

# Lorentz invariance and the Global Positioning System

Robert J. Buenker

*Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität  
Wuppertal, Gausstr. 20, D-42119 Wuppertal, Germany*

Phone: +49-202-439-2511/2774

Fax: +49-202-439-2509

E-mail: [bobwtal@yahoo.de](mailto:bobwtal@yahoo.de), [buenker@uni-wuppertal.de](mailto:buenker@uni-wuppertal.de)

Blog: <http://alternativelorentztransformation.blogspot.com/>

## Abstract

An example is considered (clock riddle) which demonstrates that opposite results are obtained for the length of an object depending on whether FitzGerald-Lorentz contraction (FLC) is assumed or instead the determination is based on the elapsed time required for a light pulse to traverse between its endpoints. This lack of internal consistency in relativity theory is traced to an undeclared assumption Einstein made regarding a normalization factor appearing in his original derivation of the Lorentz transformation (LT).

*Keywords: postulates of special relativity, Lorentz transformation (LT), velocity transformation (VT), Global Positioning System (GPS), alternative Lorentz transformation (ALT)*

## 1. Introduction

The first version of the Lorentz transformation was published in 1887 by Voigt [1,2]. A decade later Lorentz himself pointed out that a relativistic space-time transformation that leaves Maxwell's equations invariant can only be specified to within a common (normalization) factor on the basis of then-existing experimental data [3]. The currently accepted version that Einstein derived in 1905 [4] satisfies the condition of Lorentz invariance, and leads to a number of predictions such as time dilation and FitzGerald-Lorentz length contraction (FLC). This version of relativity theory is not only based on Einstein's two postulates, but also depends on an *undeclared assumption* to fix the value of Lorentz's normalization factor. However, an example is discussed below that demonstrates that this choice introduces a contradiction into the theory since it leads to opposite predictions depending on how it is applied in this case.

## 2. The Clock Riddle and its Ramifications

One of the basic goals of relativity theory is to establish the relationship between the measured values of a given quantity obtained by two observers in relative motion to each other. In the following example the two observers (O and O') are initially at rest in inertial system S. They each measure the diameter of a sphere and agree that it has a value of D m. O' then places the sphere on his rocket ship and moves away from O. After some time he assumes a constant relative velocity  $v$  in the common  $x$ - $x'$  direction so that he is now at rest in inertial system S'. He then repeats the length measurements on the sphere and finds in accordance with the relativity principle that its diameter still has a value of D m in all directions. According to the FLC, O finds that the sphere has contracted along the  $x$  direction, but that its dimensions along all perpendicular directions have remained the same. Thus,

$$\Delta y = \Delta y' = D. \quad (1)$$

There is another way to carry out these measurements, however, namely to take advantage of Einstein's light-speed postulate [4]. Indeed, the modern-day definition of the meter [5] as the distance traveled by a light pulse in  $c^{-1}$  s ( $c=2.99792458 \text{ ms}^{-1}$ ) requires that the diameter be measured using clocks that are at rest in S and S', respectively. The theory assumes that the two clock rates are

not the same because of time dilation on the rocket ship and therefore that the measured elapsed times for the light to traverse the sphere satisfy the relation

$$[\gamma = (1 - v^2 c^{-2})^{-0.5}]:$$

$$\Delta t' = \gamma^{-1} \Delta t. \quad (2)$$

Accordingly, the above distance values have the following relation:

$$\Delta y' = c \Delta t' = c (\gamma^{-1} \Delta t) = \gamma^{-1} c \Delta t = \gamma^{-1} \Delta y = D. \quad (3)$$

The conclusion is that the two observers must *disagree* on their measured values for the diameter of the sphere and *by increasingly larger amounts* depending on how close their relative speed  $v$  approaches  $c$ , i.e.  $\Delta y = \gamma D$ . This is in clear contradiction to what was determined in eq. (1) on the basis of the FLC.

It needs to be emphasized that all of the above values come directly from application of Einstein's theory [4]. There is never a question about how the corresponding measurements to obtain the various quantities mentioned in eqs. (1-3) are actually carried out in practice. For example, it might be thought that the contradiction can be removed by simply arguing that the various results are not obtained at the same time. The problem with that approach is that  $S$  and  $S'$  move with constant relative velocity and thus there is no reason to expect that any of the measured values will change with time. The above example has been referred to in earlier work [6] as the “clock riddle” to distinguish it from the far better known “clock paradox” used to illustrate the essential role of acceleration in time dilation [7].

Comparison of the two theoretical methods for length measurements in the direction parallel to  $\mathbf{v}$  also uncovers a discrepancy. According to the FLC [4], the length of the sphere should contract on the rocket ship ( $S'$ ):

$$\Delta x' = \gamma \Delta x = D. \quad (4)$$

Since the rates of clocks are independent of orientation, one expects a perfectly analogous prediction to eq. (3) in this case, namely:

$$\Delta x' = c \Delta t' = c (\gamma^{-1} \Delta t) = \gamma^{-1} c \Delta t = \gamma^{-1} \Delta x = D. \quad (5)$$

Instead of observing a contraction in the parallel direction,  $O$  actually finds that the sphere's diameter has *increased* by the same fraction as above ( $\Delta x = \gamma D$ ). The conclusion is that *isotropic length expansion* accompanies time dilation in  $S'$ ,

not the type of anisotropic length contraction expected from application of the FLC.

All of the above results follow directly from the Lorentz transformation (LT) [4]. It should be obvious that any theory that is self-contradictory is not acceptable as a valid representation of physical events. The most reasonable answer to the question of why the LT has nonetheless received overwhelming support from the physics community for more than a century is that it has had such a good record when it comes to its many other predictions. To understand how this state of affairs has come about, it is best to revisit Einstein's derivation of the LT in 1905.

### 3. The alternative Lorentz transformation (ALT)

As mentioned in the Introduction, Lorentz noted as early as 1899 [2,3] that there was an undefined degree of freedom in the most general space-time transformation that leaves Maxwell's equations invariant. He expressed this relationship by inserting a *normalization* factor  $\varepsilon$  in the equations below:

$$t' = \gamma\varepsilon(t - vxc^{-2}) = \gamma\varepsilon\eta^{-1}t \quad (6a)$$

$$x' = \gamma\varepsilon(x - vt) \quad (6b)$$

$$y' = \varepsilon y \quad (6c)$$

$$z' = \varepsilon z, \quad (6d)$$

with  $\eta = (1 - vxt^{-1}c^{-2})^{-1} = (1 - vu_xc^{-2})^{-1}$ . Exactly the same equations were derived by Einstein [4] based on his two postulates of relativity, except that he used a slightly different notation than Lorentz (he referred to the normalization factor as  $\varphi$  instead of  $\varepsilon$ ). He eliminated the uncertainty posed by the degree of freedom in the general version of the LT by asserting (see p. 900 of Ref. 4) that “ $\varphi$  is a temporarily unknown function of  $v$ .” He then showed on the basis of symmetry considerations that  $\varphi/\varepsilon = 1$  is the only allowed value for the normalization function under these circumstances, thereby producing the LT upon substitution in eqs. (6a-d). However, it is important to understand that a clear assumption is involved in the above statement. *It amounts to a third postulate of relativity theory.*

The fact that Einstein did not declare it as an additional postulate is at the very least an interesting fact of history, but this would be an insignificant development if the assumption were true or at least supported by experimental data. However, the example of the “clock riddle” discussed in the previous section shows unequivocally that it is in fact false. No theory can be fully valid when it leads to opposite predictions for the same event depending on how it is applied. Fortunately, it is not difficult to see how the problem can be corrected, and to do so in a way that does not affect the many other confirmed predictions of Einstein’s original theory.

The first step in this direction is to note that the undefined normalization factor  $\varepsilon/\varphi$  is of no consequence in deriving the relativistic velocity transformation (VT). The VT is obtained directly from eqs. (6a-d) by dividing the three special equations by its temporal counterpart, with the result ( $u'_x = x't'^{-1}$  etc.):

$$u'_x = (1 - vu_x c^{-2})^{-1} (u_x - v) = \eta(u_x - v) \quad (7a)$$

$$u'_y = \gamma^{-1} (1 - vu_x c^{-2})^{-1} u_y = \eta\gamma^{-1} u_y \quad (7b)$$

$$u'_z = \gamma^{-1} (1 - vu_x c^{-2})^{-1} u_z = \eta\gamma^{-1} u_z. \quad (7c)$$

It is obvious that  $\varepsilon$  is simply cancelled out in each of the divisions and therefore does not appear at all in the VT [note also that  $\eta$  appears in all three equations by virtue of eq. (6a) of the general version of the LT]. A number of the most important results of relativity theory actually result directly from the VT and thus do not rely in any way on Einstein’s assumption about  $\varphi$ . These include the theory of optical aberration (starlight at the zenith) [8] and the Fresnel light-drag experiment [9], both of which experiments were quite important in Einstein’s thought process [10]. The VT also guarantees compliance with the light-speed postulate. It is used directly in the derivation of the Thomas precession of a spinning electron [11,12] and thus the LT is not essential in this case either. Moreover, the proof that Maxwell’s equations are invariant to the general form of the LT in eqs.(6a-d) demonstrates that the value chosen for  $\varepsilon$  is inconsequential for this purpose as well. Indeed, it was this fact that caused Lorentz to introduce the normalization factor  $\varepsilon$  in the general transformation in the first place [3].

The two main predictions of Einstein's theory [4] that do depend on the value of the normalization factor are time dilation and the FLC. The first experimental test of time dilation was carried out by Ives and Stilwell in 1938 with their study of the transverse Doppler effect [13]. The results confirmed Einstein's prediction [4] that the frequency of light  $\nu_r$  observed in the laboratory would always be less than the value of the emitted frequency  $\nu_e$  from a moving source [14]:

$$\nu_r = \gamma^{-1} \nu_e. \quad (8)$$

However, it is important to see that the above equation implies that the measurement process is *subjective*. In the Ives-Stilwell experiment, the light source was accelerated in the laboratory where the receiver is at rest. According to eq. (8), a decrease in frequency would also be observed if the tables were turned and light emitted from the laboratory were observed in the rest frame of the original moving source. In other words, each observer would find that it was the other's clock that had been slowed by time dilation as a result of their relative motion. This result was believed to be the inevitable consequence of the relativity principle. Since the Ives-Stilwell study was only a "one-way" experiment, it was incapable of verifying this aspect of Einstein's prediction.

This situation was remedied with the high-speed rotor experiments carried out by Hay et al. in 1960 using the Mössbauer technique [15]. In this case it was the absorber/detector rather than the light (x-ray) source that was subject to acceleration since it was mounted on the rim of the rotor. The empirical findings for the shift in frequency  $\frac{\Delta\nu}{\nu}$  are summarized by the formula:

$$\frac{\Delta\nu}{\nu} = (R_a^2 - R_s^2) \frac{\omega^2}{2c^2}, \quad (9)$$

where  $R_a$  and  $R_s$  are the respective distances of the absorber and x-ray source from the rotor axis ( $\omega$  is the circular frequency of the rotor). It shows that a shift to higher frequency (blue shift) is observed when  $R_a$  is greater than  $R_s$ , as in the present case. The corresponding result expected from eq. (8) would be:

$$\frac{\Delta\nu}{\nu} = \gamma^{-1} (|R_a - R_s| \omega) - 1 \approx -(R_a - R_s)^2 \frac{\omega^2}{2c^2}, \quad (10)$$

i.e. a red shift should be observed in all cases in accordance with the symmetric interpretation of time dilation. However, the results shown in eq. (9) indicate on the contrary that the effect is *anti-symmetric*, in clear contradiction to both eq. (8) and the LT. Hay et al. [15] nonetheless declared that their results were consistent with Einstein's theory [4] without mentioning the difficulty with the prediction of the LT. They also noted that eq. (9) can be derived from Einstein's equivalence principle [16], which equates centrifugal force and the effects of gravity. Subsequent experiments by Kündig [17] and Champeney et al. [18] also found that their results were summarized by eq. (9). Kündig stated explicitly that the results confirmed the position that it is *the accelerated clock that is slowed by time dilation*, thereby asserting that the measurement process is *objective* in this experiment, contrary to the prediction of eqs. (8) and (10).

A more detailed discussion of the transverse Doppler experiments and their relation to the LT may be found in a companion publication [19]. The main conclusion in the context of the search for an internally consistent version of the Lorentz transformation is that *the amount of the time dilation increases with the speed*  $v_{i0}$  of the x-ray source relative to the axis. Specifically, it is proportional to  $\gamma(v_{i0})$  [20-21]. A completely analogous result was obtained in the Hafele-Keating experiments with atomic clocks located on circumnavigating airplanes [22-23], which show clearly that it is the speed  $v_{i0}$  relative to the earth's center of mass that ultimately determines their rates. In that case the elapsed time  $\tau_i$  on a given clock satisfies the relation:

$$\tau_1 \gamma(v_{i0}) = \tau_2 \gamma(v_{20}). \quad (11)$$

The Hafele-Keating experiments also provide the basis for the methodology of the Global Positioning System (GPS). It is assumed that the rates of satellite clocks satisfy eq. (11) as well as a comparable relation for the gravitational red-shift [24]. In particular, it is found that the satellite clocks run slower than their counterparts on the ground when gravitational effects are excluded. Thus, *the symmetry principle predicted by the LT is contradicted by the everyday operations of GPS*. Exactly the same formula [19] applies to the Hay et al. experiments [15], in which case the axis of the rotor serves as reference for the speeds of the absorber and x-ray source that are to be inserted in the  $\gamma$  factors. Expansion of eq. (11) with  $v_{i0} = R_i \omega$  and  $\tau_i = \nu_i^{-1}$  leads directly to the empirical formula given in eq. (9).

Thus, eq. (11) can be called the *Universal Law of Time Dilation* (ULTD).

It is a simple matter to convert this equation into the form of the general Lorentz equation for time dilation given in eq. (6a):

$$t' = Q^{-1}t. \quad (12)$$

In this equation  $Q$  is the ratio of clock rates in  $S$  and  $S'$  as determined by the ULTD. Accordingly,  $Q > 1$  if the clock at rest in  $S'$  runs more slowly, and by virtue of the fundamental objectivity of the revised theory,  $Q < 1$  if it runs faster than that in  $S$ . In the typical case where the clock at rest in  $S'$  has been accelerated relative to  $S$  before returning to a state of uniform translation,  $Q = \gamma(v)$ . Eq. (11) is more general since it also accounts for the situation when both clocks being compared are moving relative to the original rest frame  $S_0$ . It is clearly necessary in applying eq. (11) to first identify the above rest frame; it has been referred to as the objective rest system (ORS) in earlier work [25]. The relative speed  $v$  of  $S$  and  $S'$  is not directly involved in the ULTD, thereby eliminating the subjective character of the measurement process otherwise inherent in Einstein's LT.

Combining eq. (12) with eq. (6a) allows one to calculate the value of the normalization factor  $\varepsilon$ :

$$\varepsilon = \eta(\gamma Q)^{-1}. \quad (13)$$

Substitution of this value of  $\varepsilon$  into the general version of the Lorentz transformation in eqs. (6a-d) then gives the alternative version of the Lorentz transformation (ALT) that satisfies the ULTD as well as the VT and Einstein's two postulates of relativity for any fixed value of  $Q$ :

$$t' = Q^{-1}t \quad (14a)$$

$$x' = \eta Q^{-1}(x - vt) \quad (14b)$$

$$y' = \eta(\gamma Q)^{-1}y \quad (14c)$$

$$z' = \eta(\gamma Q)^{-1}z, \quad (14d)$$

whereby eq. (14a) is by construction the same as eq. (12). The same result can be obtained somewhat more directly by combining eq. (12) with the VT of eqs. (7a-c) by multiplying the various velocity components with the corresponding times in  $S$  and  $S'$ , respectively. The clear distinction between the ALT and the LT is that there is *no space-time mixing* in the former set of equations. The arguments



in Einstein's version of relativity for *remote non-simultaneity of events* as a necessary condition for satisfying the light-speed postulate are therefore *negated* by the ALT [26]. There is also no possibility of forcing a violation of Einstein causality through time reversal [27] since the constant  $Q$  is necessarily positive. The FLC is also no longer a consequence of relativity theory, thereby removing the contradiction that results when the LT is applied to the solution of the clock riddle [6,28]. Other distinguishing features of eqs. (14a-d) are discussed in the following section.

#### 4. Lorentz invariance and the amended relativity principle (ARP)

While it is clear that the ALT satisfies the light-speed postulate because of its direct relationship to the VT, it still remains to show that the choice for the normalization constant in eq. (13) also satisfies the other of Einstein's relativity postulates, the relativity principle [4]. This question is closely tied up with the condition of Lorentz invariance that is a key feature of the LT. Squaring and adding the four relations in eqs. (6a-d) leads to the following result:

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = \varepsilon^2 (x^2 + y^2 + z^2 - c^2 t^2). \quad (15)$$

The value for the normalization function of  $\varepsilon / \varphi = 1$  assumed by Einstein [4] to obtain the LT leads to the aesthetically pleasing and transparently symmetric form that is so familiar to theoretical physicists. Most importantly, Einstein's version of eq. (15) satisfies the relativity principle since it looks exactly the same from the vantage point of both observers. It is less obvious how any other choice of  $\varepsilon$  can satisfy the latter requirement, and this has been used to justify adopting the value of unity in deriving the LT. Specifically, the question arises as to whether the choice of  $\varepsilon = \eta(\gamma Q)^{-1}$  that leads to the ALT is also consistent with the relativity principle.

To consider this possibility it is helpful to first write down the corresponding result obtained from the inverse of eqs. (6a-d), which can be obtained most simply by algebraic manipulation of eq. (15):

$$x^2 + y^2 + z^2 - c^2 t^2 = \varepsilon^{-2} (x'^2 + y'^2 + z'^2 - c^2 t'^2). \quad (16)$$

To satisfy the relativity principle, the latter equation must be consistent with an alternative form of eq. (15) that is obtained by switching the roles of the two

inertial systems and the respective observers in these rest frames. This is accomplished by simply exchanging all primed and unprimed subscripts and changing the sign of their relative speed from  $v$  to  $-v$ , with the result:

$$x^2 + y^2 + z^2 - c^2 t^2 = \varepsilon'^2 (x'^2 + y'^2 + z'^2 - c^2 t'^2). \quad (17)$$

The relativity principle therefore demands that a) the new normalization factor  $\varepsilon'$  be defined in a completely analogous manner as  $\varepsilon$  and b) that the following relation between  $\varepsilon$  and  $\varepsilon'$  be satisfied:

$$\varepsilon^2 \varepsilon'^2 = 1. \quad (18)$$

It is obvious that Einstein's value of  $\varepsilon = \varepsilon' = 1$  satisfies both of the above requirements. Indeed, a value of  $\varepsilon = \varepsilon' = -1$  also is not excluded by these conditions.

The value of  $\varepsilon$  in eq. (13) that is used to derive the ALT of eqs. (14a-d) leads to the following relation when substituted in eq. (18):

$$\eta^2 (\gamma Q)^{-2} \eta'^2 (\gamma Q')^{-2} = 1, \quad (19)$$

whereby  $\eta'$  must be obtained from  $\eta = (1 - v u_x c^{-2})^{-1} = (1 - v x t^{-1} c^{-2})^{-1}$  in the standard way, i.e. consistent with condition b) above, by exchanging corresponding primed and unprimed values and changing  $v$  to  $-v$ :  $\eta' = (1 + v u'_x c^{-2})^{-1} = (1 + v x' t'^{-1} c^{-2})^{-1}$ . The value of  $\gamma$  remains the same because it is a function of  $v^2$ , and the value of  $Q' = Q^{-1}$  is fixed by forming the inverse of eq. (14a), i.e.  $t = Q'^{-1} t' = Q t'$ . Thus,  $Q$  and  $Q'$  bear a reciprocal relationship to one another, as one expects from an objective theory of measurement.

Substitution in eq. (19) thereby simplifies the condition of relativistic invariance to:

$$\eta^2 \eta'^2 \gamma^{-4} = 1. \quad (20)$$

From the definitions of  $\eta$  and  $\eta'$ , it follows that

$$\eta \eta' = \gamma^2 \dots \quad (21)$$

This result is obtained by using eq. (7a) of the VT to define  $u'_x$  in  $\eta'$  in terms of  $u_x$ . It is obviously compatible with eq. (20), as required by the relativity principle. It also removes the sign ambiguity referred to above in connection with Einstein's original derivation of the LT [4].

The above discussion demonstrates *that space-time mixing is not essential to satisfy the relativity principle*. The direct proportionality assumed in eq. (14a) between the respective clock rates in S and S' is quite consistent with experimental findings, including the GPS methodology, but it also seemingly conflicts with the conventional view that all inertial systems are equivalent and therefore indistinguishable [29]. Galileo's original arguments when he introduced the relativity principle in 1623 shed considerable light on this issue. He used the example of passengers locked in the hold of a ship who were trying to determine whether they were still located at the dock or were moving on a perfectly calm sea [30]. His main point was that it would be impossible for them to make this determination on the basis of their purely *in situ* observations. More interesting in the present context, however, is that this argument does not exclude the possibility that objects on the ship, including the passengers themselves, did not undergo changes in their physical measurements as a result of the ship's motion. Rather, the assertion is that *all such changes must be perfectly uniform*, and that this is the fundamental reason why no distinction can be observed without carrying out measurements outside the ship's hold. That interpretation is also consistent with Einstein's original work [4] in which he concluded that acceleration of a clock leads to a decrease in its rate. After the acceleration phase is concluded and a new state of motion is reached, it seems reasonable to assume that the clock's rate continues to be slower than in its original state. The relativity principle simply states that the rates of all clocks are altered in the same proportion when they make the transition between the same two inertial systems. Similarly as with the First Law of Thermodynamics, it does not matter which intermediate states were reached in the process as long as the initial and final states are identical [31].

Another way of describing the above effect is to say that the *unit of time* has changed as a result of the acceleration. Based on exclusively *in situ* measurements, it is impossible for a co-moving observer to recognize this change. Atomic frequencies and the rates of decay processes continue to have the same values according to his measurements, even though they have changed in a well-defined and experimentally verifiable manner from the vantage point of a stationary observer in the original rest frame. Moreover, there is every reason to

assume that analogous changes occur in the values of other physical properties such as lengths and inertial masses.

In view of these considerations it is advisable to expand upon the usual definition to obtain the following amended relativity principle (ARP): *The laws of physics are the same in all inertial systems, but the physical units in which they are expressed vary in a well-defined manner from one system to another.* The constant  $Q$  in the ALT of eqs. (14a-d) can thus best be seen as a conversion factor between the different sets of units employed in  $S$  and  $S'$ , respectively. Each observer believes that his various units have standard values, but they are in fact not the same as the corresponding values in another inertial system. It needs to be emphasized that such an interpretation in terms of different fixed systems of rational units is not feasible when the LT is used because of its fundamentally subjective description of the measurement process. Finally, a comprehensive discussion of how such conversion factors can be determined for a wide range of physical properties can be found in a companion publication [32]. Values are given both for changes in the state of motion of the observer as well as for his position in a gravitational field.

## 5. Conclusion

The Lorentz transformation (LT) lacks internal consistency and is therefore not a valid component of physical theory. It allows for two distinct methods of measuring distances on a moving object, one based on FitzGerald-Lorentz length contraction (FLC) and the other making use of elapsed times for light to pass between the endpoints of the object. The corresponding results of the theory do not generally agree. For example, when the distance is measured parallel to the direction of motion, no change is expected. Yet the elapsed time (on a neutral clock) for light to pass between its endpoints should increase with the relative speed of the object, thereby indicating that the distance has *increased* in direct proportion to the amount of time dilation. An even larger discrepancy between the two methods occurs for parallel length measurements. This state of affairs has been referred to as the “clock riddle” of relativity theory. It shows that Einstein’s LT is self-contradictory and must be rejected.

A review of Einstein's derivation in 1905 shows that he made an undeclared assumption in addition to his two postulates of relativity. He removed the inherent degree of freedom in the general form of the Lorentz transformation by concluding that the relevant normalization constant (referred to as  $\varphi$  in his work) may only have a value of unity. This choice leads directly to the FLC, thereby causing the discrepancy revealed in the clock riddle. Making the choice instead on the basis of experimentally verified characteristics of time dilation that are assumed in the methodology of the Global Positioning System (GPS) leads to an alternative version of the Lorentz transformation (ALT) that is internally consistent. Specifically, it is assumed that clock rates in different inertial systems (located at the same gravitational potential) are strictly proportional to one another:  $t' = Q^{-1}t$ . This assumption fixes the value of the normalization constant but also guarantees that the two postulates of relativity are satisfied. The ALT therefore leads to the same relativistic velocity transformation (VT) as in Einstein's original work [4] because the latter is completely independent of the choice of  $\varphi$ . Consequently, the revised theory predicts many of the same effects such as aberration of starlight at the zenith, Fresnel light-drag and Thomas spin precession as the original since all of the latter can be derived exclusively on the basis of the VT. *Indeed, there is no confirmed experimental result of Einstein's theory which does not also result from application of the ALT instead of the LT.*

The fundamental deficiency of the LT with regard to its description of GPS is its failure to predict that the clocks satellites are slowed more by time dilation than their identical counterparts on the earth's surface. It predicts instead that it is simply a matter of perspective which set of clocks runs slower. If this were true, satellite clocks would have to be adjusted to have a *lower* rate due to time dilation (after correction for the gravitational red shift) from the vantage point of an onboard observer instead of increasing their rate as is actually required to bring them into synchronization with the clocks on the ground. This subjective characterization of the measurement process which results from application of the LT is not supported by actual experimental tests of time dilation that have been carried out over the past 50 years with rotors, meta-stable particles and circumnavigating airplanes. Instead, all such results are accurately described by the Universal Law of Time Dilation (ULTD) of eq. (11). Accordingly, it is not

the relative speed  $v$  of  $S$  and  $S'$  that determines the amount of time dilation but rather their respective speeds relative to a well-defined reference frame, which is the axis of the rotor in the former and the earth's center of mass in the other two cases mentioned. Time dilation is completely distinct from the gravitational red shift, for which the respective locations of the clocks in a gravitational field are rate-determining. The constant  $Q$  in the ALT eqs. (14a-d) is determined by taking the appropriate ratio of the respective  $\gamma$  factors in the ULTD for the two rest frames being compared. Clock rates are simply proportional in the ALT [eq. (14a)]; *there is no space-time mixing as foreseen by the LT*. The strict proportionality of elapsed times excludes the occurrence of remote non-simultaneity and it also rules out the possibility of time reversal under any circumstances. The ALT is also inconsistent with the FLC, thereby removing the contradiction which results from application of the LT to the clock riddle. Distances are to be measured with clocks, consistent with the modern definition of the meter and the everyday applications of GPS.

The ALT satisfies the relativity principle, but not the condition of Lorentz invariance. Instead, it conforms to a more subtle form of space-time invariance that takes into account differences in the units employed in the respective rest frames and the orientation of their relative velocity to that of the object of the measurement ( $-vu_x$  and  $vu'_x$  in the  $\eta$  factor of the VT). The simple proportionality of clock rates in eq. (14a) of the ALT is not compatible with the highly symmetric nature of conventional Lorentz invariance, just as it is also not consistent with the symmetry of the measurement process prescribed by the LT by virtue of Einstein's  $\phi = 1$  assumption. For the sake of clarity it is important to amend the usual statement of the relativity principle (ARP) to emphasize that the system of units in which the laws of physics are expressed are generally not the same in different inertial systems/rest frames. One doesn't need the LT to transform these laws, rather a well-defined prescription for quantitatively determining the appropriate conversion factor for each physical unit needed to accomplish this goal. Detailed analysis [32] shows that all such conversion factors are integral powers of the constant  $Q$  in the ALT for purely kinematic changes, whereas an analogous situation holds for the constant employed for predicting the corresponding variations caused by gravitational effects. Normal

algebraic manipulations suffice to deduce the values of conversion factors for a given quantity from its composition in terms of the fundamental units of distance, inertial mass and time. These details illustrate that the present revised theory of relativity is based on a purely objective theory of measurement. Since there has never been a confirmed experiment that supports the subjective character of Einstein's original version, this aspect of the present theory is quite positive and complements the decisive advantage it holds by virtue of its internal consistency with regard to distance determinations.

(January 7, 2013)

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