# Voigt's Conjecture of Space-time Mixing: Contradiction between Non-simultaneity and the Proportionality of Time Dilation

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**Abstract**

The history of the development of the Lorentz transformation (LT) is reviewed, starting with the original suggestion of Voigt in 1887 for a modification of the longstanding classical (Galilean) relationship between space and time coordinates. His conjecture has led to the currently accepted view by theoretical physicists that space and time are inextricably mixed and are thus merely two components of a single entity "spacetime." The LT itself, which retains the space-time mixing characteristic, was first introduced by Larmor. He recognized that Voigt's transformation needed to be amended in order to conform to the requirements of the Relativity Principle (RP). It is pointed out that Newton's First Law indicates that clock rates must remain fixed in the absence of unbalanced external forces, which therefore implies that the ratio of two such rates in different inertial rest frames should be time-independent as well ($\Delta t'=\Delta t/Q$). It is critical in this discussion to note that the non-simultaneity of events demanded by space-time mixing is not consistent with the proportionality of the time-dilation prediction of the LT; it is impossible for $\Delta t$ and $\Delta t'$ to be proportional to one another without both of them vanishing at the same time. This contradiction removes the LT from contention as a physically valid transformation. Lorentz showed at the end of the 19th century that there was a degree of freedom in the definition of the LT that could be explored to eliminate this inconsistency. By choosing a particular value for a normalization constant, it is possible to obtain a different transformation (GPS-LT) which eliminates space-time mixing while still satisfying both of Einstein's two postulates of relativity and remaining consistent with Newton's First Law. The *asymmetric* time dilation observed in many experiments and assumed in the operation of the Global Positioning System indicates that clock-rate proportionality should be an essential component of relativity theory, in agreement with the GPS-LT assumption of a strict proportionality between the rates of clocks in different inertial systems.

*Keywords: Time dilation, remote non-simultaneity, Lorentz transformation (LT), Universal Time-dilation Law (UTDL), alternative Global Positioning System-Lorentz transformation (GPS-LT)*

## I. INTRODUCTION

Confusion ran high among physicists in the latter half of the 19th century because of their inability to explain the results of a number of experiments that had been recently carried out with light waves [1]. It had started with the Fresnel light-drag experiment, which not only showed that light is slowed as it moves through a transparent medium but, by extrapolation of the value of the medium's refractive index n to a unit value, that the observed light speed in the laboratory should be completely independent of the speed v of the medium in the limit of free space

[ $c(v) = c$ ]. Maxwell's theory of electricity and magnetism published in 1864 also indicated that the speed of light had the same constant value c in each rest frame in which it is observed. This result was clearly at odds with the traditional application of the classical space-time (Galilean) transformation which indicates that speeds should be additive and therefore that $c + v \neq c$. This led to a frantic search for an "ether" which serves as a rest frame for the light waves analogous to that known for sound waves. Michelson and Morley [2] used their newly developed interferometer to test this theory, but it merely verified the conclusion that the speed of light is independent of the rest frame through which it moves, in particular that it is directionally independent at all times of the year.

Voigt [3] then stepped into the fray with what in retrospect must be seen as both a daring and ingenious proposition. He speculated in 1887 that the problem lay with the Galilean transformation itself. He attempted to resolve the issue by using nothing more than a free parameter and a little algebra. The resulting transformation was ultimately rejected on other physical grounds, namely it violates Galileo's Relativity Principle (RP), but it is nonetheless deserving of more than just a footnote in history. This is because it introduced for the first time the concept of space-time mixing, which remains to the present day to be a dogmatic principle of theoretical physics. It contradicts one of Newton's most cherished beliefs, which held sway with the physics community for several centuries, namely that space and time are completely separate entities, one measured with a yardstick and the other with a clock. The consequences of this aspect of Voigt's conjecture will be discussed in the following.

**II. Derivation of the Voigt transformation**

The starting point of Voigt's derivation is the classical or Galilean transformation (GT). It relates the measured values of space (x,y,z) and time (t) for a given object obtained by two observers in relative motion to one another. It is assumed that the two observers are separating with constant speed v along the common x,x' axis of the their respective coordinate systems. The relationship between their measured values is given below in terms of their respective coordinates, x,y,z,t and x',y',z',t', whereby it is assumed that the two systems are coincident at t=t'=0:

$$t' = t \tag{1a}$$

$$x' = x - vt \tag{1b}$$

$$y' = y \tag{1c}$$

$$z' = z\,. \tag{1d}$$

By definition, the velocity of the object in each coordinate system is obtained by division of the space and time coordinates at any instant. Using eqs. (1a-b), one therefore obtains the key relationship between the measured speeds of the object when it moves along the x,x' axis:

$$\frac{x'}{t'} = u'_x = \frac{x}{t} - v = u_x - v\,. \tag{2}$$

There is thus a linear relation connecting the two values of the speed of the object. More generally, the GT predicts that the corresponding velocities **u** and **u'** are related by vector addition when the object travels in a direction which is not parallel to the separation velocity of the two observers. According to eq. (2), if a light wave has the speed $u_x = c$, it follows that the corresponding value for the other observer is $u'_x = c - v$. It is exactly this expected relationship that is not supported by the above experiments. Voigt's solution [3] to this problem is simply to add an extra term to eq. 1(a) which depends on x and which also contains a free parameter a to be determined by requiring that $u_x = u'_x = c$ for a light wave moving along the x axis:

$$t' = t + ax\,. \tag{3}$$

Combining this relation with eq. (1b) of the GT, one obtains the following equation for the two values of the speed of light:

$$\frac{x'}{t'} = c = \frac{x - vt}{t + ax} = \frac{\frac{x}{t} - v}{1 + \frac{ax}{t}} = \frac{c - v}{1 + ac}, \qquad (4)$$

from which one concludes that $a = -vc^{-2}$ in eq. (3). The Voigt replacement for eq. (1a) of the GT is thus determined to be:

$$t' = t - vc^{-2}x \qquad (5).$$

The above derivation needs to be extended to apply to motion of the light waves in an arbitrary direction. This can be done most simply by first forming the quantity ( $x'^2 - c^2t'^2$ ) using both eqs. (1b) and (5), which is seen to have the following result:

$$\left(x'^2 - c^2t'^2\right) = \left(1 - v^2c^{-2}\right)\left(x^2 - c^2t^2\right) = \gamma^{-2}\left(x^2 - c^2t^2\right), \qquad (6)$$

where $\gamma = \left(1 - v^2c^{-2}\right)^{0.5}$. Eq. (6) is seen to not only verify the above light-speed relation for motion along the x axis, but also to give a clear indication of how to obtain the desired generalization for the case when the light waves travel in a different direction than along the x axis. By assuming instead of eqs. (1c-d) that $y' = \gamma^{-1}y$ and $z' = \gamma^{-1}z$, one arrives at the following relation between the two sets of coordinates for light waves moving in an arbitrary direction:

$$\left(x'^2 + y'^2 + z'^2 - c^2t'^2\right) = \gamma^{-2}\left(x^2 + y^2 + z^2 - c^2t^2\right). \qquad (7)$$

Both sides of this equation vanish for a light wave regardless of its direction in space, which was the goal of Voigt's derivation [3]. The corresponding transformation is thus:

$$t' = t - vc^{-2}x \qquad (8a, 5)$$

$$x' = x - vt \tag{8b, 1b}$$

$$y' = \gamma^{-1} y \tag{8c}$$

$$z' = \gamma^{-1} z \,. \tag{8d}$$

It can be seen that this set of equations reduces to the GT of eqs. (1a-d) in the limit of null relative velocity of the two observers, i.e. if we ignore the fact that the equations are useless in this case (with $v = 0$). More significant is the fact that the same equations reduce to the GT when c is assumed to have an infinite value. One can say then without qualification that the classical transformation (GT) contains the implicit assumption that the speed of light is infinite.

## III. Taking the Relativity Principle into account

The space-time transformation that Voigt [3] presented is successful in satisfying the light-speed constancy condition, but it fails on other grounds. This can be seen by evaluating the inverse transformation, obtained by Gauss elimination from eqs. (8a-d):

$$t = \gamma^2 \left( t' + vc^{-2} x' \right) \tag{9a}$$

$$x = \gamma^2 \left( x' + vt' \right) \tag{9b}$$

$$y = \gamma y' \tag{9c}$$

$$z = \gamma z' \,. \tag{9d}$$

According to Galileo's Relativity Principle (RP), the inverse transformation should be obtained by simply exchanging the primed and unprimed subscripts in the forward set of equations and substituting −v for v. This is a mathematical procedure that mimics the situation when the observers change positions; it will be referred to as Galilean inversion in the following. It is clear that eqs. (9a-d) do not satisfy this relationship relative to eqs. (8a-d), hence showing that the Voigt transformation is not consistent with the RP and thus must be rejected as a physically valid set of equations.

It is nonetheless a simple matter to modify the transformation in a way which satisfies both the RP and the light-speed constancy condition. Before doing this, it is helpful to make a change in variables to intervals for two different events: $\Delta x = x_2 - x_1$, $\Delta x' = x'_2 - x'_1$ etc. This change allows each observer to choose his own coordinate system without the necessity of having it coincide at some point with the other coordinate system. Intervals are of course required in order to compute speeds, which remains the center of attention in this discussion. This being done, one merely needs to multiply each of the corresponding eqs. (8a-d) by a factor of γ on the right, with the result:

$$\Delta t' = \gamma\left(\Delta t - vc^{-2}\Delta x\right) = \gamma\eta^{-1}\Delta t \tag{10a}$$

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right) \tag{10b}$$

$$\Delta y' = \Delta y \tag{10c}$$

$$\Delta z' = \Delta z\ , \tag{10d}$$

with $\eta = \left(1 - vc^{-2}\dfrac{\Delta x}{\Delta t}\right)^{-1}$ in eq. (10a).

This space-time transformation was first presented by Larmor [4]. It is what we know today as the Lorentz transformation (LT). The inverse set of equations is obtained by the Galilean inversion procedure as well as by Gauss elimination:

$$\Delta t = \gamma\left(\Delta t' + vc^{-2}\Delta x'\right) = \gamma\eta'^{-1}\Delta t' \tag{11a}$$

$$\Delta x = \gamma\left(\Delta x' + v\Delta t'\right) \tag{11b}$$

$$\Delta y = \Delta y' \tag{11c}$$

$$\Delta z = \Delta z', \tag{11d}$$

[ $\eta' = \left(1 + vc^{-2}\frac{\Delta x'}{\Delta t'}\right)^{-1}$ ] in eq. (11a); note that η' and η are related by Galilean inversion. The same procedure as for eq. (7) for the Voigt transformation when applied to the LT gives:

$$\left(x'^2 + y'^2 + z'^2 - c^2t'^2\right) = \left(x^2 + y^2 + z^2 - c^2t^2\right). \tag{12}$$

This relationship is referred to as Lorentz invariance. On this basis it is obvious that the LT satisfies both the light-speed constancy requirement and the RP.

The relativistic velocity transformation (RVT) is easily obtained from the LT by dividing the three equations for Δx', Δy', and Δz' in eqs. (10b-d) by Δt' in eq. (10a):

$$u'_x = \left(1 - vu_xc^{-2}\right)^{-1}\left(u_x - v\right) = \eta\left(u_x - v\right) \tag{13a}$$

$$u'_y = \gamma^{-1}\left(1 - vu_xc^{-2}\right)^{-1}u_y = \eta\gamma^{-1}u_y \tag{13b}$$

$$u'_z = \gamma^{-1}\left(1 - vu_xc^{-2}\right)^{-1}u_z = \eta\gamma^{-1}u_z, \tag{13c}$$

The same definitions for η and γ are used as in the LT; $u'_x = \frac{\Delta x'}{\Delta t'}$, $u_x = \frac{\Delta x}{\Delta t}$, etc. Note that the *same set of equations* results from the Voigt transformation when the analogous divisions are made using eqs. (8a-d).

The corresponding inverse of the RVT can be obtained using Galilean inversion, demonstrating that these equations also satisfy the RP. The following identity is useful in proving the inverse relationship [5]: $\eta\eta' = \gamma^2$. The proof given below relies on eq. (13a):

$$\eta\eta' = \left[\left(1 - u_xvc^{-2}\right)\left(1 + u'_xvc^{-2}\right)\right]^{-1} = \left[\left(1 - u_xvc^{-2}\right)\left(1 + \eta vc^{-2}\left(u_x - v\right)\right)\right]^{-1}$$

$$= \left[\left(1 - u_xvc^{-2}\right)\frac{1 - u_xvc^{-2} + vc^{-2}u_x - v^2c^{-2}}{1 - u_xvc^{-2}}\right]^{-1} \tag{14}$$

$$= \left(1 - \mathrm{v}^2\mathrm{c}^{-2}\right)^{-1} = \gamma^2$$

This identity will also prove useful in Sect. IV.

Lorentz [6] took note of the experience of Voigt and Larmor with relativistic space-time transformations, especially that both results are consistent with the light-speed constancy requirement. He pointed out that there is a degree of freedom [7] in the definition of such transformations that can be expressed in terms of a type of normalization constant which he referred to as ε. The resulting general transformation (GLT) is given below:

$$\Delta t' = \gamma\varepsilon\left(\Delta t - v\Delta x c^{-2}\right) = \gamma\varepsilon\eta^{-1}\Delta t \tag{15a}$$

$$\Delta x' = \gamma\varepsilon\left(\Delta x - v\Delta t\right) \tag{15b}$$

$$\Delta y' = \varepsilon\Delta y \tag{15c}$$

$$\Delta z' = \varepsilon\Delta z \tag{15d}$$

The original transformation given by Voigt [3] is obtained from the GLT by setting $\varepsilon = \gamma^{-1}$ , whereas Larmor's LT results for $\varepsilon = 1$. The fact that this degree of freedom exists is easily understandable because light-speed constancy only puts a restriction on the ratio of space and time variables. Any proportionality constant therefore suffices to fulfill this condition. As a consequence, the RVT can also be obtained from the GLT by appropriate division of its space and time variables.

The inverse of the GLT is obtained by Galilean inversion:

$$\Delta \mathrm{t} = \gamma\varepsilon'\left(\Delta \mathrm{t}' + \mathrm{v}\mathrm{c}^{-2}\Delta \mathrm{x}\right) = \gamma\varepsilon'\eta'^{-1}\Delta \mathrm{t}' \tag{16a}$$

$$\Delta \mathrm{x} = \gamma\varepsilon'\left(\Delta \mathrm{x}' + \mathrm{v}\Delta \mathrm{t}'\right) \tag{16b}$$

$$\Delta \mathrm{y} = \varepsilon'\Delta \mathrm{y}' \tag{16c}$$

$$\Delta z = \varepsilon' \Delta z' \tag{16d}$$

which also defines the relationship between ε and ε' in the two sets of equations. It is easy to show that this relationship places a condition on the value of ε required for a particular transformation to also satisfy the RP, namely $\varepsilon = \frac{1}{\varepsilon'}$. This condition is not satisfied by the Voigt transformation of eqs. (8a-d), in which case both ε and ε' are equal to $\frac{1}{\gamma}$. On the other hand, it is obvious that the LT satisfies the RP since $\varepsilon = \varepsilon' = 1$ in that case.

The degree of freedom in the GLT raises the key question of whether the value of $\varepsilon = 1$ is a unique solution for determining the desired relativistic space-time transformation. Poincaré [8] at least made an attempt to justify the latter choice. He pointed out that the set of LT 4x4 matrices for different relative speeds v forms a mathematical group. Multiplication of any two of them leads to a third which has the same form and a different value of the relative speed which agrees with the RVT. This argument is still a popular justification for the LT value of $\varepsilon = 1$, but there is a problem with it nonetheless. The group character only results when the two relative velocities are in the same direction, a fact which Thomas [9] used 20 years later to predict the precession of electronic spins. Another argument for the $\varepsilon = 1$ choice is based on the assertion that distances measured transverse to the direction of relative motion of the two observers should be the same for both, as in eqs. (10c,d). Yet, the analogous argument for the corresponding RVT eqs. (13b-c) does not hold; the ratio of two such components is equal to $\eta\gamma^{-1}$ in the general case.

Einstein [10] derived the same set of GLT equations as Lorentz [6,7] but referred to the normalization constant as φ instead of ε. He tackled the problem of justifying the LT choice of $\varphi = 1$ as follows. He stated that "φ is a temporary undefined function of v." He then used a symmetry argument to prove that *under these circumstances* the only allowed value of φ is unity, thereby arriving at the definition of the LT in eqs. (10a-d). It has gone largely unnoticed, however, that Einstein's assertion that φ can only depend on v is an *assumption* that also requires justification. In short, the derivation of the LT, which is the basis for the strongly held belief that space and time are inextricably mixed, rests on an unproven assumption about the value of a normalization constant in the GLT of eqs. (15a-d).

**IV. Contradiction between remote non-simultaneity and Einsteinean time dilation**

Is space-time mixing the inevitable consequence of the empirical fact of light-speed constancy, as originally proposed by Voigt [3] in 1887? Is the LT the unique solution to the goal of merging the two concepts in the relativistic theory of kinematics? Both questions posed a clear challenge to Newtonian mechanics, and so there is merit in considering what light can be shed upon each of them by the classical theory.

A good place to start with is Newton's First Law of Kinematics (Law of Inertia). Both the observers and the object of their measurement to be described in terms of the space-time transformation are assumed to move with constant speed and direction. This assumption is consistent with the Law of Inertia, and is a result of the complete absence of unbalanced forces acting on them. Under the circumstances, what should one expect for the properties of the object of the measurement? In accordance with the Law of Causality, it can be assumed that all these properties *should remain unchanged* for an indefinite amount of time, *including the rates of its stationary clocks*. The same conclusion holds for all other inertial systems, no matter what their speed and direction might be. The respective rates of the two sets of clocks can be different, however, but their *ratio* must then be a *constant* as well. On this basis, one can deduce a clear relationship between the elapsed times Δt and Δt' measured by inertial clocks for any given event, namely $\Delta t' = \frac{\Delta t}{Q}$, where Q is a proportionality constant which does not depend in any way on the object of the measurement. This result is clearly at odds with eq. (10a) of the LT.

Does the above proportionality relationship prove that Newton's First Law is in violation of the LT? Or is the opposite the case? To consider this question, it is well to recall two of the predictions about time relationships that result from the LT: remote non-simultaneity of events and time dilation. Poincaré [11] recognized that eq. (10a) indicates that pairs of events that are simultaneous for one observer might not be so for another moving relative to him, and on this basis he began to question the traditional belief that everything occurs at the same time throughout the universe. For example, suppose that observer O finds that two events occur at the same time A on his stationary proper clock ($\Delta t = 0$), but that observer O' finds instead that they occur at different times B and C on his stationary proper clock, i.e. $B \neq C$ and $\Delta t' \neq 0$. This situation is allowed according to eq. (10a).

The prediction of time dilation is derived [10,12] by considering the relationship between elapsed times Δt and Δt' for the same event that are measured by two observers in relative motion with speed v along the x axis. Attention is centered on a clock that is stationary in the rest frame of one of the observers ($\Delta x' = 0$). The other observer (O) finds that this clock moves during the measurement from his vantage point, specifically that the change in its position is $\Delta x = v\Delta t$. Substitution of the latter relationship into eq. (10a) of the LT leads to the following equation:

$$\Delta t' = \gamma(v)\left(\Delta t - v^2 c^{-2} \Delta t\right) = \gamma^{-1} \Delta t \qquad [17]$$

According to eq. (17) the elapsed times in the above example are always strictly proportional to one another. Thus for the first event, since observer O found it to occur at time A on his stationary clock, the corresponding value obtained by the other observer (O') must be

$B = \frac{A}{\gamma}$. Similarly for the second event, eq. (17) leads one to conclude that $C = \frac{A}{\gamma}$. In other words, $B = C$ according to this argument. This equality stands in contradiction to the original inequality based on the non-simultaneity assumption, namely $B \neq C$. The same result can be obtained more directly by considering time differences for the two events. Since they occur at the same time for $O(\Delta t = 0)$, it is impossible according to eq. (17) that the corresponding time difference for O'(Δt') not be equal to zero.

Both the predictions of remote non-simultaneity and proportional time dilation are obtained from eq. (10a) of the LT. *The inescapable conclusion is therefore that the LT is not a physically valid space-time transformation.* It is simply unacceptable that the same theory gives diametrically opposite answers for the same question. The conflict between Newton's First Law and the LT mentioned at the beginning of this section is thus resolved in favor of the former. The question is thus open as to whether the First Law is itself compatible with light-speed constancy and the RP. It will be shown in the following that there is nothing standing in the way of this possibility.

**V. An alternative version of the Lorentz transformation**

The GLT of eqs. (15a-d) offers a surprisingly easy means of incorporating the clock-rate proportionality implied by Newton's Law of Inertia into relativistic theory. One simply needs to evaluate the normalization constant under the condition of Δt'=Δt/Q derived in Sect. IV. Starting with eq. (15a) one arrives at the following equation [13-16]:

$$\Delta t' = \gamma(v)\varepsilon\left(\Delta t - vc^{-2}\Delta x\right) = \gamma\varepsilon\eta^{-1}\Delta t = \frac{\Delta t}{Q} \qquad (18)$$

Upon solving for ε the result is:

$$\varepsilon = \frac{\eta}{\gamma \mathrm{Q}}. \tag{19}$$

Inserting this value in the GLT equations then gives the desired space-time transformation, originally referred to as the Alternative Lorentz Transformation (ALT [13-16]):

$$\Delta t' = \eta \frac{\Delta t - vc^{-2}\Delta x}{\mathrm{Q}} = \frac{\Delta t}{\mathrm{Q}} \tag{20a}$$

$$\Delta x' = \eta \frac{\Delta x - v\Delta t}{\mathrm{Q}} \tag{20b}$$

$$\Delta y' = \left(\frac{\eta}{\gamma \mathrm{Q}}\right)\Delta y \tag{20c}$$

$$\Delta z' = \left(\frac{\eta}{\gamma \mathrm{Q}}\right)\Delta z \tag{20d}$$

Does the ALT satisfy the RP, however? The condition for that is $\varepsilon\varepsilon' = 1$ [see the discussion after eqs. (16a-d)]; hence, from eq. (19):

$$\frac{\eta\eta'}{\gamma^2 \mathrm{QQ}'} = 1, \tag{21}$$

where Q' is the constant obtained from Q by Galilean inversion. After applying eq. (14), this reduces to simply $\mathrm{QQ}' = 1$. Since there is no other restriction on the choice of Q, this condition is easily met. One can think of the constant Q as a *conversion factor* between the clock rates in the two inertial systems, so taking Q' to be its reciprocal is exactly what one expects for the corresponding conversion factor in the reverse direction. The inverse set of equations for the ALT is obtained by Galilean inversion because the RP is satisfied:

$$\Delta t = \eta' \frac{\Delta t' + vc^{-2}\Delta x'}{\mathrm{Q}'} = \frac{\Delta t'}{\mathrm{Q}'} \tag{22a}$$

$$\Delta x = \eta' \frac{\Delta x' + v\Delta t'}{Q'} \tag{22b}$$

$$\Delta y = \frac{\eta'}{\gamma Q'} \Delta y' \tag{22c}$$

$$\Delta z = \frac{\eta}{\gamma Q'} \Delta z' \tag{22d}$$

One can also derive the ALT by combining the time relation in eq. (20a) with the RVT of eqs. (13a-c). By definition, $u'_x = \frac{\Delta x'}{\Delta t'}$, for example, so multiplication of the left-hand side of eq. (13a) by $\Delta t'$ gives eq. (20b) after multiplying the right-hand side with $\frac{\Delta t}{Q}$. In some ways the RVT is the most important transformation in relativity theory because the corresponding space-time transformation in eqs. (20a-d) can so easily be derived from it. It has the additional advantage of being completely independent of the clock-rate ratio connecting the two inertial systems. It also should be noted that the RVT "automatically" satisfies the RP since it can be derived from either the LT, which does satisfy the RP, or the Voigt transformation, which does not. It is obvious that any experiment which is consistent with the RVT is also described correctly by the ALT. This is an important observation since many of the greatest successes of relativity theory can be obtained from the RVT without making use of the LT. The derivation of the aberration of starlight at infinity [1, 17] can be obtained directly by applying the RVT, for example, whereby the corresponding derivation based on the LT is notably more complicated [18]. The RVT is also sufficient for obtaining the Fresnel light-drag expression [19], so the ALT is also compatible with this result. The same is true for Thomas spin precession [9], since that relationship is also independent of the ε normalization constant in the GLT.

The Lorentz invariance condition in eq. (12) is also invalid because it suffers from the same contradictory relationship as the LT. The corresponding invariance relation obtained from eqs. (20a-d) is:

$$\left(x'^2 + y'^2 + z'^2 - c^2t'^2\right) = \left(\frac{\eta}{\gamma Q}\right)^2 \left(x^2 + y^2 + z^2 - c^2t^2\right). \tag{23}$$

Multiplication by $\frac{\eta'}{\gamma Q'}$ leads to the more symmetric invariance relation given below:

$$\left(\frac{\eta'}{Q'}\right)\left(x'^2 + y'^2 + z'^2 - c^2t'^2\right) = \left(\frac{\eta}{Q}\right)\left(x^2 + y^2 + z^2 - c^2t^2\right). \tag{24}$$

The distinction between the ALT and the LT becomes critical when times are measured. A prime example is the study of atomic clocks carried out by Hafele and Keating [20, 21] in 1971 with circumnavigating airplanes. They found that the elapsed times on the various clocks employed in the experiment decreased with their speed relative to the "non-rotating" polar axis, or simply the earth's center of mass (ECM). The corresponding empirical formula expressed in terms of the present notation is:

$$\Delta t'\gamma\left(u'\right) = \Delta t\gamma\left(u\right), \tag{25}$$

where u and u' are the speeds of two such clocks and $\Delta t$ and $\Delta t'$ are the respective measured elapsed times over the same distance. Because of the earth's rotation, the clocks on the eastward-flying airplane returned to the airport of origin with less elapsed time than those left behind there, which in turn had more elapsed time than the clocks on the other airplane flying in the westerly direction. The same relationship was found earlier in experiments [22-24] with an x-ray source and absorber mounted on a high-speed rotor. In this case frequencies were measured and a blue shift was found when the absorber is mounted farther from the rotor axis than the x-ray source. The relationship in eq. (25) can thus be looked upon as the Universal Time-dilation Law

(UTDL [25]). In order to evaluate it, it is first necessary to determine a rest frame (objective rest system or ORS [26]) relative to which the speeds of the various timing devices are measured. The UTDL can be used directly to compute the value of Q in the ALT of eqs. (20a-d), namely as

$$Q = \frac{\gamma(u')}{\gamma(u)}.$$

It is easy to confuse the clock-rate proportionality relationship in eq. (20a) with the LT time-dilation formula in eq. (17). The latter states expressly that the ratio of the two clock rates depends on the speed v by which the observers are moving relative to one another. The proportionality constant is fixed by the LT to be $\gamma(v)$ in all cases. Nonetheless, it is completely unclear which of the two clocks runs at the slower rate. The derivation of eq. (17) can be changed by reversing the roles of the two observers, in which case:

$$\Delta t = \gamma^{-1} \Delta t' . \qquad [17']$$

Eqs. (17,17') are not related by algebra. The indication from the LT instead is that it is always the "moving clock" that runs slower than that in the rest frame of the observer [23]. Time dilation is supposedly symmetric, and therefore *subjective*, in this respect.

On the other hand, the time dilation indicated by eq. (20a) is perfectly *asymmetric*, in accord with Newton's Law of Inertia. The constant Q is fixed for all time, at least until some unbalanced external force is applied to the clocks. It can take on any value, greater or less than unity, consistent with the UTDL of eq. (25). Accordingly, as shown above, Q is a ratio of $\gamma$ factors in the usual case and is thus a characteristic of the relationship between the two rest frames, and therefore is not necessarily equal to $\gamma(v)$. Timing experiments of different kinds have always been in agreement with the UTDL and therefore with eq. (20a). It was recognized by Sherwin [27] that the LT prediction of symmetric time dilation was violated by the results of the x-ray frequency measurements [22-24]. His explanation, which can be recognized as *ex post facto* in nature, was that the existence of a unique inertial rest frame in the experimental arrangement, namely the rotor axis, changed the way in which the LT was to be used to arrive at its prediction. The same argument was used later by Hafele and Keating [20]. The implication is that in the absence of unbalanced external forces the situation would be different, that it would become "ambiguous," to use Sherwin's term, which of the two clocks runs slower. *This type of symmetric time dilation has never been observed.*

The engineers who developed the Global Positioning System (GPS) recognized the importance of asymmetric time dilation of the atomic clocks carried onboard satellites. A "pre-correction factor" is applied [28-29] to the standard frequency of the clocks prior to launch to insure that their rates are nearly the same in orbit as for their counterparts on the earth's surface. This procedure allows for the accurate measurement of the time required for a light signal to travel between the two positions, and this is essential for obtaining the high accuracy for distance measurements required. The correction for time dilation can be determined using the UTDL (another correction is required for the effect of gravity on clock rates). For this reason a more descriptive name for the relativistic space-time transformation in eqs. (20a-d) is the Global Positioning System - Lorentz transformation (GPS-LT). It is the only transformation that satisfies both of Einstein's postulates of relativity [10] and is also consistent with Newton's First Law.

**VI. Conclusion**

Voigt's answer to the problem of finding a consistent explanation for the surprising results of experiments carried out in the 19th century has had lasting results. He suggested that the difficulties could be traced directly to the Galilean transformation. What he then proposed is a classic example of how to amend a theory to describe new results without affecting its previous successful predictions. To do this, however, he broke with the traditional Newtonian view that space and time are distinct entities. This space-time mixing still remained after Larmor further modified the coordinate transformation so that it (the LT) also satisfies the Relativity Principle (RP) as well as the light-speed constancy condition Voigt had introduced. Since that time it has been the established view among the physics community that light-speed constancy can only be incorporated into relativity theory by insisting that space and time are no longer distinct when high speeds are involved. It is ironic that Lorentz's observation that there was a degree of freedom in the definition of the relativistic space-time transformation only resulted in a series of suggestions to justify the existing version, with its assertion that the mixing of space and time is the inevitable consequence of light-speed constancy. From a purely mathematical point of view, however, it is clear that the coefficient of mixing can be eliminated by simply making the proper choice of the normalization constant $\varepsilon$ in eq. (15a).

An interesting aspect of the present discussion is the way Newton's thoughts on the subject were handled. The process began with Poincaré's observation that the mixing of space and time implies that events that are simultaneous for one observer may not be so for another. He pointed out that there had never been any definitive test that would eliminate this possibility. Yet, a few years later Einstein showed that the mixing of space and time in the LT also implies that clock rates in different inertial rest frames must be strictly proportional to one another (symmetric time dilation). It is impossible for two observers to disagree whether events are simultaneous ($\Delta t = 0$ and $\Delta t' \neq 0$) and at the same time have proportional rates, *so there is an inherent contradiction in these two LT predictions.*

On the other hand, Newton's First Law asserts that a clock moving under the absence of external unbalanced forces will do so with constant speed and direction. It defies both common sense and experiment to believe that the rate of the same clock will not remain constant as well. The rates of two such clocks in different inertial rest frames might not be the same under these circumstances, *but the ratio of their rates has to be a constant*, thereby insuring that space and time are *not* mixed for them. *Clock-rate proportionality is the antithesis of remote non-simultaneity*. There is strong evidence for this conclusion from experience with the Global Positioning System. It would make no sense to adjust the rates of satellite and earth-bound clocks to be equal if events did not occur simultaneously for them.

One of Einstein's greatest advances was the prediction of time dilation, the slowing down of clocks because of their motion/acceleration. The version derived from the LT is flawed, however, because it claims that two clocks in motion can both be running slower than one another at the same time. Experiment has consistently shown instead that the elapsed times of clocks satisfy the Universal Time-dilation Law of eq. (25) and therefore that time dilation is an *exclusively asymmetric* phenomenon. As with Newton's Laws of Kinematics and the three Laws of Thermodynamics, the UTDL cannot be derived from some theory. Rather, it summarizes the results of all relevant observations and serves as a stimulus for new experiments to further test its reliability.

A simple way of describing the results of this phenomenon is in terms of a unit of time; a rest frame with systematically slower clock rates has a larger unit of time than its counterpart. Such an organizing principle cannot be used to describe the symmetric time dilation of the LT because it is meaningless to speak of units when there is no agreement about which clock is slower. This situation is in complete agreement with Galileo's original statement of the RP. He foresaw that it would be impossible based solely on *in situ* observations alone for passengers locked in the hold of a ship moving on a perfectly calm sea to distinguish their state of motion from that at the dock from which it departed. He did not rule out the possibility that clocks on the ship run at a different rate than those on shore, however, only that it is not possible to measure this difference without looking outside. There is merit in making an addendum [13, 30] to the RP on this basis: *The laws of physics are the same in all inertial rest frames, but the units in which they are expressed can and do differ from one frame to another.*

The GPS-LT of eqs. (20a-d) subscribes to this version of the RP through the introduction of the constant Q in its definition. The latter serves as a conversion factor between the two units of time for a given pair of rest frames. Conversion factors for all other physical properties can be determined as integral powers of Q [31]. An analogous set of relationships can be formulated [32] for conversion factors describing the effect of gravitational forces on physical properties.

The present revised theory has many potential applications. Primary among these are tests of the UTDL of eq. (25). For example, it would be interesting to measure the ratio of the rates of clocks subject to different ORSs, such as is the case when one is in the gravitational field of the moon while the other is earth-bound. The main purpose of the present work is not technological in nature, however. It contains an iron-clad proof that the Lorentz transformation does not conform to physical reality. Since this set of equations is the cornerstone of Einstein's version of relativity, this means that every standard textbook dealing with this subject needs to be revised in an essential way. The answer to the question in the title of ref. 28 is clearly "No." *A significant part of Einstein's legacy is as stake.* After a century of hearing unflinching support from experts in this field, it is high time to challenge them to either refute the present claim or acknowledge that it is correct once and for all time.

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