Degree of Freedom in the Lorentz Transformation

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Abstract

Lorentz pointed out in 1899 that a relativistic space-time transformation that leaves Maxwell's equations invariant could only be defined to within a common factor on the basis of then-existing experimental data. The present Note calls attention to the fact that Einstein's subsequent derivation of the Lorentz transformation (LT) is based on an *undeclared assumption* regarding the functional dependence of this factor. Consequently, many of the unconfirmed predictions of Einstein's special theory of relativity (SR), such as remote non-simultaneity of events and Fitzgerald-Lorentz length contraction, would lose their validity if the above assumption is shown to be incorrect. At the same time, any other choice for Lorentz's common factor is seen to be consistent with the relativistic velocity transformation (VT). Accordingly, predictions of SR that are derived exclusively from the VT would not be affected by such a change in relativity theory. On this basis, an alternative Lorentz transformation (ALT) is defined that also satisfies Einstein's two postulates of relativity, but incorporates the strict proportionality of the rates of moving clocks assumed in the operation of the Global Positioning System (GPS) rather than invoking the inextricable mixing of space and time coordinates predicted by the LT.

Keywords: postulates of special relativity, Lorentz transformation (LT), velocity transformation (VT), alternative Lorentz transformation (ALT)

The theoretical justification for the conclusion that superluminal velocities are unattainable is based on the Lorentz transformation (LT) [1] for the space and time coordinates of a given object as measured by observers moving with constant velocity in two different inertial systems. The equations are listed below in standard notation [2] similar to that used in Einstein's original derivation [1] (c is the speed of light in free space and v is the relative speed of the two inertial systems S and S'):

$$t' = \gamma\,(t - vxc^{-2}) = \gamma\,\eta^{-1}\,t \tag{1a}$$

$$x' = \gamma\,(x - vt) \tag{1b}$$

$$y' = y \tag{1c}$$

$$z' = z, \tag{1d}$$

with $\gamma = (1-v^2c^{-2})^{-0.5}$ and $\eta = (1 - vxt^{-1}c^{-2})^{-1} = (1 - vu_xc^{-2})^{-1}$. From eq. (1a), it is seen that the two values of t and t', respectively, can be of opposite sign (time inversion) if the component of the object's speed $u_x = xt^{-1}$ in the parallel (x,x') direction is greater than the light speed c. The latter condition, i.e. $t't^{-1}<0$, is satisfied if $\eta<0$ in eq. (1a), which is possible for at least some value of the relative speed $v<c$ of the two observers whenever $u_x > c$. Such a result would mean that the time-order of a pair of events can be opposite for the two observers and would therefore constitute a violation of Einstein causality, thereby ruling out the possibility of super-luminal motion occurring in nature.

The relativistic velocity transformation (VT) is obtained by combining eqs. (1a-d) of the LT ($u_x' = x't'^{-1}$ etc.):

$$u_x' = (1 - vu_xc^{-2})^{-1}(u_x - v) = \eta\,(u_x - v) \tag{2a}$$

$$u_y' = \gamma^{-1}\,(1 - vu_xc^{-2})^{-1}\,u_y = \eta\,\gamma^{-1}\,u_y \tag{2b}$$

$$u_z' = \gamma^{-1}\,(1 - vu_xc^{-2})^{-1}\,u_z = \eta\,\gamma^{-1}\,u_z. \tag{2c}$$

Note that η appears in all three equations. Unlike the LT, the VT does not place any restrictions on the maximum speed attainable by the object. Instead, it divides the relationships between the measured speeds of the observers into three categories [3]: if $u<c$, then $u'<c$ as well; if $u=c$, then $u'=c$ (Einstein's second postulate of relativity [1]); and if $u>c$, then $u'>c$.

The VT is quite important in physics because it leads to a number of experimentally verified predictions in relativity theory independently of the LT. Examples include the aberration of starlight at the zenith [1] and the Fresnel light drag formula [4], and it is obviously consistent with the light-speed postulate itself. However, other predictions that result from application of the LT cannot be obtained from the VT. These include Fitzgerald-Lorentz length contraction (FLC), remote non-simultaneity of events, the inextricable mixing of space and time and the symmetric nature of the measurements of observers in different inertial systems (fundamental disagreement as to which of their proper clocks is running slower, for example), and, as already mentioned, the impossibility of super-luminal motion. Each of the latter is obtained directly from eqs. 1(a-d), but cannot be derived on the basis of the VT alone. The space-time predictions of Einstein's theory [1] can therefore be divided *into two mutually exclusive categories,* those which require the LT in their derivation, and those which can also be derived from the VT and therefore do not rest specifically on the LT.

The latter observation is interesting in an historical context. Lorentz pointed out as early as 1899 [5] that there is a *degree of freedom* in obtaining a space-time transformation that leaves the Maxwell equations invariant. He wrote down a more general transformation that takes account of this indeterminacy [6]:

$$t' = \gamma\, \varepsilon\, (t - vxc^{-2}) = \gamma\, \varepsilon\, \eta^{-1}\, t \qquad (3a)$$

$$x' = \gamma\, \varepsilon\, (x - vt) \qquad (3b)$$

$$y' = \varepsilon\, y \qquad (3c)$$

$$z' = \varepsilon\, z. \qquad (3d)$$

It is easy to show that any value for the free parameter ε in these relations leaves the Maxwell equations invariant. It is also clear that the same situation holds in arriving at the VT and satisfying Einstein's light-speed postulate. Dividing x', y', z' by t' leads to a cancellation of ε as long as it is present in each of the above equations, making it clear that the VT of eqs. (2a-c) is compatible with any choice for this parameter.

Einstein was aware of the degree of freedom in the general Lorentz transformation (GLT) of eqs. (3a-d). He gives the same equations as Lorentz in his 1905 paper [1], simply renaming the parameter φ instead of ε. On p. 900 of Ref. 1, he then proceeds by stating "… and φ is a temporarily unknown function of v." He gives no explanation for why φ should only depend on the single variable v, the relative speed of the two participating inertial systems. It is an *undeclared assumption* in his derivation, but one with important consequences.

By making this restriction on the functionality of φ, he was able to show on symmetry grounds that its only possible value is $\varphi = 1$, which in turn leads directly to the LT of eqs. (1a-d).

At essentially the same time, Poincaré [7] also derived the LT from the GLT of eqs. (3a-d), making the same assumption for the functionality of Lorentz's ε as did Einstein [1]. He did give a justification for this assumption, however, stating [8] that "the ensemble of all these transformations, together with the ensemble of all spatial rotations must form a group." He then pointed out that the only way to achieve this condition was to assume that $\varepsilon = 1$. In a subsequent paper in the same year [9], he gave a proof for this choice, but to do so he also had to assume that ε is only a function of v. However, Poincaré's argument about the group theoretical requirements expected for the LT *is itself an assumption* that requires its own justification. Moreover, it is important to note that the aforementioned group properties for the LT *only* hold when the velocities in the two transformations in eqs. (1a-d) are in *the same* direction.

In summary, the LT is not the inevitable consequence of Einstein's two postulates of relativity, but rather rests squarely on an additional assumption he used to eliminate the degree of freedom in the GLT of eqs. (3a-d), namely that φ/ε must be a function of the single variable v. In this connection, it is important to recall the two distinct categories of predictions based on Einstein's relativity theory alluded to above. The second of these consists of results that can be derived directly from the VT and thus do not depend in any way on the choice of ε in the GLT. By contrast, the first set, which includes predictions of the FLC and remote non-simultaneity, as well as the conclusion that super-luminal speeds are forbidden in nature, *can only be derived from the LT*.

The above discussion emphasizes the fact that it possible to satisfy the two postulates of relativity without making Einstein's assumption regarding the functionality of the parameter ε in the GLT of eqs. (3a-d). The question therefore arises whether another choice for ε might lead to a viable alternative to the LT. For example, there has been a widespread consensus that the simple relationship for measured times in the classical Galilean transformation (GT), i.e. t'=t, is ruled out by the relativity postulates. The degree of freedom in the GLT in fact makes it possible to counter this view in a thoroughly straightforward manner.

Consideration of eq. (3a) shows that one does arrive at the GT result for timing measurements if one defines ε to be equal to $\eta \gamma^{-1}$. The corresponding transformation obtained from the GLT is thus:

$$t' = t \tag{4a}$$

$$x' = \eta (x - vt) \tag{4b}$$

$$y' = \eta \gamma^{-1} y \tag{4c}$$

$$z' = \eta \gamma^{-1} z. \tag{4d}$$

The new transformation replaces the simple y = y' and z= z' relations in the LT by the more complicated eqs. (4c-d), and in turn obtains a simpler version of the LT's eq. (1a) *in which space and time are no longer mixed*.

Although the transformation in eqs. (4a-d) satisfies the light-speed postulate, it still has a problem with experiments [10-12] that have shown conclusively that clock rates vary between rest frames (time dilation). The studies of atomic clocks on board airplanes [12] indicate that their rates differ from those on the ground in a strictly proportional manner, however, and this experience is used to good advantage in the Global Positioning System (GPS) methodology. The rates of GPS satellite clocks are adjusted while on the ground by a constant factor so that they run at the same rate after they reach orbiting speed as do identical clocks left behind on the earth's surface [13]. It is nonetheless possible to accommodate time dilation within the framework of the GLT by simply inserting the relevant proportionality factor (Q) explicitly in the corresponding transformation. This is accomplished by setting $\varepsilon = \eta (\gamma Q)^{-1}$ in eqs. (3a-d), with the result:

$$t' = Q^{-1} t \tag{5a}$$

$$x' = \eta Q^{-1} (x - vt) \tag{5b}$$

$$y' = \eta (\gamma Q)^{-1} y \tag{5c}$$

$$z' = \eta (\gamma Q)^{-1} z. \tag{5d}$$

The fact that Q is variable allows one to use this transformation for any object and observer, unlike the case for the LT, which predicts that a moving clock will always run γ times slower than that of the observer. The latter result has been found to be contradicted in the airplane experiments [12]. They show that clocks traveling in a westerly direction run faster than those at the airport of departure, for example.

Eqs. (5a-d) are consistent with both of Einstein's postulates and the VT. This alternative Lorentz transformation (ALT) therefore allows one to account for the second category of experimental predictions mentioned above such as the aberration of starlight from the zenith. However, there is a complete lack of agreement between the ALT and the LT for the other set of predictions such as remote non-simultaneity and the FLC. Eq. (5a) of the ALT demands that the measured elapsed times Δt and $\Delta t'$ of the two observers also occur with the same ratio Q. Consequently, if events are simultaneous for one of the observers ($\Delta t = 0$), they also must be simultaneous for the other ($\Delta t' = 0$). *This shows that it is incorrect to claim that non-simultaneity is the inevitable consequence of Einstein's two postulates.* The same equation precludes the possibility of an inversion in the time-order of events since the ratio of clock rates Q has to be positive. Thus the ALT of itself, like the VT, does not preclude superluminal motion.

The FLC also is not consistent with eqs. (5a-d). For example, it can no longer be claimed that distances measured in a perpendicular direction ($y=y'$) are the same for both observers. Instead, one can use the light-speed postulate to show that the ratio of measured distances must be the same as the ratio of their respective clock rates ($\Delta y = c\Delta t \neq \Delta y' = c\Delta t'$). The latter conclusion is consistent with the modern definition of the meter [14] as the distance traveled by a light pulse in free space in c^{-1} s; the slower one's clock, the farther the light will travel in this amount of elapsed time. That means that *isotropic length expansion* accompanies time dilation (the slowing down of proper clocks), not the anisotropic length contraction demanded by the LT and the FLC.

The symmetry principle implied by the LT is also not supported by the ALT. Eq. (5a) requires that there be no ambiguity which of the proper clocks of moving observers is running slower; if $Q>1$, then O's clocks are faster, and if $Q<1$, the opposite is true. This type of relationship is assumed in the GPS methodology, and it is consistent with the measurements of clock rates in the Hafele-Keating experiments [12].

In summary, the general form of the Lorentz transformation contains a degree of freedom that has generally been overlooked since Einstein's original work on relativity theory was published over a century earlier. By choosing a different value for the free parameter ε in the general Lorentz transformation (GLT), it is possible to satisfy both of the postulates of relativity while avoiding many of the unverified predictions of Einstein's LT such as remote non-simultaneity of events and the impossibility of superluminal motion. Once this purely mathematical point is accepted, it should be clear that there is *no irrefutable basis* for any of the LT's predictions unless they are also derivable directly from the VT (for example, the explanations for the aberration of starlight from the zenith and for Fresnel's light-drag experiment). Once the LT is seen to be invalid, the basis for the FLC and Einstein's symmetric, and *therefore subjective*, theory of measurement is lost, as well as that for the aforementioned conclusions about remote non-simultaneity and superluminal motion.

The degree of freedom in the GLT can be put to good use to develop a slightly amended version of Einstein's theory. A different Lorentz transformation, the ALT, super-cedes the LT while still remaining consistent with Einstein's postulates and the VT. The keystone of this new theory is the fundamental objectivity of measurement and the related concept of absolute remote simultaneity of events. This is possible simply by removing Einstein's original undeclared assumption for the free parameter ε in the GLT, and replacing it with the experimentally viable condition of strict proportionality between rates of clocks in uniform motion to one another.

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