately. One also might say that the discovery of the neutrino's involvement in nuclear reactions supports the charge-independence assumption in view of this particle's lack of either electric charge or magnetic moment.

Probably the most influential conclusion based on the above experimental findings was drawn by Heisenberg[42] in 1932. He postulated that the neutron and proton were simply two different states of the same system (nucleon) which would be perfectly degenerate in the (hypothetical) absence of the electromagnetic interaction. Out of this hypothesis ultimately evolved the theory of isospin,[43,44] which has had a very great impact in both nuclear physics and the corresponding study of elementary particles.

Some of the first attempts to describe the nuclear force employed a non-relativistic Schrödinger equation[45] and an empirical potential of the type indicated by the scattering data. The most famous of these treatments employs the Yukawa exponential potential[46,47] $V(r)=-V_o(\alpha/r)\, e^{-r/\alpha}$, where r is the distance between nucleons (protons or neutrons), $V_0$ characterizes the depth of the potential and $\alpha$, its range. The non-relativistic kinetic energy $p^2/2M$ is deemed

appropriate for this purpose in view of the relatively large rest masses of the nucleons. By varying the two parameters $V_0$ and $\alpha$, it is possible to fit a certain number of experimental results on the basis of the corresponding solutions of the Schrödinger equation, but the Yukawa procedure has never evolved into a truly quantitative theory of nuclear binding. Nonetheless the simplicity of the model on which it is based has led to much valuable insight into the subject. In particular, it emphasizes the extremely short range of the effect through the exponential form of the corresponding potential. For $r \gg \alpha$, the potential is effectively damped to zero, whereas for $r \leq \alpha$ it varies predominantly as $r^{-1}$. In the latter respect it is similar to the Coulomb potential, but the large magnitude of $V_0$ gives it a much stronger weight in the short internuclear distance region where the binding between nucleons is at its greatest. True to the creation-annihilation concept, no recognition is given to the possibility that either electrons or antineutrinos are involved directly in the nuclear binding process.

Another approach to the nuclear problem is modeled after quantum electrodynamics, focusing on the idea that the Coulomb force involves the exchange of photons between the interacting charged species. Using arguments based on the Heisenberg uncertainty principle, Yukawa postulated[47] that the particle regulating the: nuclear force must have a correspondingly short range, which in turn keeps it from being directly observable under normal circumstances. By assuming a velocity close to the speed of light, Yukawa was led to predict a rest mass for the new particle which was intermediate between that of an electron and a proton (200 $m_{oe}$ was first proposed). The Coulomb force's range is thereby thought to be infinite, consistent with the null rest mass of the photon. The theory received support from experiment a decade later when the $\pi$ mesons ($\pi^+$ and $\pi^-$ , with a rest mass of 273.13 $m_{oe}$ each) were discovered[48] and shown to interact with nuclei. Attempts to construct a quantitative theory analogous to quantum electrodynamics were not generally successful,[49] however. The use of low-order perturbation theory to compute the mesonic interactions was concluded to be inadequate,[50] despite the fact that a similar approach in quantum electrodynamics is found to be quite accurate. The anomalous spin magnetic moment of the electron amounts to a deviation of only one part per thousand[51] from the uncorrected value of Dirac theory, and can be predicted to an accuracy of six to seven significant figures with the perturbative inclusion of quantum electrodynamical effects. By contrast, the proton magnetic moment was measured by Stern and coworkers[9,52] to be 2.5 times the theoretical value deduced on the basis of the respective masses of the proton and electron. Nonetheless, the

π meson theory accounts for the known observations at least qualitatively, and the concept of a cloud of such (virtual) particles surrounding the proton has received broad acceptance, especially after it was found to be quite consistent with the results[53] of elastic scattering of high-energy electrons by nucleons.

Another theory which has had a great impact in this field is the nuclear shell model proposed by Goeppert Mayer[54] and Jensen.[55] This work concentrates on certain regularities in the properties of nuclei as a function of the numbers of their constituent protons and neutrons, particularly the values of the nuclear spin and of the electric quadruple and magnetic dipole moments. It is possible to predict trends in these data by assuming that protons and neutrons are added in shells according to an *Aufbau* principle similar to that used in atomic structure theory to explain observed trends in the periodic table of the elements. The shell model traces its origins to the independent-particle or Hartree model[56] and also to the Wigner coupling scheme,[57] which was successful in classifying nuclear states with the aid of a spin-independent zero-order Hamiltonian. Each shell can be identified with definite angular momentum quantum numbers l and j, with their familiar degeneracy factors determining degrees of maximum occupancy in accordance with the Pauli principle. Isomorphic pairs of shells are reserved for the individual protons and neutrons, whereby the stability of the proton shells always appears to be somewhat lower, reflecting the effects of their participation in the repulsive Coulomb interaction. On the basis of these considerations it was possible to explain the occurrence of certain "magic numbers" which had been noted empirically for the atomic and mass numbers of nuclei in surveying the aforementioned properties.[39,54,55]

For the present discussion the most interesting aspect of the shell model is the conclusion that forces are at work which are akin to the spin-orbit coupling interaction familiar in the classification of fine-structure effects observed in atomic spectra. Terms of this nature arise from a perturbative reduction of the Dirac equation,[58,59] as shown by Breiti,[60] Pauli[61] and others.[62]. The origin of the spin-orbit interaction can be traced to the **v x E** term which appears as an adjunct to the magnetic field[63] upon application of a Lorentz transformation to the classical Maxwell equations.[64] The form of the spin-orbit interaction involved in nuclear binding is quite distinct[65] from the latter, however, because the magnitude of the effect is found to be much larger than one would expect on the basis of the small magnetic moments of nucleons.

The fact that the quantum numbers employed in such models are typical for a central potential has sometimes been remarked[66] to be inconsistent (or at least not obviously consistent) with the picture of the nucleus as a collection of protons and neutrons, i.e. without a fixed center comparable to that known for atoms. The success with which the experimental data can be ordered on this basis belies such criticism, but the *ad hoc* nature of its central potential assumption combined with its inability to obtain truly quantitative agreement with measured results have tended to prevent the nuclear shell model from being a complete triumph.[67] Nonetheless, the ability of such methods to account in an extremely detailed manner for the observed regularities in the properties of complex nuclei is a fact which must be reckoned with in trying to construct a more quantitative theory of nuclear structure in the future.

It is impossible to do justice to the progress made in theoretical nuclear physics with such a brief survey, but at least one more series of developments needs to be considered in the present context. The liquid drop model, which was proposed by Bohr[68] in 1936, points out similarities between nuclear matter and the liquid state. This model is based first and foremost on experimental measurements of radii and binding energies of a large series of nuclei, and as such is reminiscent of the theory of the chemical bond introduced into molecular physics and chemistry by Pauling.[69] Probably the most important result of an experimental nature is the finding that the magnitudes of nuclear radii are approximately proportional to the third root of the atomic mass number,[70] or alternatively, that the nuclear volume increases proportionately to the number of constituent nucleons. This is a very simple result which gives the impression that, even more than in molecular systems, for which a similar model is already very successful, the components of nuclei are not at all compressible. If one knows the volume of one nucleon (or better, the volume of two nucleons that are tightly bound together), it is possible to extrapolate this result to obtain the size of the entire nucleus. This close-packing condition is another important characteristic that must be derivable from the nuclear potential. To this can be added that the interactions between nucleons in the same shell are found to be much stronger than between those in different shells.[71]

What do the theories of nuclear structure tell us about the forces involved in positronium decay? If the pervasive view that nuclear forces occur primarily between elemental protons and neutrons is correct, the answer must be that there is little or no relation. On the other hand, if the neutron is looked upon as a compound of its decay elements, the electron, proton and

antineutrino, a more significant relationship can be anticipated. For one thing, it then becomes essential to account for interactions between electrons and protons inside the nucleus, in which case it is not difficult to imagine that there might be similarities between these forces and those which might conceivably cause the electron and positron to be bound together much more tightly than in positronium. *Moreover, once this path is taken, it is only consistent to assume that the antineutrino also interacts strongly with either the electron or proton or both.*

The above possibilities will be explored in subsequent sections, but for now it is worthwhile to consider how such a neutron hypothesis would affect the simple bookkeeping procedures used to describe nuclei in terms of the number of constituent protons (Z) and neutrons (N). In the first place, the number N becomes identical with the number of electrons in the nucleus, as well as the number of anti-neutrinos. The number of protons is no longer Z, but Z+N, while the total number of constituent particles is not Z+N, but Z+3N. The latter difference is 2N and thus is always even. This fact means there is no alteration in nuclear statistics as a result of such an assumption, a point used by Pauli[7] in postulating the existence of the neutrino in the first place. Otherwise, particle balance in nuclear equations is ensured in this approach (even in cases for which β-decay occurs in one form or another) if one inserts the appropriate particle-antiparticle binary systems as needed. When photons are involved, this means adding mass-less $e^+e^-$ species as well. Conservation of energy can be handled in the usual way, in accordance with STR.

The resulting elemental balance in the description of such processes would be perfectly in line with the atomistic theories originated thousands of years before, as well as the more precise formulations proposed by Boyle and Dalton much later. What is clearly missing in the discussion to this point is a quantitative specification of the possible forces which are involved, one that would allow an accurate prediction of the differences in the masses of products and reactants in such processes. In its simplest form, this would mean the *ab initio* computation of the binding energy of the deuteron relative to its constituent proton and neutron separated to infinity, or even more fundamentally, with respect to two protons, an electron and an antineutrino at rest in their respective elemental states.

## V. METASTABLE PARTICLES

The hypothesis that the neutron is a tri-atomic compound composed of each of its decay elements has far-reaching consequences beyond the realm of nuclear physics. There are many other meta-stable systems which are listed among the elementary particles, the smallest of which are the two muons and the three pions. They differ from the neutron mainly in that their lifetimes are much shorter ($10^{-10}$ - $10^{-16}$s) and that their decomposition energies are far greater (over 100 MeV in the case of the muons). In addition, their decay products do not include a proton, although electrons and neutrinos are emitted. Beyond this there are a large number of other meta-stable particles known, referred to as mesons and baryons on the basis of their rest masses.

In 1964 Gell-Mann[72] and Zweig[73] suggested that all of them could be interpreted as being composed of hypothetical fermions called quarks (or aces[74]) and their antiparticles. Probably the most novel aspect regarding this suggestion is the assumption that the new particles should possess non-integral electric charges, either $\pm\frac{1}{3}$e or $\pm\frac{2}{3}$e. In the meantime the number of quarks and anti-quarks has grown to twelve[72] and the idea has achieved wide acceptance. Fermi and Yang[75] and Sakata[76] tried to identify a common denominator of elements in terms of the proton, neutron and lambda from which to construct all other known particles. This approach had only been partially successful, primarily because certain combinations which appear reasonable in this theory have never been observed. The quark concept is clearly consistent with the view that all matter consists of elements, but there is still some question as to whether particles of non-integral electric charge actually exist. One reads in the literature, to be sure, that individual quarks have been discovered experimentally, but the evidence is always indirect, referring to a successful prediction derived from the theoretical model.

The idea that all observed meta-stable particles might be composed of their decay products, either as elements or as other combinations thereof, is not necessarily in conflict with the quark model, although it is also not obviously consistent with it either. Just because a proton might have a quark composition, in other words, in no way precludes the possibility that a quantitative theory which employs protons as elements might not exist. The crucial test is whether the properties of the various meta-stable particles can be successfully computed with the help of any such theory. In the present context this would mean computing the rest masses of the muons and pions, for example, just as for the neutron, deuteron and other heavier nuclei, in a similar manner

as one calculates binding energies and ionization potentials of atoms and molecules with the help of quantum electrodynamics.

There is a new aspect introduced by the study of the other meta-stable particles, however, namely that they exhibit multiple decay channels. Whereas the neutron is always observed to produce a proton, electron and antineutrino upon decomposition, several sets of decay products are known for most other meta-stable systems. *Does this not speak against the concept of a unique composition for them in terms of certain elements?* Not necessarily, because if mass-less particle-antiparticle binaries exist, other possibilities must be taken into account. One only has to look at a typical β decay process discussed earlier in Sect. II to see how the addition of $e^+e^-$, $p^+p^-$ or $\nu\,\bar{\nu}$ species can result in a particle balance in reactions which are conventionally interpreted in terms of the creation-annihilation hypothesis. The situation can be compared to that faced by the early chemists several centuries ago when a new substance was to be analyzed. A major distinction between the two cases exists, however, namely that when very large decay energies are involved, the masses of the participating systems cannot be used unambiguously to confirm a given structure. In other words, the concept of molecular weight is much less useful in elementary particle physics than it is in chemical investigations. All one knows with certainty is that if the rest mass of the combined system is less than that of its components if it is a bound system, otherwise it must be meta-stable. The experimental fact that *all* the particles which decay into protons, electrons or neutrinos *have rest masses which are larger than the total for their respective fragments* again speaks clearly for the analogy with molecular excimers mentioned earlier. It remains to be seen whether a system of forces can be found which is capable of temporarily binding electrons and neutrinos together in muons and pions and the like, even though the total energy /mass in the relevant states is much higher than that for the corresponding decay particles at rest.

## VI. MAGNETIC PHENOMENA: NEUTRINO INTERACTIONS

The charge-to-mass ratios appearing in the Breit-Pauli relativistic corrections are closely related to the magnetic moments of the corresponding particles. The Bohr magneton ß of a system is defined as $h/4\pi$ times the charge-to-mass ratio and the magnetic moment is usually measured in terms of this quantity. Strictly speaking, the magnetic moment is a vector proportional to the angular momentum of the particle, and in classical theory the Bohr magneton

would simply be the corresponding proportionality constant. Magnetic measurements on electrons in atoms have shown that such a straightforward relationship is oversimplified, and have led to the introduction of a supplemental factor, the gyromagnetic ratio g. The discovery of spin angular momentum[77] brought with it the equally surprising result of g = 2 for the electron's spin magnetic moment.[63,78] Later on, it was found that the correct value is actually slightly greater than this result,[51,79,80] however. Moreover, the corresponding value for the proton[9,52] was measured to be several times larger than that of the electron.

The question that arises in the present context is whether the observed g values should be included explicitly in the Hamiltonian used to describe the particle-antiparticle states which have been suggested above. Since the scaling results discussed in the last section no longer hold if one multiplies the q/m values in the Breit-Pauli terms with a different factor for each type of particle, however, it seems clear that the measured g values should not appear in such a Hamiltonian. Moreover, this choice is also consistent with the accepted explanation[80] for the observed non-integral g values for spin magnetic moments, namely that they arise because the corresponding particle is affected by external factors (virtual photons or virtual mesons) which prevent the measurement of its properties in a pure (bare) state.

Having made this decision, it can be noted that the only experimental constants appearing in the damped Breit-Pauli Hamiltonian are the electric charges and rest masses of the interacting particles. It is therefore clear how to define a specific Hamiltonian for a system involving given numbers of electrons and protons and their respective antipartic1es, particularly the $e^+e^-$ and $p^+p^-$ combinations of primary interest. There remains an important question, however, namely *how to deal with neutrino interactions.* The obvious approach is to simply substitute the corresponding values for the charge and rest mass of such particles and to solve the corresponding Schrödinger equation, but the observed electrical neutrality of the neutrino seems to prec1ude any chance of success for such a procedure. There is one possibility worth considering, however, *provided the rest mass of the neutrino is also exactly zero,* as seems possible based on experimental observations. In that case, the charge-to-mass ratio cannot simply be computed by dividing the above two quantities because the quotient of zero with itself is undefined. From a purely theoretical standpoint this eventuality would seem to invalidate the conclusion that the Breit-Pauli terms necessarily vanish for the charge-less neutrino, i.e. because $q_\nu/m_{o\nu}$ itself might still be different from zero. Such a possibility seems to clash with the experimental observations

bearing on the magnetic moment of the neutrino, however, which indicate that it is also of vanishingly small magnitude,[11] thereby implying that the corresponding charge-to-mass ratio is still effectively zero.

The idea that the neutrino might possess a non-zero magnetic moment which could help to explain its observed activity in nuclear reactions is not new, and was in fact proposed by Pauli[81] as a general property of mass-less neutral particles based on the Dirac theory.[82] He later stated[7] that such a hypothesis "doesn't seem to me at all well founded." The history of the relationship between the magnetic moment and the charge-to-mass ratio of a given particle has not been without its surprises, however. As already noted, the gyromagnetic ratio, for example, is an experimental quantity *invented* for the expressed purpose of accounting for discrepancies between observations and overly simplistic theoretical models. It is therefore wise to exercise some care in making conclusions about a particle's charge-to-mass ratio based on magnetic measurements alone.

In this connection it is important to recall that the Special Theory of Relativity (STR)[83] emphasizes that electric and magnetic fields are intertwined, with their respective magnitudes differing from one inertial system to another moving relative to it. Viewed from the standpoint of the affected particle, *there is never a magnetic force acting upon it,* because it is by definition stationary in its own inertial system ($v = 0$).[84] Considerations of this nature led Lorentz[85] to point out that a Galilean transformation does not leave the laws[86] of electricity and magnetism invariant, and they eventually enabled Einstein[83] to formulate STR in the first place. By this reasoning, it would seem to follow[87] that since a charge-less particle *can never experience an electrical force* according to Coulomb's law, it should consequently experience *no electromagnetic force at all in its own inertial system.* Further, because of the principle of relativity, this conclusion leads to another, namely that a charge-less particle must be unaffected by any electromagnetic field, regardless of the relative state of motion of the observer to it. If an electrically neutral system is composed of several charged particles, the analogous conclusion is negated by the possibility that its individual components move at different velocities, but for a true (charge-less) element this situation is excluded.

In order for $q/m_o$ to be non-zero for the neutrino, however, its rest mass must be exactly zero and (for $E \neq 0$; see Ref. [1]) it must move with the velocity of light ($v=c$). In this case a Lorentz transformation to a different inertial system is no longer defined since $\gamma = (1 - v^2/c^2)^{-1/2}$

is then infinite. Consequently, the above argument based on STR does not really prove that a *charge-less, mass-less* particle is unaffected by electromagnetic fields in *all* inertial systems. It seems only prudent to note, however, that the preceding discussion has been based exclusively on classical electrodynamics. It might be anticipated that a truly definitive answer to the question of whether a mass-less neutrino can possess a non-zero charge-to-mass ratio $q_\nu/m_{o\nu}$ without exhibiting a magnetic moment in the usual sense can only be obtained on the basis of a quantum mechanical formulation of this problem.

The possibility of a non-zero charge-to-mass ratio of the neutrino is interesting in several other contexts as well, however. It provides a ready explanation for the existence of both a neutrino and an antineutrino, for example. If $q_\nu/m_{o\nu}$ is non-zero, then different signs are possible for it, just as for charged species and their antiparticles. This characteristic might then be closely related to the helicity property of neutrinos measured in longitudinal polarization experiments[16,31-33]. It might also explain why there are apparently different types of neutrinos observed in neutron and pion-muon beta decays.[37] Neutrinos with distinct $|q/m_o|$ ratios would be expected to exhibit different behavior, despite their mutual lack of charge and rest mass.

In the present discussion, the point of immediate interest is whether the assumption of a non-zero $q/m_o$ value for the neutrino leads to a solution of the corresponding Schrödinger equation (with the exponentially damped Breit-Pauli interactions mentioned in Ref. [2]) for a $\nu\bar{\nu}$ complex which has zero binding energy relative to its separated particles. i.e. $-2m_oc^2$ in analogy to that of the $e^+e^-$ and $p^+p^-$ systems. Zero binding energy does not necessarily mean that a potential well does not exist. There could be a large barrier separating the two minima in this case as well as for $e^+$ $e^-$, and thus the $\nu\bar{\nu}$ system might also be characterized by a relatively small optimal inter-particle separation.[86] Calculations investigating this point will be discussed in the following sections.

## VII. EXTENDING THE XBPS MODEL TO NEUTRINO INTERACTIONS

In Sect. VI it was remarked that if the rest mass of the neutrino is exactly zero, its charge-to-rest-mass ratio might be non-vanishing despite its lack of electric charge. By providing the XBPS Hamiltonian [2] with such $q/m_o$ factors in the various exponentially damped Breit-Pauli terms it is then conceivable at least in theory for the neutrino to undergo attractive interactions with other particles. In this section we will test this possibility by means of explicit calculations.

The first question that arises is what value to use for $q_\nu/m_{o\nu}$. Consideration of the scaling arguments discussed in Ref. [2] shows, however, that this choice is not critical for the $\nu\,\bar{\nu}$ system itself. If we simply assume that the charge-to-mass ratios for the neutrino and antineutrino are equal but of opposite sign, we obtain a Hamiltonian (see Table I of Ref. [2]) with the following representative terms:

$$H(p, r, q_\nu, m_{o\nu}) = p\alpha^{-1} - \left(\frac{q_\nu}{m_{o\nu}}\right)^2 \alpha^2 r^{-3} F(p, q_\nu, m_{o\nu})\,. \qquad \text{VII.1}$$

This operator is identical in form to that employed in Ref. [2] except that all the terms with factors of $q_i$ or $m_{oi}$ which do not appear as ratios of one another are missing because of the assumed vanishing magnitudes of the neutrino's charge and rest mass.

With this special form of the XBPS Hamiltonian it is possible to scale the coordinates in a slightly different manner than before, namely with

$$p' = \frac{q_\nu}{m_{o\nu}} \frac{Q}{M_o}^{-1} p$$

$$\text{and } r' = \left(\frac{q_\nu}{m_{o\nu}}\right)^{-1} \frac{Q}{M_o} r\,, \qquad \text{VII.2}$$

so as to obtain the following result:

$$H(p, r, q_\nu, m_{o\nu}) = \left(\frac{q_\nu}{m_{o\nu}}\right)^{-1} \frac{Q}{M_o}$$

$$x\left(\frac{p'}{\alpha^{-1}} - \frac{Q^2}{M_o{}^2} \alpha^2 r'^{-3} F(p', Q, M_o)\right) \qquad \text{VII.3}$$

$$= \left(\frac{q_\nu}{m_{o\nu}}\right)^{-1} \frac{Q}{M_o} H(p', r', Q, M_o)$$

where H (p′, r′, Q, $M_o$) is the corresponding Hamiltonian in the primed coordinate system for particles with a different value of $|q/m_o|$. The operations are exactly the same as in the mass scaling procedure of Ref. [2], but the relative charge-to-mass ratios are involved instead of relative masses.

It must be recalled, however, that the simple relation between the two different Hamiltonians in this case only holds for $q_\nu = m_{o\nu} = 0$. Under these conditions the lowest possible energy eigenvalue for the $\nu\,\bar{\nu}$ system, i.e. $2m_{o\nu}c^2$. is exactly zero as well. As a result, a wave-

function $\psi(r)$ satisfying the corresponding Schrödinger equation with this eigenvalue can be converted by means of the above coordinate transformation [eq. VII.2] into an eigenfunction $\psi(r')$ of the primed Hamiltonian, which by virtue of the above scaling property also possesses the desired vanishing energy. In short, if we can find a Hamiltonian of this type which gives E = 0 as the lowest eigenvalue for the $\nu\bar{\nu}$ system with a given $|q_\nu/m_{o\nu}|$ value, we can easily repeat the process for any other charge-to-mass ratio. It can be noted that for this situation to hold, it is essential that the argument of the damping exponential operator F also contain a $q/m_o$ factor.

The next point that merits discussion is whether it makes sense to talk of a bound system with the same energy as its separated products, in this case E=0 . The answer is clearly yes as long as the combined and dissociated systems are separated by a reasonably large energy barrier. With reference to Fig. 1 of Ref. [2] for the $e^+e^-$ system, it is easy to construct an analogous total energy curve for the $\nu\bar{\nu}$ system which satisfies this requirement (Fig. 1). Because of the absence of the long-range Coulomb interaction in this case, we must expect that the energy first goes up as the neutrino and antineutrino approach each other from a long distance. The kinetic energy itself rises more slowly with decreasing separation than for the $e^+e^-$ system because it varies linearly with momentum for systems with no rest mass. As a result, one doesn't expect the atomic-like potential minimum for the $e^+e^-$ system in this case, but rather that the energy should increase steadily until very short inter-particle separations of roughly $r \cong \alpha^{3/2}$ (for $q_\nu/m_{o\nu} = 1.0$ a.u.). A maximum seems likely at that point, similarly as for $e^+e^-$ , because the Breit-Pauli short-range interactions start to change more rapidly than the kinetic energy. After this point it is simply necessary that the exponential damping halt the attractive tendency when the total energy has again reached the value of zero. The kinetic energy begins to dominate once more at still smaller separations, so that a potential well can be formed in which the $\nu\bar{\nu}$ system can exist indefinitely in the absence of external forces.

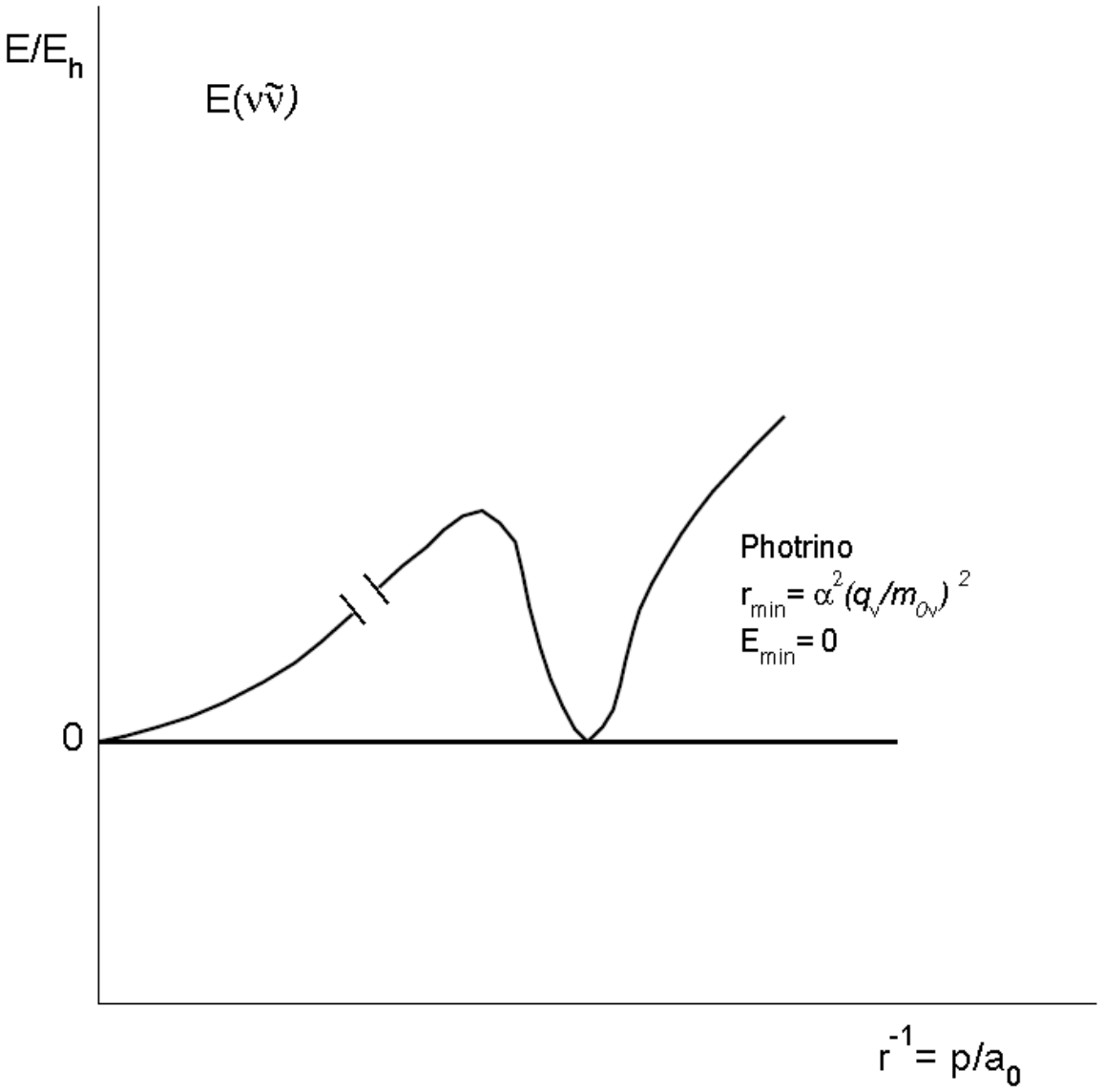


FIG. 1. Schematic diagram showing the proposed variation of the $\nu\,\overline{\nu}$ system's internal energy as a function of the reciprocal of the distance between the two constituent particles. Only one minimum is expected, in contrast to the $e^+e^-$ case shown in Fig. 1 ofRef. [1], at which point the total energy vanishes exactly, i.e. corresponding to a binding energy of $2m_{o\nu}/c^2$ for the tightly bound $\nu\,\overline{\nu}$ (photrino) system.

The present scaling argument tells us that it really does not matter what value is chosen for $|q_\nu/m_{o\nu}|$ to demonstrate this effect, but to obtain the most straightforward possible comparison with the $e^+e^-$ system, it is reasonable to take it to be unity. If the former 5s,5p basis is employed (see Ref. [2]) in the corresponding XBPS calculations, it is found that the zero-energy

minimum occurs for a damping constant A which is only slightly smaller than the corresponding $e^+e^-$ and $p^+p^-$ value (Fig. 2). This result is easily understood if one uses the optimum scale factor and A value derived from the $e^+e^-$ calculations, for which the lowest-energy eigenvalue is $-2m_{oe}c^2$. This means that the damped Breit-Pauli terms (Table I of ref. [2]) will have the same magnitudes for the $\nu\bar{\nu}$ system (with $|q_\nu/m_{o\nu}|$=1.0 a.u.) as for the $e^+e^-$ system. The Coulomb component is missing, however, as are the $m_oc^2$ terms in the kinetic energy. The two electronic $m_{oe}c^2$ terms appearing with minus signs in the kinetic energy expression for $e^+e^-$ make up the entire difference in the assumed binding energies of the two systems, so we can effectively ignore them in the following discussion.

This means that there are only two effects remaining which can cause a non-zero binding energy for $\nu\bar{\nu}$ when employing the same damping constant as before for $e^+e^-$. One of these favors the latter system (Coulomb attraction), while the other favors $\nu\bar{\nu}$ [i.e. pc vs. $(p^2c^2 + m^2c^4)^{1/2}$]. Of these, the Coulomb effect is larger in magnitude for the inter-particle distance range in question, but the energy difference (2680 hartree) is relatively small compared to the $e^+e^-$ binding energy. Thus to obtain an exactly zero binding energy value for the $\nu\bar{\nu}$ system in the same basis set, it is only necessary to lower the value of the damping constant A by 0.0007 a.u..

This result ignores translational effects, however, and so it is interesting to carry out the optimizations based on the $E_o$ expectation value, <H>-<T>, rather than on that of the total energy alone. As before with $e^+e^-$, when this is done the required value of A must increase in order to obtain the same binding energy as before. If the same value of A (1.7725 a.u.) is employed which gives the correct $2m_{oe}c^2$ binding energy value of -37557.7 hartree for $e^+e^-$ using the $<E_o>$ criterion, a total internal energy of 897.2 hartree results for $\nu\bar{\nu}$, which is about three times smaller than in the analogous treatment employing the optimal A value (1.0775 e ) for $e^+e^-$ based on the <H> computations.

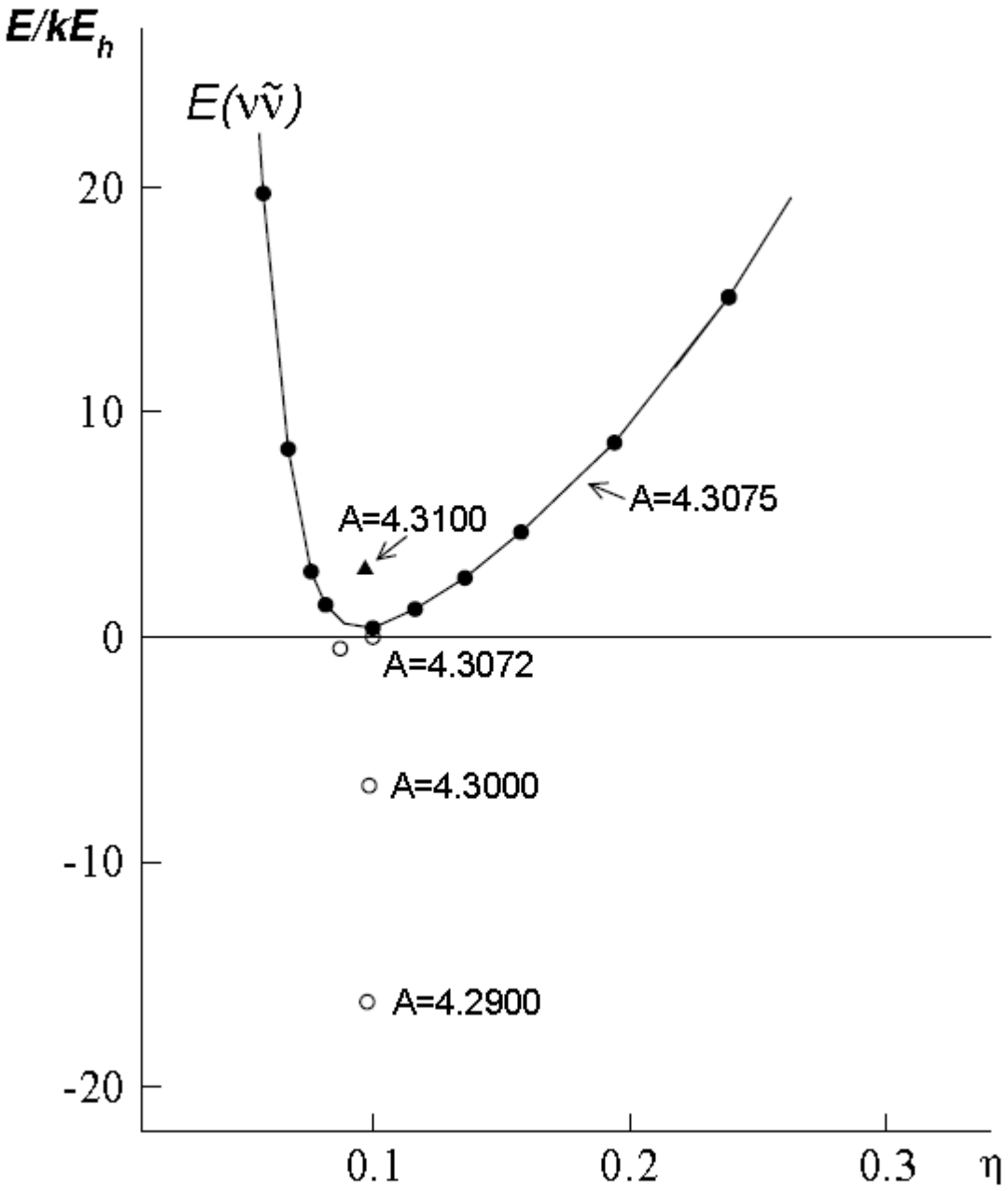


FIG. 2. Variation of the computed total energy (in khartree) of the $\nu\,\overline{\nu}$ system as a function of the 5s,5p basis set scaling factor η in the XBPS treatment for various values of the damping constant A. The horizontal line at the center of the diagram corresponds to the null rest energy ($2m_{o\nu}c^2$) of the system. A value of A is sought which leads to this energy result for the optimum choice of η. Results for several other A values are also shown for comparison. Note that the values of A in the diagram are 4.0 times larger than those given in original text due to a difference in definition.

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In view of the fact that the present full CI calculations fall short of providing exact solutions of the Schrödinger equations under consideration, it seems justified to conclude that for all practical purposes the same damping constant in the XBPS Hamiltonian leads to minimal binding energies for the three particle-antiparticle binary systems studied which are each equal to $-2m_oc^2$, in accordance with the expectations of STR. This result is exact for $e^+e^-$ and $p^+p^-$, but only accurate to a good degree of approximation for the $\nu\bar{\nu}$ system based on the calculations carried out thus far. Moreover, the second scaling theorem discussed above [eq. VII.3] shows that this situation holds for any choice of the charge-to-mass ratio assumed for the neutrino. The latter finding thus offers a means of describing more than one kind of neutrino in the present model, consistent with what has been inferred from the results of various experiments[37] and other theoretical considerations[38] (see Sect. III).

## VIII. NON-IONIZING PROPERTY OF NEUTRINOS

The total energy curve of Fig. 1 for a $\nu\bar{\nu}$ interaction derives its attractive characteristics from the short-range Breit-Pauli terms in the XBPS Hamiltonian by virtue of an assumed non-zero charge-to-mass ratio for the constituent particles. This assumption needs to be reconciled with the observation that neutrinos have essentially no magnetic moment. In at least one sense the total energy variation shown in Fig. 1 is consistent with the experimental findings related to this issue, however, namely with the lack of ionization exhibited by neutrinos and especially their ability to penetrate essentially unhindered through dense matter.[14]

For a particle to cause ionization it is necessary that it be attracted to an electron (or other charged system) to a close proximity. Because of the lack of a Coulomb interaction, the total energy for the hypothetical $\nu\bar{\nu}$ system should rise as the two constituents come closer together. Before the attractive Breit-Pauli terms can reverse this trend, it is necessary that the neutrinos approach each other very closely, to at least $r = \alpha$. The possibility of undergoing a long-range attractive force thus clearly distinguishes electrons (cf. Fig. 1 of Ref. [2]) from neutrinos (Fig. 1). If we look upon ionization as requiring some kind of orbital motion for the colliding (point) particles, this distinction can be crucial since it means that the centrifugal force must exceed any opposing attractive force in the $e^-\bar{\nu}$ or $\nu\bar{\nu}$ interactions at all but extremely small inter-particle separations. In this case, a hyperbolic trajectory would be expected almost universally. There is always a large centrifugal barrier which needs to be overcome, and the particles must come very

close to one another before this is feasible, even if a fairly large $q/m_o$ value is assumed for the neutrino.

An automobile with such a total energy profile could race down the highway with reckless abandon, certain to be repelled whenever it came very close to any other object. The only danger would be that it might approach something so closely at extremely high speed that the short-range attractive force would finally be able to overcome the repulsive force. Such a picture is very reminiscent of the behavior of neutrinos in the Reines-Cowan experiment,[14] in which an extremely small cross section for neutron formation of $\sim 10^{-43}$ cm$^2$ is observed.

The conclusion that a non-zero $q/m_o$ value necessarily implies a non-zero magnetic moment would also seem to overlook the possibility that the mass of a neutrino is only zero at rest. The ratio of the charge to the *relativistic* mass is a more relevant quantity in determining the results of magnetic interactions, and this is exactly zero whenever the neutrino is in motion because no corresponding change in its electronic charge occurs as a result. The dominant formula used in describing cyclotron dynamics, $p = mv = qBr$, leads to the conclusion that a charge-less particle cannot achieve orbital motion with a finite radius, for example. The key assumption in formulating the XBPS Hamiltonian is that the coupling constants are ratios of charge to rest mass, and without this provision the short-range minimum in either Fig. 1 or 2 would not be possible. In general, it should not be forgotten that virtually everything one knows about electromagnetism stems from experiments with individually charged particles (see also Sect. VI), even if the macroscopic system being observed possesses zero *net* electric charge.

Support for this view also comes from the classical experimental result that the magnetic force is no more short-ranged than its electric counterpart. The force per unit length on an electric wire, for example, varies as the product of its current and that of a neighboring wire and is inversely proportional to the first power of the distance between them.[88] There is little hint of a short-range effect in this behavior. The theoretical explanation for this result lies in the (nearly) constant velocities of the electrons involved in such experiments, which implies that the quantity $mvr/r \equiv l/r$ is constant as well. In quantum mechanical interactions the more typical situation is that l (or s) is constant, so that an energy term of order $l^2r^{-3}$ varies as the inverse cube of the inter-particle distance rather than as $r^{-1}$. The fact that particles don't always move at constant velocities opens up the possibility that related short-range interactions also exist, however, as suggested by the appearance of the Breit-Pauli operators.[62] For the present, however, we can conclude that a

non-zero charge-to-rest-mass ratio for the neutrino does offer a way of rationalizing its extreme penetrability through dense matter, while at the same time leaving open the possibility of its undergoing a significantly strong interaction under certain well-defined circumstances.

## IX. RESULTS OF CALCULATIONS FOR OTHER BINARY SYSTEMS

There are 21 distinct binary systems that can be formed from the proton, electron and neutrino and their respective antiparticles. Of these three are of the particle-antiparticle type already discussed, while the other 18 divide into pairs related to one another by charge conjugation. One such example is $p^+e^-$, which has the same Hamiltonian as $p^-e^+$ by virtue of the fact that the charges of the respective constituents appear as products of one another in all interactions (or as an absolute value in the exponential damping functions). Six of the remaining pairs are diagonal cases involving two identical particles. No binding is expected as a result, in accord with the results of explicit calculations. The various Breit-Pauli terms are attractive for certain angular momentum states, so this is not such an obvious result. The strength of the attraction is never enough to outweigh the kinetic energy contributions, however. Another three pairs are easily dispensed with as well, namely $p^+\nu$, $p^+\bar{\nu}$ and $p^+e^+$ and their charge-conjugated partners. If we assume a negative $q/m_o$ value for $\nu$, the $p^+\nu$ system is seen to be quite similar to $p^+e^-$, at least as long as the absolute value of the charge-to-mass ratio is close to unity. In all cases it is assumed that the damping constant A in a given basis is the same as has been found in the $e^+e^-$ calculations discussed in Ref. [2].

This leaves two such pairs of binary systems unconsidered, $e^-\nu$ and $e^-\bar{\nu}$ and their charge-conjugated positron systems. Of these, only the ones with $q/m_o$ values of opposite sign for their constituents are interesting in view of the experience with the other binary systems. Because of the fact that electrons and antineutrinos commonly appear together as decay products of neutrons, it is tempting to associate a positive $q/m_o$ value with $\bar{\nu}$. The similarity to $e^+e^-$ is expected to be particularly great if this quantity is assumed to be unity, in which case the results of Table 3 in Ref. [2] again become pertinent. Especially if the translational energy is excluded from consideration, it has been found above that $e^+e^-$ and $\nu\bar{\nu}$ are characterized by very nearly the same wave-function in their respective lowest-energy states. The maximum binding energy of the $e^-\bar{\nu}$ system can thus be anticipated to fall midway between the corresponding values for $e^+e^-$ and $\nu\bar{\nu}$. This expectation is very nearly fulfilled in explicit calculations with the XBPS

Hamiltonian in which it is assumed that $q/m_o$=1.0 a.u. for the $\bar{\nu}$ species. A binding energy of -15872.194 hartree, i.e. nearly one-half of the e+e- binding energy, is computed when the same 5s,5p basis is employed which was found to be optimal for $e^+e^-$ and $\nu\bar{\nu}$ (see Fig. 2).

Since it seems certain that no such bound $e^-\bar{\nu}$ system actually exists (it would correspond to the mutual annihilation of the two particles in the conventional description for particle-antiparticle pairs), it can be concluded that such a high value for the $\bar{\nu}$ charge-to-mass ratio is unacceptable. We can eliminate any computed binding with the electron by decreasing the above $q/m_o$ value, however, and repeating the scaling optimization procedure for the corresponding $e^-\bar{\nu}$ system (while again maintaining the damping constant A at its previous value). The resulting minimal energies are plotted in Fig. 3 as a function of the assumed $q/m_o$ value. There is a simple relationship between this quantity and the optimum scaling parameter $\eta$, namely they are found to be inversely proportional to one another. This result is consistent with the second scaling theorem discussed in Sect. VII, which shows that the optimal value of $\eta$ is inversely proportional to the square of the $|q/m_o|$ value assumed for the neutrino in a treatment of the hypothetical $\nu\bar{\nu}$ system.

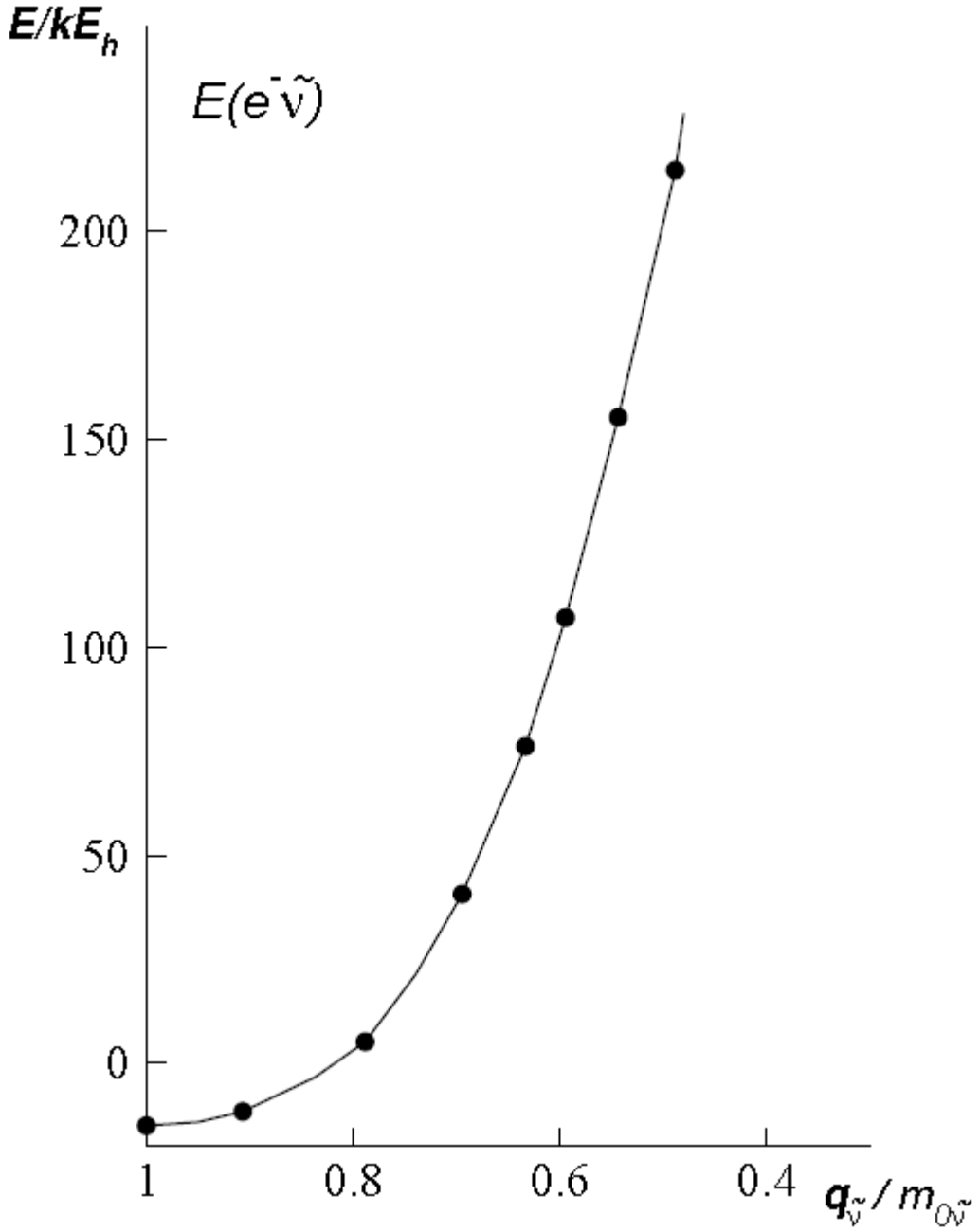


FIG. 3. Variation of the minimum value of the computed total energy (in khartree) of the $e^- \bar{\nu}$ system (obtained by optimizing the scale factor η) in the XBPS treatment employing a 5s,5p basis set, given as a function of the antineutrino charge-to-rest-mass ratio $q/m_0$. The value of the exponential damping constant A assumed throughout (1.0775 a.u.) is taken from the results of the analogous $e^+e^-$ calculations in the same basis.

On the basis of the above considerations it appears likely that the $|q/m_o|$ value assumed for the neutrino and antineutrino cannot be greater than 0.7 a.u. and still avoid the prediction of a bound $e^- \bar{\nu}$ system in nature. A value exceeding zero by a good margin is still tenable on this

basis, however, since it would allow for a $\nu\bar{\nu}$ system with a zero binding energy which is nonetheless separated by a large barrier from its dissociation products. The above scaling arguments show that the inter-particle distances involved must decrease as the value of $|q_\nu/m_{o\nu}|$ is lowered. It also should be noted that the symmetry of the lowest-energy $e^-\bar{\nu}$ state is also found to be $0^-$, so that the analogy with the $e^+e^-$ tight-binding state appears to retain its validity over a wide range of $|q_\nu/m_{o\nu}|$ values. There is a tendency to emphasize the $p_{1/2}$ character of the electronic function over $s_{1/2}$ as the antineutrino charge-to-rest-mass ratio decreases, as seems reasonable from the nature of the Breit-Pauli terms and the requirement of minimizing the total energy for the resultant $e^-\bar{\nu}$ system (the charge-conjugation symmetry is clearly missing for this binary system).

## X. INITIAL CALCULATIONS OF THE $p^+e^-\bar{\nu}$ SYSTEM; NUCLEAR BINDING IN THE XBPS MODEL

The questioning of the creation-annihilation hypothesis considered in Refs. [1,2] has led to an alternative interpretation of particle-antiparticle interactions in terms of a Schrödinger equation whose Hamiltonian contains momentum-dependent short-range operators. According to this model the electron and positron can be bound so strongly to one another that there is a total loss of mass relative to their respective free-particle states. Since nuclear binding processes are well known to be accompanied by distinctions in the total masses of products and reactants, in accordance with the predictions of STR, it is natural to speculate that the same types of interactions might be involved in the $e^-e^+$ interaction. The exponential form of the attractive potentials employed in the XBPS Hamiltonian follows at least partially from this line of reasoning.

There is another similarity connecting these two types of phenomena as well, however, which again is tied up with the supposition that matter can be created and destroyed by the gain or loss of energy. As discussed in Ref. [1], the accepted view of the role of the electron and antineutrino in nuclear interactions is that they are created whenever a neutron decays but that they are not present in the bound nuclei themselves.[10] The alternative interpretation which will be pursued in the present chapter is that the neutron is akin to a tri-atomic molecule, i.e. it is composed of a proton, an electron and an antineutrino. It has *negative* binding energy and is thus analogous to an excimer system commonly encountered in molecular physics studies. A key

element in the ensuing theoretical model is that neutrinos are capable of being strongly attracted by other particles, but only at very short range. The $e^- \bar{\nu}$ model system treated in the last section is the starting point of this investigation.

The XBPS Hamiltonian (Table 1 of Ref. [2]) has a single free parameter, the exponential damping constant A, and it has been fixed by the requirement that the binding energy of each of the three particle-antiparticle binary systems be $2m_oc^2$ in a full CI treatment employing a given one-particle basis set. Otherwise, all that is needed to completely define a system's Schrödinger equation are the electric charges and rest masses of the component particles. In the case of the antineutrino $\bar{\nu}$ these are assigned to be zero in each instance, but the possibility that their ratio has a non-zero value is left open as a means of explaining this particle's role in the nuclear binding process. The actual value for the antineutrino's charge-to-rest-mass ratio has not been specified as yet, primarily because it has been found that the requirement of a vanishing binding energy for the $\nu\bar{\nu}$ binary system can *be satisfied for any choice of this quantity's magnitude.* A potential solution to this problem is provided by the "tri-atomic hypothesis" for the neutron's structure, however. Accordingly, it will simply be required that the magnitude of the antineutrino's $q/m_o$ value be such so as to lead to the experimental total energy of the neutron when solving the XBPS for the $p^+e^-\bar{\nu}$ system which is assumed to correspond in the present model. Specifically, the binding energy of the latter system should be equal to -28758 hartree (-0.7825 MeV) This value corresponds to the difference in the rest masses of the proton-electron combination and that of the meta-stable neutron.

The first series of calculations carried out to investigative this hypothesis employs the simple 2s,2p Gaussian basis mentioned in Sect. 3.3. of Ref. [2] (exponents of $2.0 \times 10^8$ and $1.0 \times 10^8$ $a_o^{-2}$ in both cases). A scale factor $\eta$ is then defined as before to be optimized so as to obtain the minimum energy possible for a given choice of $q_\nu/m_{o\nu}$. Strictly speaking, only a local minimum is of interest in this case, since the desired energy is greater than that of the separated products, but the range of $\eta$ involved is anticipated to be in the same neighborhood as for the $e^+e^-$ calculations considered earlier. The minimization is carried out in terms of the total energy, although the magnitude of the corresponding translational energy will also be determined. Accordingly, the value chosen for A is 1.0540 a.u., since it gives the correct binding energy in the analogous treatment for the $e^+e^-$ system. For all $q/m_o$ values of $\bar{\nu}$ it is found that the lowest energy eigenvalue in the interesting range of $\eta$ occurs for a state of $1/2^-$ symmetry. A minimum

with close to the desired total energy (+28900 hartree) occurs for a scale factor η of 0.18 and a $q/m_o$ value of + 0.5733 a.u. The corresponding variation of energy with η is shown in Fig. 4, while the dependence of the minimal total energy as a function of the assumed $q/m_o$ value for $\bar{\nu}$, i.e. as obtained by optimizing η in each case, is given in Fig. 5.

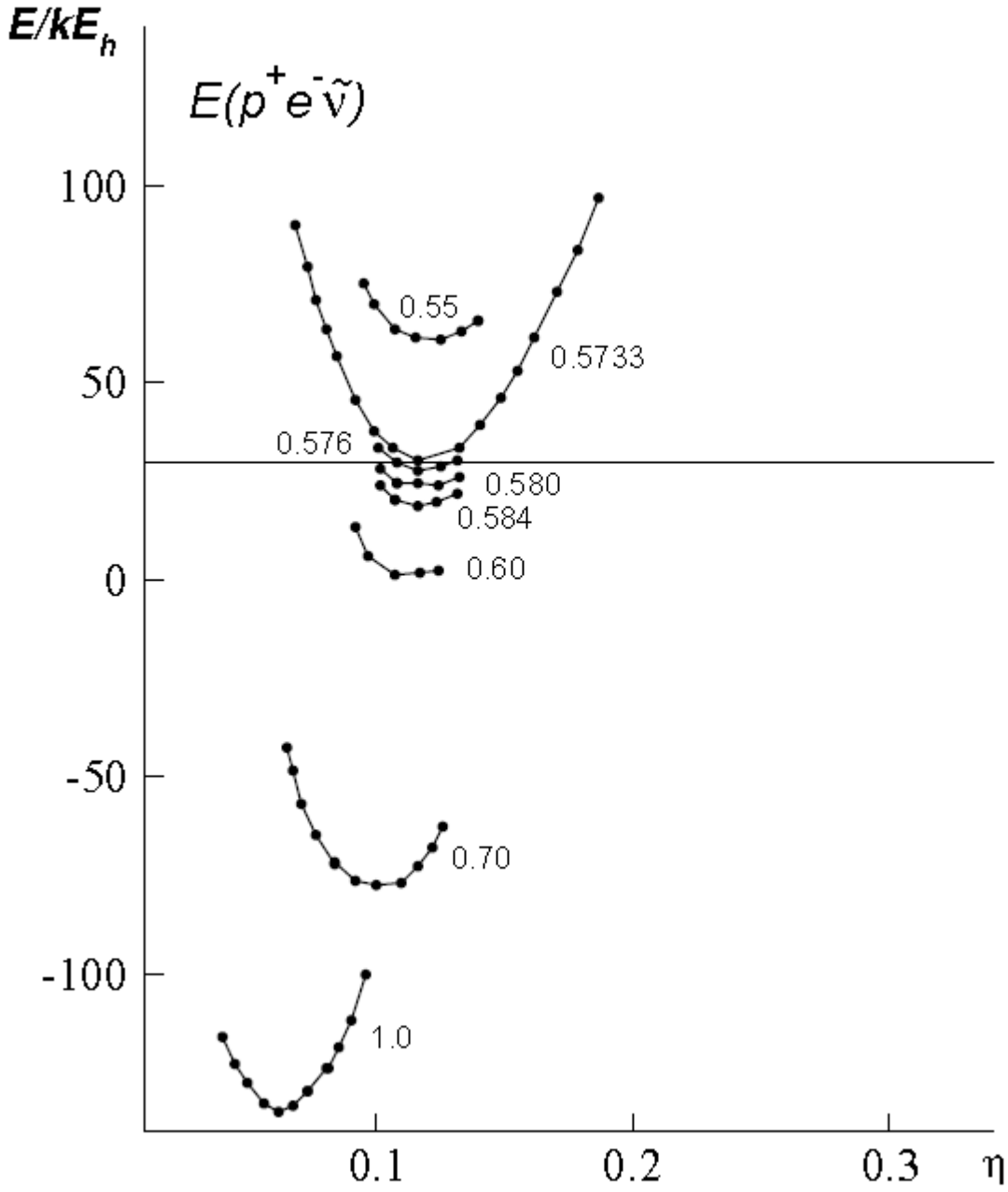


FIG. 4. Variation of the computed total energy (in khartree) of the $p^+e^-\bar{\nu}$ (neutron) system as a function of the 2s,2p basis set scaling factor η in the XBPS treatment for various values of the antineutrino charge-to-rest-mass ratio $q/m_o$. The horizontal line at the center of the diagram corresponds to the experimental total energy of the neutron at rest. The value of the exponential damping constant A assumed in each case (1.054 a.u.) is taken from the results of the analogous $e^+e^-$ calculations in the same basis.

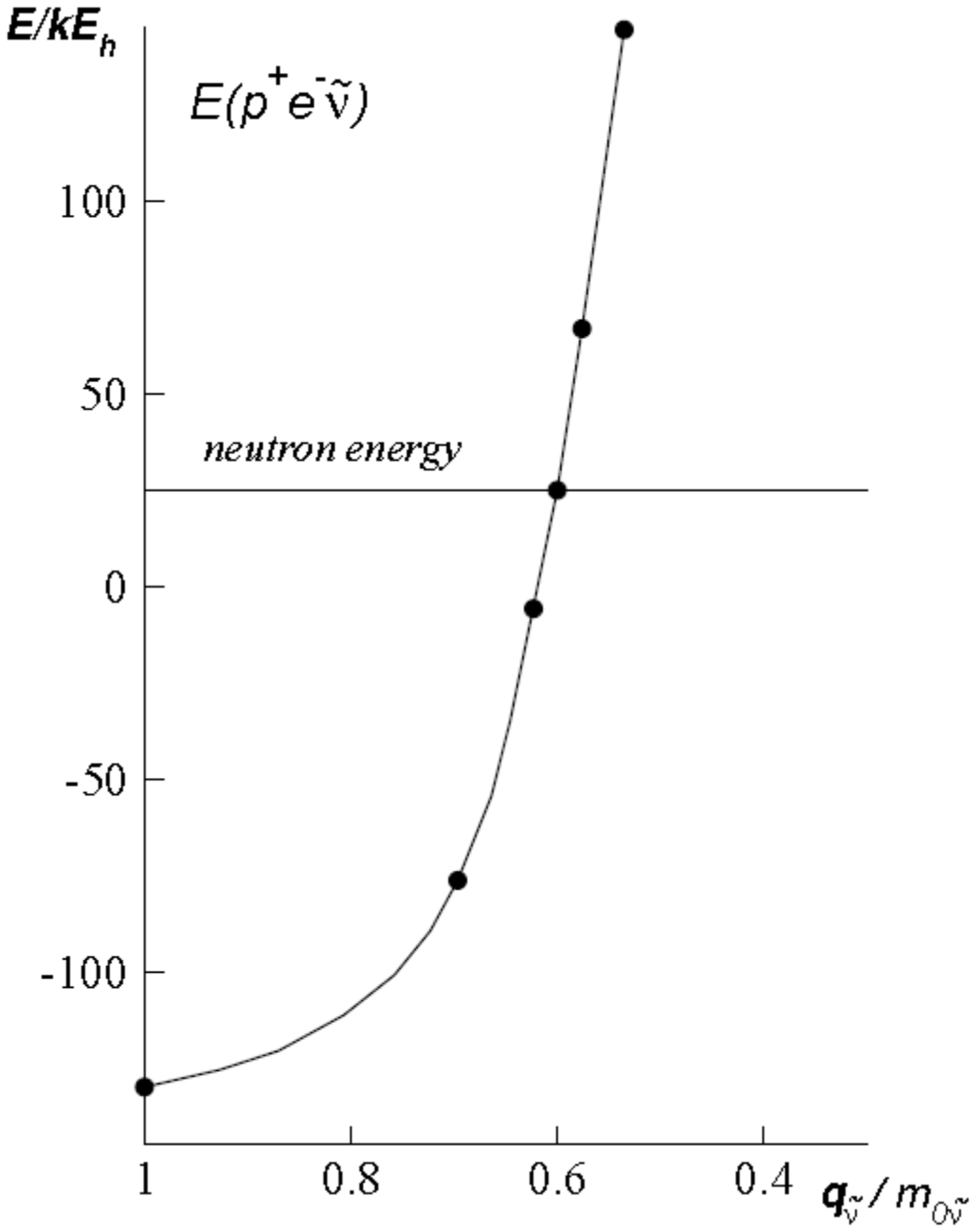


FIG. 5. Variation of the minimal $p^+e^-\bar{\nu}$ system's computed total energy (taken from Fig. 4, 2s,2p basis) as a function of the antineutrino charge-to-rest-mass ratio $q/m_o$. The horizontal line at the center of the diagram corresponds to the experimental total energy of the neutron at rest.

Although the above calculations are fairly crude because of the small number of functions in the one-particle basis employed, they nonetheless illustrate a number of features of the present theoretical model which can be expected to be retained as the level of computational treatment is

improved. To begin with, the fact that a minimum in total energy is always found demonstrates that the short-range potential employed is capable of binding the above three particles together within a relatively small volume. The computed binding energy is relatively sensitive to the choice of the $\bar{\nu}$ $q/m_o$, and indicates that for values close to +1.0 a.u, the $p^+e^-\bar{\nu}$ system would be more stable than the neutron actually observed experimentally. The corresponding optimal $\eta$ value increases rather quickly as $q/m_o$ decreases. There is thus a tendency for the constituent particles to draw more closely together at the same time that the total energy is increasing during the same variation. This is very similar to the experience noted for the $e^-\bar{\nu}$ system discussed in Sect. IX (see Fig. 3), which in turn can be anticipated based on the coordinate-scaling arguments of Sect. VII. It is also clear from comparing Figs. 3 and 5 that the rate of increase in total energy caused by lowering the antineutrino's $q/m_o$ value is somewhat greater for the $e^-\bar{\nu}$ binary system than for tri-atomic $p^+e^-\bar{\nu}$.

The wavefunction corresponding to the experimental neutron binding energy is shown for the present basis in Table I (SCF orbitals and full CI coefficients). It is relatively easy to analyze these results because the proton is seen to occupy a single s-type orbital almost exclusively in the small basis employed. For each product of three spatial orbitals, there are two doublet states possible, constructed from the three spin products ($M_J$=+1/2): ααß, αßα and ßαα (spins given in order of $p^+e^-\bar{\nu}$, α for $m_J$=1/2 and ß for $m_j$=-1/2 for both $s_{1/2}$ and $p_{1/2}$ spin-orbitals. In the calculations the two doublets are represented by the linear combinations: $\chi_1=(2/3)^{1/2}$ααß–$6^{-1/2}$(αßα+ßαα) and $\chi_2=2^{-1/2}$ (αßα-ßαα). In this basis the preferred spin combination is $(3/4)^{1/2}$ $\chi_1$+(1/2)$\chi_2$, which thus reduces to $\chi_{opt}=2^{-1/2}$ (ααß-αßα). In essence the proton therefore almost always has α spin, while the $e^-\bar{\nu}$ pair forms a singlet (αß-ßα) combination which is very reminiscent of the $0^-$ state preferred by the $e^+e^-$ system and the other particle-antiparticle binaries studied, as well as $e^-\bar{\nu}$ itself. The $p_{1/2}$ $s_{1/2}$ configurations are again preferred for the $e^-\bar{\nu}$ complex, just as in the absence of the proton, but the corresponding $s_{1/2}$ $p_{1/2}$ product also makes a substantial contribution in each case. This type of polarization of the wavefunction is greater for the $p^+$ $e^-\bar{\nu}$ system, however. Clearly the $M_J$=-1/2 component of the $1/2^-$ state can be obtained by inverting all α (and ß spins), which in effect means that the resulting proton with ß spin is then bound to the same $0^-$ $e^-\bar{\nu}$ structure as before.

TABLE 1. Self-consistent field (a) and selected CI (b) coefficients for the XBPS calculation (order 176) of the $1/2^-$ ground state of the $p^+e\bar{\nu}$ neutron system employing a 2s,2p basis ($\alpha_1$ = 0.36 x $10^8$ $a_0^{-2}$ and $\alpha_2$ = 0.18 x $10^8$ $a_0^{-2}$) with scale factor $\eta$=0.18, exponential damping constant A=1.054 a.u. and $\bar{\nu}$ $q/m_o$ value of 0.5733 a.u.

a)

| Orbital | SCF Coefficients | | | | | |
|---|---|---|---|---|---|---|
| | $p^+$ | | $e^-$ | | $\bar{\nu}$ | |
| | $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ |
| $1s_{1/2}$ | 1.62047 | -1.88386 | -1.23308 | 2.15739 | -0.01823 | -2.48485 |
| $2s_{1/2}$ | -0.72534 | 2.37670 | 1.99702 | -1.47876 | 1.01666 | 2.26743 |
| $1p_{1/2}$ | 1.52029 | -1.26858 | -0.47744 | -1.92162 | -0.55928 | 1.89942 |
| $2p_{1/2}$ | -0.67147 | 1.86271 | 1.38257 | 1.41742 | 1.44199 | -1.35692 |
| $1p_{3/2}$ | 1.52029 | -1.26858 | -0.76754 | 1.82523 | -0.23080 | -1.96655 |
| $2p_{3/2}$ | -0.67147 | 1.86271 | 1.58427 | -1.18771 | 1.19238 | 1.58076 |

b)

| Configuration (orbital occupations) | | | CI ($1/2^-$) Coefficients * | |
|---|---|---|---|---|
| $p^+$ | $e^-$ | $\bar{\nu}$ | | |
| $1s_{1/2}$ | $1p_{1/2}$ | $1s_{1/2}$ | 0.572975 | 0.331858 |
| $1s_{1/2}$ | $2p_{1/2}$ | $1s_{1/2}$ | -0.211011 | -0.122144 |
| $2s_{1/2}$ | $1p_{1/2}$ | $1s_{1/2}$ | 0.082410 | 0.047579 |
| $1s_{1/2}$ | $1p_{1/2}$ | $2s_{1/2}$ | -0.242427 | -0.140535 |
| $1s_{1/2}$ | $1s_{1/2}$ | $1p_{1/2}$ | 0.274353 | 0.158440 |
| $1s_{1/2}$ | $1s_{1/2}$ | $2p_{1/2}$ | 0.141213 | 0.081532 |
| $1s_{1/2}$ | $2s_{1/2}$ | $1p_{1/2}$ | 0.255488 | 0.147615 |
| $1s_{1/2}$ | $2s_{1/2}$ | $2p_{1/2}$ | 0.279461 | 0.161382 |
| $1s_{1/2}$ | $2p_{1/2}$ | $2s_{1/2}$ | 0.196547 | 0.113694 |
| $1p_{1/2}$ | $1p_{1/2}$ | $1p_{1/2}$ | -0.023543 | -0.040888 |
| $2s_{1/2}$ | $1s_{1/2}$ | $1p_{1/2}$ | 0.058872 | 0.033968 |
| $2s_{1/2}$ | $2s_{1/2}$ | $1p_{1/2}$ | 0.061997 | 0.035741 |
| $2s_{1/2}$ | $2s_{1/2}$ | $2p_{1/2}$ | 0.055680 | 0.032099 |
| $1p_{3/2}$ | $1s_{1/2}$ | $1s_{1/2}$ | -0.054191 | |
| $1p_{3/2}$ | $2s_{1/2}$ | $1s_{1/2}$ | -0.057859 | |
| $1p_{3/2}$ | $1p_{1/2}$ | $1p_{1/2}$ | -0.067169 | |
| $1p_{3/2}$ | $1p_{1/2}$ | $2p_{1/2}$ | -0.060622 | |

*Two configuration-state functions are needed to span the spaces corresponding to (1/2) (1/2) (1/2) occupations (as defined in text), whereas only one such function is needed for the corresponding (3/2) (1/2) (1/2) species; all configurations with at least one CI coefficient whose absolute value exceeds 0.04 are listed.

## XI. THE MECHANISM FOR PROTON BONDING IN THE p+e-$\bar{\nu}$ SYSTEM

A comparison of Figs. 3 and 5 helps to illustrate the origin of the proton binding process in the present model. The energy difference results for $e^-\bar{\nu}$ and $p^+e^-\bar{\nu}$ in the same (2s,2p) basis are plotted against the assumed $q/m_o$ value for $\bar{\nu}$ in Fig. 6, and show that the proton binding energy increases as this charge-to-mass ratio is reduced from its original value of +1.0 a.u. Because of the small $q/m_o$ value of the proton it is to be expected that only terms in the XBPS Hamiltonian (Table 1 of Ref. [2]) which do not involve this quantity will be important. An analysis of the $p^+e^-\bar{\nu}$ total energy for the $\bar{\nu}$ $q/m_o$ values of 1.0 and 0.5733 a.u. is given in Tables 2 and 3 respectively, inc1uding corresponding results for optimal treatments of the $e^-\bar{\nu}$ system in its lowest-energy $0^-$ state. The proton's kinetic energy contribution is relatively small because of its large mass, but at the interpartic1e distances involved, this quantity still works effectively against net binding. For the $\bar{\nu}$ $q/m_o$ value of 0.5733 a.u. which leads to the experimental neutron total energy for the $p^+e\bar{\nu}$ system, it is found, for example, that the proton kinetic energy is 36401.887 hartree, compared with a value of the proton-electron Coulomb attraction contribution of -5189.462 hartree. The latter result corresponds to a mean electron-proton distance of 3.62 $\alpha^2$ bohr (10.2 fermi). There are two other XBPS terms for the proton which do not involve its own $q/m_o$ value, however, which play a decisive role in the binding process, namely the spin-same-orbit and Darwin terms involving only the squares of either the $q/m_o$ values for the electron or antineutrino (Table 1 of Ref. [2]). Consequently, these terms are of the same order of magnitude as their counterparts in the $e^+e^-$ calculation (see Table 3 of Ref. [2]).

The magnitudes of these contributions to the $p^+e^-\bar{\nu}$ energy depend very much on the characteristics of the various occupied spin-orbitals, however. A strong contribution toward proton binding comes from its spin-same-orbit interaction with the electron when the latter occupies a $p_{1/2}$ orbital while the proton occupies $s_{1/2}$ (see Table IX). This attractive interaction is countered by the corresponding term involving $\bar{\nu}$, however, because the latter also occupies $p_{1/2}$ heavily and its $q/m_o$ value is assumed to be of opposite sign to that of the electron. The Darwin term also is characterized by large contributions to the $p^+e\bar{\nu}$ total energy in which the proton is directly involved, however. In this case it is the antineutrino which provides an attractive force for the proton, however, which is countered only partially by the corresponding repulsive $p^+e^-$ Darwin term contribution (Table 1 of Ref. [2]).

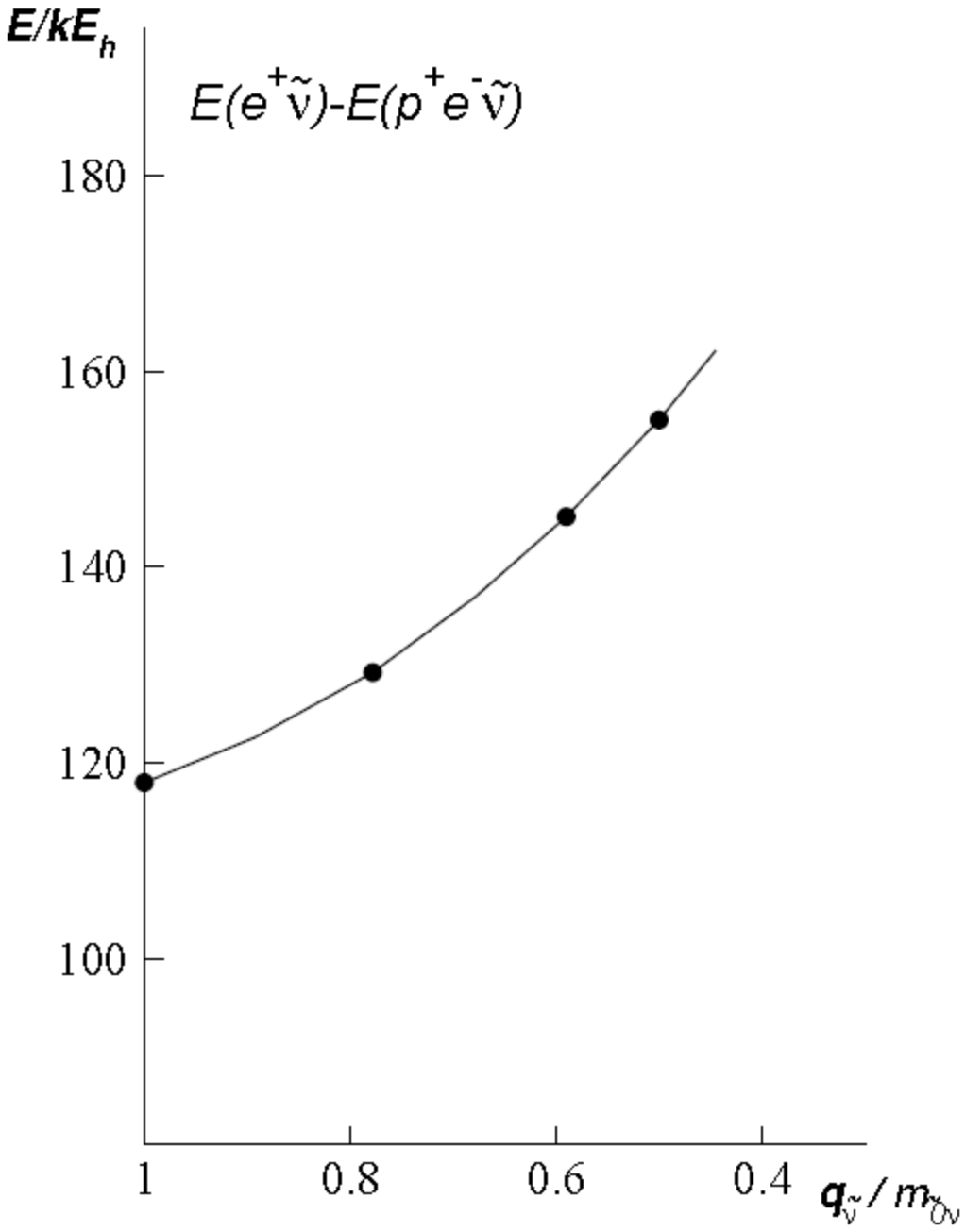


FIG. 6. Difference of the computed total energies (in khartree) of the $e^- \overline{\nu}$ and $p^+e^- \overline{\nu}$ systems (with separately optimized scale factors η) as a function of the antineutrino charge-to-rest-mass ratio $q/m_o$ (2s,2p basis).

TABLE 2. Energy contributions (in hartree) of various operators (see Table 1 of Ref. [2] for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+e^-\bar{\nu}$ neutron system and (b) the $0^-$ state of the isolated $e^-\bar{\nu}$ binary obtained by employing the 2s,2p basis with scale factor $\eta = 0.115$, exponential damping constant A = 1.054 a.u. and antineutrino $q/m_o$ value of 1.0 a.u. for the XBPS Hamiltonian.

| a) Operator | $p^+e^-$ | $p^+\bar{\nu}$ | $e^-\bar{\nu}$ | Total ($p^+e^-\bar{\nu}$) |
|---|---|---|---|---|
| Kinetic Energy | 23276.671 ($p^+$) | 1117472.413($e^-$) | 1133826.572 ($\bar{\nu}$) | 2274575.656 |
| Coulomb | -4305.801 | 0.000 | 0.000 | -4305.801 |
| Spin-same-orbit | -172833.434 | 48470.883 | -395254.117 | -519616.667 |
| Spin-other-orbit | -31.731 | 27.278 | -875528.194 | -875532.646 |
| Darwin Term | 92782.662 | -228814.351 | 19549.016 | -116482.672 |
| Orbit-orbit | -44.084 | 129.057 | -450397.290 | -450312.316 |
| Spin-spin | -0.550 | -0.132 | -447017.344 | -447018.026 |
| Spin-spin $\delta$ | -0.223 | -1.938 | 7381.956 | 7379.795 |
| Total Energy | | | | -131312.679 |

| b) Operator | Total ($e^-\bar{\nu}$) |
|---|---|
| Kinetic Energy | 2247935.736 |
| Coulomb | 0.000 |
| Spin-same-orbit | -411800.053 |
| Spin-other-orbit | -916668.797 |
| Darwin Term | 8317.250 |
| Orbit-orbit | -480109.984 |
| Spin-spin | -461579.506 |
| Spin-spin $\delta$ | 0.000 |
| Total Energy | -13905.355 |

There is thus a fairly complex system of interactions available among the Breit-Pauli terms whose net energy contribution depends on two major factors: a) the character of the orbitals occupied respectively by the three particles and b) the assumed value of the antineutrino's charge-to-rest-mass ratio [since the corresponding spin-same-orbit and Darwin interactions with the proton depend on the square of this quantity (Table 1 of Ref. [2])]. The easiest means of grasping the influence of these various factors in the proton binding energy is to first examine the situation for $q/m_o$ ($\bar{\nu}$) = +1.0 a.u. If the orbital occupations of the $e^-$ and $\bar{\nu}$ particles were exactly equivalent, all the proton spin-same-orbit and Darwin interactions would exactly cancel one another. That would effectively leave only the proton kinetic energy and proton-electron Coulomb terms to determine the binding energy, which would mean that the $p^+e^-\bar{\nu}$ system is decidedly unstable relative to $e^-\bar{\nu}$. By assuming a wavefunction in which the electron has more $p_{1/2}$ character than $\bar{\nu}$, it is possible to shift the balance toward a more favorab1e binding

situation, however, by virtue of the fact that the attractive proton-electron spin-same-orbit term then outweighs the repulsive $p^+ \overline{\nu}$ contribution of the analogous type.

TABLE 3. Energy contributions (in hartree) of various operators (see Table I of Ref. [2] for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+e\overline{\nu}$ neutron system and (b) the $0^-$ state of the isolated $e^-\overline{\nu}$ binary obtained by employing the 2s,2p basis with scale factor $\eta = 0.18$, exponential damping constant A = 1.054 a.u. and antineutrino $q/m_o$ value of 0.5733 a.u. for the XBPS Hamiltonian.

| a) Operator | $p^+e^-$ | $p^+\overline{\nu}$ | $e^-\overline{\nu}$ | Total ($p^+e^-\overline{\nu}$) |
|---|---|---|---|---|
| Kinetic Energy | 36401.887 ($p^+$) | 1346815.079 ($e^-$) | 1453755.882 ($\overline{\nu}$) | 2836972.849 |
| Coulomb | -5189.461 | 0.000 | 0.000 | -5189.461 |
| Spin-same-orbit | 208544.881 | 50249.141 | -465129.910 | -623425.649 |
| Spin-other-orbit | -58.267 | 54.775 | -1007066.189 | -1007069.681 |
| Darwin Term | 107014.110 | -262604.473 | 17161.765 | -138428.597 |
| Orbit-orbit | 200.486 | -58.064 | -523108.311 | -522965.888 |
| Spin-spin | -1.096 | -0.248 | -516918.459 | -516919.802 |
| Spin-spin $\delta$ | -0.387 | -4.109 | 5928.285 | 5923.790 |
| Total Energy | | | | 28897.559 |

| b) Operator | Total ($e^-\overline{\nu}$) |
|---|---|
| Kinetic Energy | 2789426.035 |
| Coulomb | 0.000 |
| Spin-same-orbit | -481697.661 |
| Spin-other-orbit | -1063806.525 |
| Darwin Term | 11500.864 |
| Orbit-orbit | -558243.900 |
| Spin-spin | -531802.164 |
| Spin-spin $\delta$ | 1127.425 |
| Total Energy | 166504.076 |

Examination of Table 2 shows clearly that heavy net attractive contributions actually arise for both the spin-same-orbit and Darwin terms as a result of such a polarization. As a result, the total binding energy of the proton to the $e^-\overline{\nu}$ complex is computed to be 117400 hartree (i.e. . 131312.679-13905.355 hartree). By comparison, if only the proton kinetic energy and proton-electron Coulomb contributions were counted, a negative proton binding energy would result.

There is another key factor in the $p^+e^-\overline{\nu}$ binding process, however, which is also important in the formation of tri-atomic systems in the realm of molecular physics. Some adaptation of the charge distribution of a diatomic component of the molecule is almost always crucial to the production of a stable tri-atomic system.[89] Generally speaking, the more stable the diatomic system, the higher the price to be paid for altering its charge distribution to accommodate bonding with a third atom. In the present context, it is important to recall that a quite small AO

basis has been employed in these exploratory calculations, however, and so it can be anticipated that at least some of the $e^- \bar{\nu}$ system's affinity to polarize its charge distribution is a consequence of its relatively poor representation at this level of theoretical treatment.

When the $q/m_o$ value is decreased from unity for the antineutrino, several additional effects emerge. The $e^- \bar{\nu}$ complex gradually loses its stability in the process, and it thus becomes easier for the proton to bind to it as a direct consequence. The cancellation of the Breit-Pauli $p^+e^-$ and $p^+ \bar{\nu}$ interactions is no longer perfect even if both $e^-$ and $\bar{\nu}$ have equivalent orbital occupations because the key $(q/m_o)^2$ weighting factors now favor the electron. In addition, polarization effects are less likely to destabilize the $e^- \bar{\nu}$ complex, so that the types of changes in the isolated system's wavefunction which maximize binding with the proton are accomplished with less resistance than before. For the $q/m_o$ value of 0.5733 a.u. required to obtain the experimental neutron rest mass for the $p^+e^- \bar{\nu}$ calculations employing the 2s,2p basis, it is found that the proton binding energy increases to 137600 hartree, over 20000 hartree greater than the value which results when a unit value for this quantity is employed.

The results of Table 3 show further that both the spin-same-orbit and Darwin interactions again produce a net attraction for the proton, namely 158296 and 155590 hartree, respectively (obtained by adding the corresponding $p^+e^-$ and $p^+ \bar{\nu}$ results). These results are accomplished to a large extent by polarization of the $e^- \bar{\nu}$ complex, as can be seen from the following comparison. For this $q/m_o$ value the minimal total energy obtained for the isolated $e^- \bar{\nu}$ system is 166504 hartree, whereas the corresponding energy value obtained employing the polarized $p^+e^- \bar{\nu}$ wavefunction is 311438 hartree (obtained by adding the $e^-$ kinetic energy of 1346815.079 hartree to the $e^- \bar{\nu}$ results including the $\bar{\nu}$ kinetic energy), an increase of 144934 hartree. Again, while it is very likely that such polarization effects are greatly exaggerated by the use of such a small one-particle basis set in the present calculations, it at least seems conceivable that the combination of changes of this nature in the $e^- \bar{\nu}$ charge distribution along with the use of a $|q/m_o|$ value for the antineutrino which is smaller than that of the electron could lead to sufficient proton binding to produce a meta-stable $p^+e^- \bar{\nu}$ complex with the properties of the experimentally observed neutron. The effect of this polarization increases with the $\bar{\nu}$ charge-to-mass ratio, so these considerations make it at least qualitatively understandable why the proton binding energy increases as $q/m_o$ decreases, as seen clearly from the result of Fig. 6.

There is another important aspect of the proton binding process yet to be discussed, however, namely the influence of the exponential damping factors in the XBPS Hamiltonian (Table 1 of Ref. [2]). These factors play a decisive role in obtaining an energy minimum for the $e^- \bar{\nu}$ binary (Figs. 5-6), but they also have a subtle influence on the proton's interactions. As we have already seen, the Breit-Pauli terms of greatest importance for the proton are the spin-same-orbit and Darwin interactions with $e^-$ and $\bar{\nu}$ which are multiplied by the $(q/m_o)^2$ factors of the latter particles only. In these cases the corresponding damping factors are totally independent of the proton's momentum (Table 1 of Ref. [2]). By decreasing the size of the proton's orbital it is therefore possible to increase the magnitude of the un-damped Breit-Pauli expectation values, with their net attractive influence, *without producing a corresponding decrease in the magnitude of the related exponential damping factors.* As long as the charge distributions of the electron and antineutrino do not change at the same time, such an increase in the proton momentum thus generally leads to a net increase in the attractive contribution of the damped Breit- Pauli interactions to the proton-bonded system.

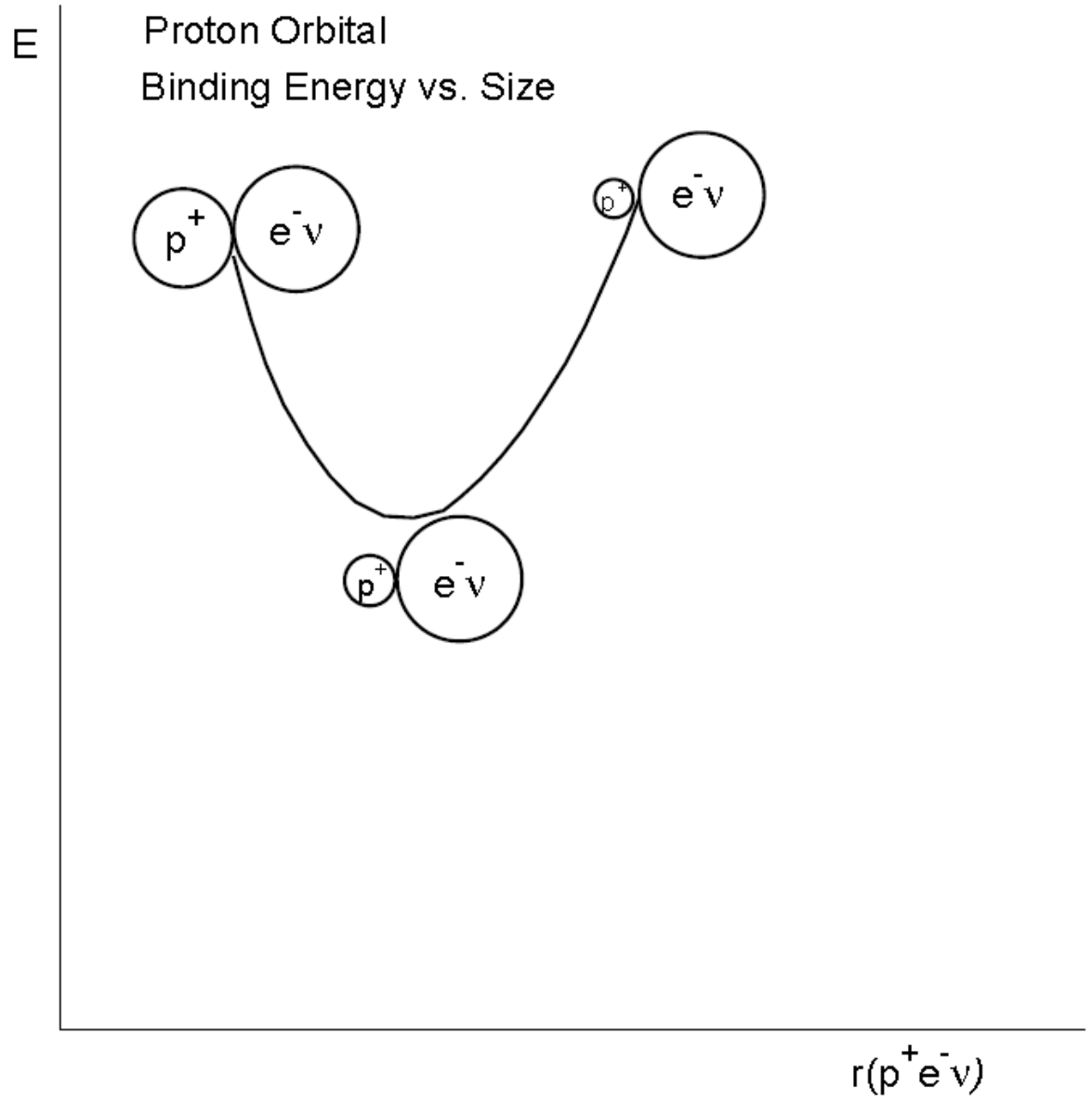


FIG. 7. Schematic diagram showing the relationship between the size of the proton orbital and the stability of the $p+e^-\bar{\nu}$ neutron system. The large mass of the proton *vis-a-vis* the $e^-\bar{\nu}$ species allows it to assume a relatively contracted charge distribution which helps to maximize the effect of its attractive short-range interactions with the lighter particles without greatly increasing. its own kinetic energy, thus leading to the total energy minimum shown.

The possibilities are not unlimited for the proton, however, because its kinetic energy also increases as its charge distribution is made more compact (see Fig. 7). Because the Breit-Pauli terms vary as $r^{-3}$ before damping effects are considered, while the proton kinetic energy increases only as $p^2 \approx r^{-2}$, it follows that in the interparticle distance range of interest ($r \approx \alpha^2$), the total

binding energy at first increases with the proton's momentum. Eventually a point of diminishing returns is reached, however, because as the proton orbital shrinks in size, the values of the Breit-Pauli integrals for fixed $e^-$ and $\bar{\nu}$ probability distributions begin to change more and more slowly, ultimately approaching the point-charge limit commonly employed in atomic calculations using the Breit-Pauli method. The proton kinetic energy continues to increase at roughly the same rate, however, and so at some point an optimum proton size is reached. Nonetheless, this effect represents a distinct advantage for the proton *vis-a-vis* lighter particles, and will be seen to play an increasingly more important role in the theory of nuclear binding which emerges from consideration of the properties of the XBPS Hamiltonian.

To illustrate the proton's tendency to assume a relatively more compact charge distribution than the electron and antineutrino, an additional series of calculations has been carried out in which two s-type Gaussian functions have been added to the original 2s,2p one-particle basis employed above. Optimization of the new exponents ($\alpha_1$ and $\alpha_2$) while holding those of the other functions fixed at their optimal values for the $e^-\bar{\nu}$ wavefunction shows that the proton prefers relatively high values for these quantities ($2.56 \times 10^8$ and $8.0 \times 10^7$ $a_0^{-2}$). These values are 8.0 and 2.5 times greater than that of the largest exponent in the optimal 2s,2p $e^-\bar{\nu}$ basis, indicating a significant contraction of the proton wavefunction, as expected. The new basis functions succeed in lowering the total $p^+e^-\bar{\nu}$ energy by nearly 60000 hartree relative to the 2s,2p result given in Table 3. In order to be consistent with the general calibration procedures adopted earlier, it is necessary to readjust the respective value of the exponential damping constant A and the antineutrino $q/m_o$ value so as to produce minimal energy results for the $e^+e^-$ and $p^+e^-\bar{\nu}$ systems of the desired values. On this basis A is increased only slightly to a value of 1.0568 a.u. while the $\bar{\nu}$ $q/m_o$ result is adjusted downward to 0.5375 a.u. A summary of the energy contributions for the $p^+e^-\bar{\nu}$ $1/2^-$ state found in this basis set are given in Table 4 for comparison with the corresponding 2s,2p data discussed first (Table 3). The shrinking of the proton charge distribution is most easily recognized from the magnitude of the kinetic energy obtained in the two treatments. This quantity increases by 79% as a result of the basis set expansion, as compared to relatively small changes

TABLE 4. Energy contributions (in hartree) of various operators (see Table 1 of Ref. [2] for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+ e^- \bar{\nu}$ neutron system and (b) the $0^-$ state of the isolated $e^- \bar{\nu}$ binary obtained by employing the 4s,2p basis with scale factor $\eta = 0.16$, exponential damping constant A = 1.0568 a.u. and antineutrino $q/m_o$ value of 0.5375 a.u. for the XBPS Hamiltonian.

| a) Operator | $p^+e^-$ | $p^+ \bar{\nu}$ | $e^- \bar{\nu}$ | Total ($p^+e^- \bar{\nu}$) |
|---|---|---|---|---|
| Kinetic Energy | 65280.208 ($p^+$) | 1339330.432($e^-$) | 1478062.82 t(\j) | 2882673.461 |
| Coulomb | -5517.636 | 0.000 | 0.000 | -5517.636 |
| Spin-same-orbit | -250929.141 | 70545.115 | -463958.811 | -644342.837 |
| Spin-other-orbit | -67.206 | 61. 902 | -984257.224 | -984262.528 |
| Darwin Term | 117676.768 | -352977.701 | 15413.212 | -219887.721 |
| Orbit-orbit | -66.116 | 228.315 | -492670.535 | -492508.336 |
| Spin-spin | -2.195 | -0.350 | -511881.100 | -511883.645 |
| Spin-spin δ | -0.474 | -10.788 | 5302.617 | 5291.355 |
| Total Energy | | | | 29562.111 |

| b) Operator | Total ($e^- \bar{\nu}$) |
|---|---|
| Kinetic Energy | 2759022.924 |
| Coulomb | 0.000 |
| Spin-same-orbit | -475143.463 |
| Spin-other-orbit | -1042436.569 |
| Darwin Term | 11692.897 |
| Orbit-orbit | -536596.507 |
| Spin-spin | -520943.516 |
| Spin-spin δ | 1286.444 |
| Total Energy | 196882.208 |

in the corresponding values for the electron and antineutrino. The net attractions of the proton spin-same-orbit and Darwin terms are also increased to 180384 and 235301 hartree respectively, representing a total of 101794 hartree additional binding to help offset the increase in proton kinetic energy. By construction, the total energies in both calculations are nearly identical, from

which it is clear that the shrinking of the proton charge distribution is also paid for at the price of additional destabilization of the $e^- \bar{\nu}$ complex.

Another key point to consider in the present $p^+e^-\bar{\nu}$ calculations is the way in which the translational energy is treated. Because of the heavy mass of the proton, it follows that the relativistic translational energy for a given $p^+e^-\bar{\nu}$ wave-function is much smaller than that for an equivalent $e^+e^-$ species. We have seen in Sect. 3.4 of Ref. [2] that the use of $\langle E_o \rangle = \langle H-T \rangle$ as criterion for the basis set optimizations leads to a notably higher damping constant A (1.7725 a.u. in the 5s,5p basis) than when only $\langle H \rangle$ is used (1.0775 a.u.). If the higher of these A values is used in the $p^+e\bar{\nu}$ calculations (or the corresponding value for the 2s,2p basis), it is impossible to obtain a low enough total energy for this system to satisfy the requirement that its rest mass be equal to that of the neutron. This situation is an artifact of the small (2s,2p) basis employed, however, because the exact solutions of the XBPS must be eigenfunctions of both H and T and the lowest total energy eigenvalue must correspond to a vanishing translational energy. Under the latter condition there would be no need to distinguish between the two different optimization procedures employed above, since they would necessarily lead to identical results ($E_{min} = E_o$), At this stage of development, however, it is necessary to deal with the fact that the approximate wavefunctions generated at the present level of treatment invariably have large expectation values of the translational momentum operator for the system at hand, at least when such compact charge distributions are involved as are known to characterize the internal structure of nuclei.

Under the circumstances, the decision to focus on the expectation value of the total energy in comparing the stabilities of systems containing different numbers of protons must carry with it the recognition that this choice favors heavier particles over lighter ones. Another way of seeing this is to recall that the value of the exponential damping constant A must be significantly smaller than variational arguments indicate it should be in order to obtain total energies for the particle-antiparticle binary systems which correspond to binding energies of $2m_oc^2$. As the quality of the basis set improves, the effective value of A must increase to maintain this condition until it ultimately assumes the ideal result corresponding to the exact solution of the XBPS for the translation-less binary systems.

To illustrate this tendency calculations have been carried out for the $e^+e^-$ system employing a 3s,2p,2d basis. They lead to a value for A of 1.2647 a.u., as compared to that of 1.0775 a.u.

obtained with the 5s,5p basis. Since the proton binding energy in the $p^+ e^- \bar{\nu}$ calculations considered thus far has been shown to arise primarily from terms which contain the corresponding damping factors in which the constant A appears, it seems highly likely that the degree of binding will be strongly influenced by such developments.

As usual, the binding energy of the $e^+e^-$ system is held at -37557.773 hartree in the new basis, for example, but the degree to which it or its $e^- \bar{\nu}$ analog can bind a proton can be expected to decrease as the value of the damping factor increases. This is tantamount to concluding that the computed proton binding energy for a given system will generally decrease as the level of theoretical treatment is improved within the XBPS model. Moreover, this expectation is also consistent with the discussion given earlier regarding the relationship between the stability of the $e^- \bar{\nu}$ complex and its susceptibility to polarization by neighboring protons. In other words, as the basis set is improved there is a growing tendency for the $e^- \bar{\nu}$ system to more strongly resist having its charge distribution altered relative to its isolated state.

There is a competing factor which tends to insure that the minimal $p^+e^- \bar{\nu}$ total energy can always be adjusted to the value corresponding to the neutron at rest, however, namely the dependence of this quantity on the $q/m_o$ value assumed for the antineutrino as the quality of the one-partic1e basis is improved. In the present case it is found, for example, that it is necessary to increase this value to 0.63 a.u. to obtain the desired energy result to a suitable approximation. The corresponding energy contributions are given in Table 5, similarly as for the other basis sets discussed earlier. Since the Gaussian exponents for the 3s,2p,2d basis have been optimized for the $e^+e^-$ system, the results of these calculations are most meaningfully compared to those employing the 2s,2p basis (Table 3), i.e. without the benefit of specially optimized proton s functions. As expected, the use of a significantly larger damping constant leads to a reduction in the net binding associated with the proton spin-same-orbit and Darwin terms (74427 and 70107 hartree respectively), 46% of the previous total. The proton's kinetic energy is correspondingly smaller in the 3s,2p,2d computations as well, 56% of the former value. The proton-electron Coulomb energy is also smaller, reflecting a general tendency to keep these two particles further apart on the average as a result of the addition of d functions to the basis set.

TABLE 5. Energy contributions (in hartree) of various operators (see Table 1 of Ref. [2] for definitions) and particle combinations for the $1/2^-$ ground state of the $p^+e\,\bar{\nu}$ neutron system obtained by employing the 3s,2p,2d basis with scale factor $\eta = 0.111$, exponential damping constant $A = 1.2648\ e^{-1}$ and aritineutrino $q/m_o$ value of 0.63 a.u. for the XBPS Hamiltonian.

| Operator | $p^+e^-$ | $p^+\bar{\nu}$ | $e^-\bar{\nu}$ | Total ($p^+\,e^-\,\bar{\nu}$) |
|---|---|---|---|---|
| Kinetic Energy | 20354.545($p^+$) | 1068402.098($e^-$) | 1140129.945($\bar{\nu}$) | 2228886.588 |
| Coulomb | -3856.027 | 0.000 | 0.000 | -3856.027 |
| Spin-same-orbit | -94307.843 | 19879.681 | -363207.789 | -437635.951 |
| Spin-other-orbit | -23.085 | 19.111 | -757175.358 | -757179.332 |
| Darwin Term | 48531. 942 | -118639.460 | 8357.925 | -61749.593 |
| Orbit-orbit | -16.656 | 145.966 | -559254.673 | -559125.363 |
| Spin-spin | -0.370 | -0.176 | -383654.253 | -383654.799 |
| Spin-spin $\delta$ | -0.179 | -1.489 | 2345.886 | 2344.218 |
| Total Energy | | | | 28029.737 |

Particularly when one considers the capacity of the proton to assume a more contracted charge distribution when extra functions are included in the basis which can be specifically optimized for it, it appears feasible to construct a suitably quantitative theory of nuclear binding on the basis of the model under discussion. At least these results show that it is far from proven that the "nuclear electrons" which Fermi spoke of in introducing his theory of beta decay[8-10] actually cannot exist within the confines of a bound nucleus. Before discussing further computations employing the XBPS Hamiltonian, however, it is well to consider other experimental information regarding the neutron which has been claimed to lend support to the hypothesis of the disappearing electron.

## XII. COMPARISON OF THE PROPERTIES OF THE $p^+e^-\bar{\nu}$ SYSTEM WITH THOSE KNOWN FOR THE NEUTRON

The calculations discussed above have been suggested by the model of a neutron as a composite system formed by its known decay elements. The lowest-energy $p^+e\,\bar{\nu}$ system is found to be a doublet, consistent with what is known for the neutron. As a product of three fermions, it is a fermionic system itself, in agreement with Pauli's original interpretation of beta decay

processes.[7] The charge distribution found to be optimal for the $p^+e^-\bar{\nu}$ system corresponds to nuclear dimensions, in fulfillment of another obvious requirement, but more extensive calculations are highly desirable in order to be more quantitative on this point. A three-way partnership is suggested in forming the neutron which is reminiscent of the excimer concept in molecular physics. The fact that the system corresponds to a local energy minimum which lies above that of its separated particles is clearly consistent with the known meta-stability of the neutron. This aspect of nuclear binding (weak interaction) will be taken up in more detail in following work dealing with the structure of the deuteron.

A key to the binding of the three particles together is the assumption of a positive $q/m_o$ value for the antineutrino, but one with a smaller absolute value than that of the electron. One has the picture of an $e^-\bar{\nu}$ system which is an imperfect copy of the $e^+e^-$ mass-less binary first discussed. Its relative instability makes it more attractive to the proton than $e^+e^-$ itself, and in effect the proton binding that results can be looked upon as an attempt to compensate for what is otherwise missing in the $e^-\bar{\nu}$ bond *vis-a-vis* either of its $e^+e^-$ or $\nu\bar{\nu}$ counterparts. The electrical neutrality of the antineutrino guarantees that the $p^+e^-\bar{\nu}$ system has no net charge, but this brings us to a far more delicate matter. The neutron possesses a magnetic dipole moment which is negative and of the order of the nuclear Bohr magneton.[7,9] As such it is far smaller in absolute magnitude than what one would expect for a system containing an electron.

To examine this point it is helpful to consider an experiment in which the hypothetical $p^+e^-\bar{\nu}$ system is subjected to a magnetic field in order to determine this quantity. If we assume that the magnetic moment of the combined system is equal to the algebraic sum of the individual moments of its constituents, the measured value must be expected to be nearly equal to that of the electron alone. This assumption works quite well in dealing with molecular properties and has also been found to be acceptable for nuclei, as for example in the comparison of the deuteron's magnetic moment with those of its constituent proton and neutron.[90] If we look more closely at the way magnetic moments are measured, however, we note that for this additivity principle to hold in the $p^+e^-\bar{\nu}$ case, it is essential that the electron be just as apt to rotate in response to the torque applied by a magnetic field in this tri-atomic system as it is in its free state, or alternatively when it is weakly bound to a positive atomic or molecular ion. Especially since the present calculations indicate that the binding process requires a particular spin orientation between all three particles in the optimum $p^+e^-\bar{\nu}$ resonance state, it seems far from

obvious that the above condition is fulfilled in the present instance. In order for the electron spin to change its orientation, it seems much more likely that the system as a whole must rotate, and that would mean that a much larger mass is involved than for a lightly bound electron located in an outer shell of an atom. A similar situation is well known in the study of the Compton effect,[91] for example. The modified wave observed in x-ray radiation from atoms *is observed only when an essentially free electron is involved in the interaction,* in which case the change in wavelength $\Delta\lambda$ is found to be inversely proportional to the electronic mass. When an inner-shell electron does the scattering, however, it is as if the whole atom is involved in the interaction. Consequently, the Compton scattering law indicates the production of an essentially unmodified wave, i.e. $\Delta\lambda \sim 0$. The calculations discussed above suggest that the electron in the $p^+e^-\bar{\nu}$ resonance system is very tightly bound to both the proton and the antineutrino constituents, and thus that the model of an inner-shell electron in the preceding analogy is far more appropriate.

On this basis one is led to expect that the $p^+e^-\bar{\nu}$ system has a magnetic moment of an extremely heavy electron, i.e. negative in sign, but of the order of a nuclear bohr magneton, which is at least approximately what is observed for the neutron.[9,52] The additivity principle should not work at all well in this case because the interaction of the particle spins is intimately involved in the internal bonding process, unlike the case when predominantly Coulomb interactions are involved. In subsequent work we will come back to this point when we discuss the muon magnetic moment, which is almost exactly what one would expect for an electron having this particle's rest mass. The overall situation is complicated for the proton and neutron, however. because of the pion cloud known to surround each of them,[47,50,53,92]. The observed neutron magnetic moment is roughly double[9,52,53] that which one would expect from the above "clamped-electron" model, but such a discrepancy is at least potentially understandable in terms of the environment otherwise assumed for the neutron. Moreover, the fact that high-energy electron scattering experiments[93] have indicated elastic form factors which correspond to a small but not necessarily vanishing neutron charge radius are also seemingly consistent with the present model's assumed tri-atomic composition for this neutral system.

## XIII. CONCLUSION

Consideration of other types of observed phenomena arising in the field of modern physics is found in no way to contradict the hypothesis discussed in Ref, [1-2] of ubiquitous massless

photons.. At the same time, this line of argumentation calls into question standard interpretations of related elementary processes which involve the creation or annihilation of individual particles. The primary example considered in this connection is that of the β decay of a neutron (Sect. II). By following through on the proposal that no material particle is ever created or destroyed in physical transformations, one is led to conclude that a strict elemental balance must characterize any reactive equation, exactly as is assumed in ordinary chemical processes. In accordance with this view a neutron must be composed of a proton, an electron and an antineutrino *prior* to undergoing β decay, because each of these particles is present at the completion of the process. An analogy is made between the neutron (and other unstable elementary particles) and excimer systems in the field of molecular physics, which are well known to correspond to meta-stable excited states whose ground state potential energy curves are completely repulsive. The energy given off in the decay of elementary particles is so large compared to that found in the operation of an excimer laser that a significant decrease in the sum of the rest masses of the products in such reactions is observed relative to the mass of the original meta-stable system. Similar arguments can be given for a whole series of elementary particles, such as muons, pions and other heavier species, all of which are known to decay with relatively short lifetimes to a collection of fragments of smaller aggregate rest mass.

In order to give quantitative substance to the above theoretical model, it is imperative that one clearly identify the nature of the interactions responsible for high-energy processes, especially the prototype example in which an electron and a positron combine to form a tightly bound binary complex with exactly zero rest mass. Emphasis is placed thereby on the fact that no corresponding state of a proton and an electron is known, i.e. the $1s_{1/2}$ state of the hydrogen atom is perfectly stable when left in an isolated condition, whereas that of positronium has only a short lifetime. Similarly, a massless $p^+p^-$ binary must also be assumed to exist based on the analogous experimental results for the interaction of a proton and an antiproton. The term “phanton” will be used in the following discussion in referring to such massless particle-antiparticle binary systems in general, and the special name of “prophoton” has been given to the $p^+p^-$ member of this family, to go along with the identification of its $e^+e^-$ counterpart as the photon.

The high energies associated with the formation of $e^+e^-$ and $p^+p^-$ suggest that similar short-range interactions are involved as in the binding of nuclei, in which case an exponential form for the corresponding potential has been deduced from scattering experiments. Because of the

participation of electrons, positrons and photons in the electromagnetic interaction, it is suggested that a good starting point in the search for such a potential is the relativistic Dirac equation or some approximation to it. It is noted that the Breit-Pauli reduction of this equation contains short-range terms varying inversely as the cube of the interparticle distance which are of the order of $\alpha^2$ ($10^{-4}$-$10^{-5}$ hartree) for typical electron-proton separations in an atomic system. The unbounded character of these terms in the limit of vanishingly small separations, particularly in relation to the corresponding relativistic kinetic energy, makes it clear, however, that the desired short-range bonding of elementary particles can only be satisfactorily described by interactions of this type if they are somehow modified to become considerably less attractive at extremely small interparticle distances.

An exponential damping of the Breit-Pauli interactions has thus been suggested in Refs. [1,2]. This is in analogy to that proposed by Yukawa fifty years earlier for the description of nuclear binding, except that in that case it was applied to an $r^{-1}$ potential. The resulting set of interactions is referred to as the exponentially damped Breit-Pauli Hamiltonian and is employed in a Schrödinger equation (XBPS) of the standard $H \Psi = E \Psi$ form. The potentially crucial advantage of this Hamiltonian is that it is *bounded from below* and thus can be treated using standard variational techniques, unlike the un-damped Breit-Pauli terms themselves. The explicit form of the XBPS Hamiltonian is given in Table 1 of Ref. [2].

The description of the decay of the neutron in the XBPS model also requires an explicit representation of the interaction of a third particle and its antiparticle, namely the neutrino and antineutrino. It is necessary, consistent with the above considerations to assume that a corresponding phanton system of zero rest mass exists with a $\nu\bar{\nu}$ composition (photrino). The fact that the neutrino possesses no electric charge prevents the use of the same scaling property of the XBPS Hamiltonian as in the case of the $e^+e^-$ and $p^+p^-$ systems , however, because it is only valid for particles having the same absolute magnitude of charge as the electron. Examination of the coupling constants in this Hamiltionian shows that the only way that it can lead to significant interactions for neutrinos is if their charge-to-rest-mass ratios are different from zero. This condition can only be met if the rest mass of the charge-less neutrino is itself exactly zero, something which is at least consistent with all experimental investigations as yet undertaken to measure this quantity. It is also clear that the use of the rest mass instead of the relativistic mass, $m = \gamma m_0$, in the coupling constants of this Hamiltonian is essential if this possibility is to have a

significant impact on the description of the neutrino interactions. Accordingly, the sign of the corresponding Breit-Pauli interactions is determined on the basis of the product of the respective $q/m_0$ values of the interacting particles rather than for the product of their electric charges alone.

Subsequent calculations for the $\nu\bar{\nu}$ system have shown that the same value for the damping constant A which gives the desired binding energy of $2m_oc^2$ for the $e^+e^-$ and $p^+p^-$ phantons also works to a good approximation for $\nu\bar{\nu}$, i.e. with zero binding energy relative to the separated neutrino and antineutrino constituents. The agreement is particularly good if translational effects are considered explicitly in the optimization procedure. A different scaling property (see eqs. VII.1-3) is found for the XBPS Hamiltonian which shows that the desired zero binding energy of the $\nu\bar{\nu}$ system holds independently of which $|q/m_o|$ value is assumed for the neutrinos. This result is consistent with the existence of more than one type (flavor) of neutrino in nature, but it also leaves the choice of this quantity's value open for other another purpose, such as to satisfy the requirement of accurately computing the measured binding energy of the neutron relative to its decay products at rest.

The corresponding $\nu\bar{\nu}$ total energy curve (Fig. 1) can be thought of as containing a double minimum at E=0, one for the tightly bound phanton system and the other for the particles separated to infinity. Because of the lack of a (long-range) Coulomb interaction for neutrinos such a theoretical energy curve climbs immediately as the particles approach each other from quite large distances, contrary to what occurs for the approach of a charged particle to its antiparticle. In the latter case the internal kinetic energy of the system varies more slowly with distance in this range than does the Coulomb attraction of the particles, an effect which is crucial to the success of the original Bohr model of the hydrogen atom. It can be argued that the failure of the neutrino to participate in similarly long-range attractive interactions is responsible for the extremely penetrating nature of its radiation, making it effectively incapable of overcoming centrifugal barriers separating it from other particles.

It is possible to understand the observed lack of conventional magnetic interactions for neutrinos on a similar basis (Sect. VI). From the point of view of a charged particle, all electromagnetic interactions are effectively purely Coulomb because magnetic contributions to the energy are always computed to be of vanishing magnitude in its own inertial system. Thus a non-zero $qJm_o$ value is not necessarily inconsistent with the measurement of a vanishingly small magnetic moment for the neutrino. At the same time, it can explain why such particles undergo

strong interactions at extremely small interparticle separations, as exemplified in the original Reines-Cowan antineutrino capture experiments. The fact that relatively high centrifugal barriers must be overcome before coming into a suitable range for which the damped Breit-Pauli terms can dominate is also consistent with the extremely small cross sections observed for such reactions. Moreover, the possibility of its participating in short-range interactions offers an explanation for the presence of an antineutrino in the meta-stable neutron, as demanded by the hypothesis of particle balance in all physical transformations. To obtain suitably strong binding with the electron, the other light component of the neutron in this view, it is necessary to assume that $\bar{\nu}$ has a positive $q/m_o$ value, i.e. opposite to that of the electron (and also to that of its antiparticle).

The known $q/m_o$ value of the proton is so small as to rule out its participation in XBPS interactions in the range of interparticle separations typical for nuclear binding for which this quantity appears as a coupling constant. This circumstance practically eliminates any chance of such Breit-Pauli terms as the spin-spin, orbit-orbit and spin-other-orbit interactions being directly involved in the binding of protons, but it leaves open a distinct possibility that the spin-same-orbit and Darwin terms with electrons and antineutrinos can affect them in a significant manner. One knows from observations of atomic spectra, for example, that the electron undergoes a spin-same-orbit interaction with the nucleus which does not depend on the mass of the heavier particle, but rather on the square of the mass of the electron itself. Preliminary calculations with the XBPS model demonstrate that such a proton-electron interaction can be enormously attractive in the desired range of interparticle separation. The opposite effect is found for the proton-antineutrino spin-same-orbit interaction, but the calculations demonstrate that the most stable state of the $p^+e^-\bar{\nu}$ system adjusts its charge distribution to obtain a significant net binding between the proton and the $e^-\bar{\nu}$ complex.

In addition to this polarization effect, another dominant factor responsible for proton binding is its attractive Darwin term interaction with the antineutrino, which easily outweighs the effect of the corresponding proton-electron repulsive contribution. By decreasing the $q/m_o$ value of $\bar{\nu}$ below that of the positron, it is found that the total energy of the most stable state of the $p^+e^-\bar{\nu}$ system can be adjusted to become equal to that known experimentally for the neutron, i.e. 28781.31 a.u. higher than that of its separated component particles. This occurs for a $q/m_o$ value of +0.63 a.u. for the antineutrino in a 3s2p2d basis, i.e. about two-thirds that of the electron

(Sect. XI). A value of unity for this quantity gives a much lower energy for the $p^+e^-\bar{\nu}$ system, showing it to be bound by 62528.34 a.u. relative to its separated constituents (employing the same basis), a change of 91309 a.u. or 2.48 MeV, so it is clear that these results are quite sensitive to the numerical value chosen for this charge-to-rest-mass ratio.

The last comparison offers a clear perception of the mechanism of proton binding in the present model. The $e\bar{\nu}$ complex can be looked upon as a clone of the prototype $e^+e^-$ massless binary, especially when $q/m_0$ for $\bar{\nu}$ is assumed to be the same as for the positron. Because of the nature of the coupling constants in the spin-same-orbit and Darwin terms in the XBPS Hamiltonian, the proton can be strongly attracted to $e^-\bar{\nu}$, especially if the charge-to-rest -mass ratio of $\bar{\nu}$ is relatively large. Reducing the value of this quantity below unity necessarily causes the stability of the $e^-\bar{\nu}$ complex to be lessened, but at the same time it increases the magnitude of the net proton binding energy to this system. The symmetry of the resulting $p^+e^-\bar{\nu}$ metastable state is consistent with this interpretation. The proton finds itself predominantly in an $s_{1/2}$ orbital, while the $e^-\bar{\nu}$ species prefers the $0^-$ character exhibited by both $e^+e^-$ and $\nu\bar{\nu}$ in their respective lowest states, giving an overall $1/2^-$ symmetry to the resulting $p^+e^-\bar{\nu}$ wave function. There is an indication that the computed proton binding energy is overestimated in the computational treatment employed, since improvements in the AO basis tend to make it more difficult for the proton to polarize the $e^-\bar{\nu}$ charge distribution. On this basis it can be expected that the $q/m_o$ value for the antineutrino needed to give the experimental neutron binding energy in the exact XBPS treatment is somewhat larger than found in the best calculations carried out in the present study. Moreover, it seems likely that as the one-particle basis becomes more flexible, that the description of the tight-binding phanton states as well as that of the $e^-\bar{\nu}$ complex will allow more effective use of the short-range Breit-Pauli attractive interactions, thereby drawing the two component fermions somewhat closer together at an improved level of treatment. This conclusion is consistent with the results of the different $e^+e^-$ optimizations discussed in Ref. [2] for which the influence of translational energy is explicitly taken into account.

The qualitative picture which emerges from these considerations is thus that the proton experiences a net attraction to the $e^-\bar{\nu}$ complex primarily because the antineutrino's charge-to-rest-mass ratio is somewhat smaller in absolute value than that of the electron. This condition makes the $e^-\bar{\nu}$ system significantly less stable than $e^+e^-$ in its lowest state, but it also makes the former binary more attractive to protons than its phanton counterpart. This attraction is

insufficient to produce a bound $p^+e^-\bar{\nu}$ system, however, i.e. to bind a single proton, and this circumstance is assumed to be responsible for the known instability of the neutron.

According to the present model, decay of the neutron occurs via a spin-flip mechanism, as will be discussed in subsequent work, whereby the predominantly singlet nature of the $0^-$ $e^-\bar{\nu}$ complex is suddenly destroyed. Instead the spins of the proton and electron begin to alternate, causing the antineutrino to be ejected, always with the same spin as that of the original neutron. This course of events is consistent with the measured spin relationships of the emitted particles in the decay of polarized neutrons. The presence of a slight triplet admixture in the $e^-\bar{\nu}$ complex is thus seen as a contributing factor to the finite lifetime of the neutron, and to have an important influence on the nature of weak-interaction decays in general. At the same time the computed strength of the electron's attraction to the proton and antineutrino helps to explain why the resulting tri-atomic system only has a magnetic moment on the order of a nuclear Bohr magneton (Sect. XII). An analogous situation can be found in the study of the Compton effect, for which it is found that collisions of photons with inner-shell electrons can only be successively described by taking the mass of the scattered system to be that of the entire atom to which it is bound.

## Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.


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