The Universal Time-Dilation Law: Objective Variant of the Lorentz Transformation

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Abstract

One of the most characteristic aspects of Einstein's special theory of relativity (SR) is its conclusion that two clocks in motion can both be running slower than the other from the vantage point of their respective observers. It is pointed out that this symmetric view of the measurement process has never been confirmed experimentally. Indeed, when investigations involving the necessary two-way communication between observers/detectors have been carried out, it has invariably been found that the rates of clocks can be unambiguously ordered on the basis of the following empirical formula: $\tau_1 \gamma(v_{10}) = \tau_2 \gamma(v_{20})$, where v_{10} and v_{20} are the speeds of the clocks with respect to a specific rest frame such as that of the earth's midpoint [$\gamma (v) = (1-v^2/c^2)^{-0.5}$]. The general attitude toward the failure of SR to anticipate the objective character of the latter timing results has been to consider them as falling outside the stated range of applicability of the theory (only uniformly translating systems). It is shown, however, that experiments such as the Ives-Stilwell study of the transverse Doppler effect and the determination of the lifetimes of rapidly moving metastable particles can also be explained quantitatively in terms of the above empirical formula. A different approach is therefore suggested in the present work, namely to eliminate an *undeclared* assumption in Einstein's derivation of the Lorentz transformation

(LT) and replace it with the condition that the above timing law be adhered to on a completely general basis. The resulting theory is shown to satisfy Einstein's two postulates while at the same time being consistent with the principle of complete objectivity in the measurement process for all physical quantities.

Keywords: Time dilation, Lorentz transformation, objectivity of measurement, standard units, velocity transformation

I. Introduction

The time-dilation phenomenon was predicted by Einstein [1] in his original work on the special theory of relativity (SR). It was a consequence of the Lorentz transformation (LT) which he derived in the same paper. On this basis he was led to conclude that the effect is characterized by a definite symmetry whereby observers in relative motion would each find that it was the other's clock that was running slower. However, the symmetry could be broken under certain circumstances in his view. He speculated that a clock located at the Equator would run slower than one at either of the Poles by virtue of the fact that it was undergoing constant acceleration due to the rotation of the earth about its axis. On this basis it was possible to distinguish between the two clocks in a way that is not possible when they are in uniform relative motion. Whether this was an operationally meaningful distinction for the time-dilation phenomenon in general was left to be decided by future experiments. In particular, Einstein pointed out that a *transverse* Doppler effect should be observed as a direct result of time dilation in the rest frame of a light source, and that this fact could be the basis for useful tests of his predictions. The development of atomic clocks has greatly facilitated efforts to obtain a comprehensive understanding of the effects of motion on the rates of clocks, but one crucial experiment of relevance to this general topic has yet to be carried out, as will be discussed in the following.

II. Quantitative Measurements of Time Dilation

Einstein's treatment of the transverse Doppler effect led to the following relation [1-3] between the emitted frequency ν_e of a light source moving with speed v relative to the observer and the corresponding measured frequency ν_r in the latter's rest frame:

$$\nu_r = \nu_e(1 - v^2/c^2)^{0.5} = \gamma^{-1}\nu_e . \qquad (1)$$

Because v is always less than the speed of light in free space c (299792458 m/s), it follows that the observed frequency is less than the emitted value in all cases ($\gamma>1$). This result is a direct consequence of the *symmetric* nature of the time-dilation phenomenon as predicted by Einstein in his 1905 paper [1]. If two observers exchange identical light signals, it follows from eq. (1) that each of them will measure a decrease in frequency (red shift). Aside from the value of the emitted frequency, the only information required in the above formula is the speed v of the light source relative to the observer. As such, eq. (1) is perfectly consistent with the general conclusion that a moving clock will always appear to have a lower rate than one in the rest frame of the observer.

Nonetheless, Einstein did not insist that this subjective view of the measurement process holds in all cases. He predicted in the same paper [1], for example, that a clock at the Equator should run slower than one located near a Pole because of its greater speed of rotation about the earth's axis. Although he did not give an explicit formula for the way in which clock rates vary with latitude on the earth's surface, it is a simple matter to derive one based on the above prediction. Accordingly, the elapsed time τ measured for a given event is inversely proportional to γ (v_0), where v_0 is the speed of the clock relative to the rest frame of the earth's midpoint. The corresponding equation for the elapsed times τ_1 and τ_2 of two clocks located at arbitrary latitudes is therefore,

$$\tau_1\gamma(v_{10}) = \tau_2\gamma(v_{20}), \qquad (2)$$

where v_{10} and v_{20} are their respective speeds relative to the above reference frame. This result can be converted into a transverse Doppler formula analogous to eq. (1) by noting that measured frequencies are inversely proportional to clock rates. Hence, in this case the receiver and emitter light frequencies are related by:

$$\nu_r/\gamma(v_{r0}) = \nu_e/\gamma(v_{e0}), \qquad (3)$$

where the speeds of the two measuring devices are again determined relative to the earth's midpoint.

It is evident that the predictions of the two Doppler formulas are quite different from one another. This is particularly evident for an exchange of light signals. Whereas on the basis of eq. (1), both observers would measure a red shift, eq. (3) indicates that a blue shift will be observed in one direction and a red one in the other since the ratio $\gamma(v_{r0})/\gamma(v_{e0})$ can be either greater or less than unity. The former situation arises when the receiver is moving faster relative to the polar axis than is the light source, whereas it never occurs upon application of eq. (1). From a practical point of view, a major distinction is that it is impossible to apply eq. (3) without first specifying a definite reference frame from which the velocities of the receiver and light source are to be measured. In the following we will refer to this frame as the *objective rest system* (ORS) to emphasize the fact that it is always possible in principle to say which of the two clocks is running at the slower rate on this basis. In the case of eq. (1), it is sufficient to know the relative speed v of the receiver and emitter, but this has the consequence that the measurement process becomes subjective: it is just a matter of one's perspective which clock is running slower.

Ives and Stilwell were the first to carry out tests of the transverse Doppler effect [4,5]. They studied the emission from excited hydrogen atoms moving in an evacuated tube in opposite directions at high speed in the laboratory. Their results were in quantitative agreement with eq. (1) to within acceptably large error bars (estimated to be 10-15% [6]). Similar results were obtained at about the same time by Otting [7]. The degree of accuracy in this experiment was improved over time, with Mandelberg and Witten [6] reporting agreement with the transverse Doppler formula to within about 5% in 1962.

However, it should be noted that the above results also are in satisfactory agreement with the other Doppler formula given in eq. (3). The speed of the light source (v_{e0}) in the study of Ref. [6] was on the order of 0.008 c, whereas the laboratory spectrometer was moving at a much lower speed (v_{r0}) relative to the earth's midpoint of about 10^{-6} c. Substitution of these values in eq. (3) gives a predicted Doppler shift of effectively the same magnitude and direction (red shift) as that obtained with eq. (1). The precision of the experiments is insufficient to allow for a meaningful decision as to which of the two formulas is more reliable. A clear distinction would be possible, however, if the conditions of the experiment could be changed so that $v_{r0} > v_{e0}$, i.e., so that the frequency of light emitted from the laboratory rest frame were to be measured with a spectrometer co-moving with the hydrogen atoms. In that case one would still expect a red shift based on eq. (1), whereas a blue shift of the same magnitude is indicated on the basis of eq. (3). It is quite understandable why such a variant of the Ives-Stilwell experiment has not been carried out to the present day, but this fact also emphasizes why *it is not possible to distinguish* between the subjective [eq. (1)] and the objective [eq. (3)] theories of the transverse Doppler effect on the basis of this type of investigation.

Another approach to studying the transverse Doppler effect was made possible by the availability of high-speed rotors and the discovery of the Mössbauer effect. Hay et al. [8] mounted a 0.86 Å ^{57}Co x-ray source near the axis of an ultracentrifuge with a ^{57}Fe absorber located close to the rim. This arrangement provided not only provided for the required transverse orientation, but also for the critical test case in which the emitter is moving more slowly in the laboratory than the receiver. Two other variants [9-10] of this experiment were reported shortly thereafter. Champeney et al. summarized their results for the Doppler shift as follows: $\Delta\nu/\nu = (v_a^2 - v_s^2)/2c^2$, where v_a and v_s are the respective speeds of the absorber/receiver and the x-ray source relative to the axis of the ultracentrifuge. First of all, it is clear that this empirical formula is in clear disagreement with the transverse Doppler prediction of eq. (1). A blue shift was recorded for the case when the detector is mounted on the rim of the ultracentrifuge ($v_a > v_s$), whereas only a red shift is expected on the basis of the latter equation. On the other hand, the objective version of Einstein's theory of the transverse Doppler effect in the form of eq. (3) *is quantitatively verified by the observed findings*. This is seen by substituting $\nu_e = \nu$ and $\nu_r = \nu + \Delta\nu$ and expanding the γ factors in Taylor series with $v_{r0} = v_a$ and $v_{e0} = v_s$ (in this case the rotor axis serves as the ORS relative to which both speeds are to be measured).

III. Sherwin's Dual-theory Approach

The authors of the three ultracentrifuge studies each noted that the observed results could be explained in terms of Einstein's equivalence principle [11] and the gravitational red shift [12]. The difference in gravitational potential $\Delta\Phi$ between two points on the rotor is computed by equating the centrifugal acceleration to a radial gravitational field [13]. The expected Doppler shift $\Delta\nu/\nu = -\Delta\Phi/c^2$ is then found to agree with the empirical formula cited above. However, it was also claimed that the same result can be obtained directly from SR. For example, Champeney et al. [10] stated that the expression for the observed frequency shifts "may be obtained either in terms of the time dilatation of special relativity or in terms of the pseudo-gravitational potential difference between source and absorber." This assessment overlooks a basic fact, however, namely that their results stand in contradiction to the prediction for the transverse Doppler effect given in Einstein's original paper [1]. In particular, interchanging the positions of the light source and the absorber on the rotor leads to a reversal in the sign of the measured frequency shift, contrary to what one expects from eq. (1). The clock which experiences the greater acceleration always runs at a slower rate. It is not just a matter of the perspective of the observer.

The surprising asymmetry of the transverse Doppler shifts was discussed in detail by Sherwin [14] shortly after the results of Hay et al. [8] had been published. He pointed out that the rotor experiments were of significance because of the "completely unambiguous nature of the result," and he attributed the fact that one clock can be uniquely identified as running slower than the other to the fact that they were not both in *uniform translation* during the course of the experiment. He made a clear distinction between inertial and non-inertial systems in this connection, asserting that in earlier experimental verifications of time dilation, such as the Ives-Stilwell study [4] and the lifetime measurements of metastable particles [15,16], the clock rates are *ambiguous* because only uniform translation is involved. He then raised a "fundamental question" as to why inertial frames are privileged above all other reference frames.

To summarize Sherwin's analysis, if at least one of the two clocks is under acceleration and is therefore in a non-inertial rest frame, one must apply the *objective* time-dilation formulas of eqs. (2-3) to predict relative rates; if, on the other hand, both clocks are in uniform translation and are therefore at rest in inertial frames, the standard (subjective) treatment of time dilation in SR must be used. In the latter case, one expects a red transverse Doppler shift in both directions based on eq. (1), whereas in the former, a blue shift will be observed from the standpoint of the clock which is more accelerated relative to some specific reference frame, such as that of the rotor axis in the Hay et al. experiments. There is one set of rules for non-inertial systems, another for purely inertial ones.

The decision to use qualitatively distinct methods to make predictions of the amount of time dilation depending on the circumstances inevitably leads to questions about "boundary" cases: what happens, for example, when an observer alternates between being in uniform translation at one moment as he makes his measurements to being slightly accelerated at another? The situation can perhaps best be illustrated with a simple *Gedanken* experiment. Consider a rocket ship overflying the North Pole. A light signal is sent to it from the ground and its frequency is measured. Let us assume for the sake of argument that the ship is slightly accelerating at this point. According to the rules discussed above, it is necessary to apply the "objective" theory under these circumstances in order to predict the amount of the transverse Doppler shift. To do this we need to take the earth's midpoint as the reference frame (ORS) from which to compute v_{r0} and v_{e0} in eq. (3). Therefore, $\gamma(v_{e0}) = 1$, whereas the γ value for the rocket can be quite large, say 10^6 in a specific case. If the emitted frequency ν_e is *c/500 nm*, this means that the Doppler-shifted value on the rocket will be only *c/500 fm* (large *blue* shift). However, at this precise moment, let us further assume that the rocket goes into cruising mode by virtue of a very slight breaking maneuver, meaning that the clocks onboard are suddenly in a state of uniform translation. As a consequence it is now necessary to switch over to the "subjective" theory for purely inertial systems, i.e. eq. (1). As a result, a large *red* shift is now predicted on the rocket (γ still has the same value as before), so that ν_r equals *c/500 mm*. However, application of a similarly small force can just as quickly bring the rocket into acceleration mode without significantly changing its velocity, in which case the predicted frequency is again c/500 fm, i.e. by virtue of eq. (3*). In effect, a hugely discontinuous change in Doppler frequency is predicted by the dual* *theory for a thoroughly continuous and miniscule variation in the rocket's velocity relative to the Pole.* Is this result at all plausible?

There is another point to be considered as well, one of an experimental nature. Sherwin bases his assertion that the SR version of time dilation in eq. (1) is well established on the basis of experiments [4, 15, 16] for which detection is made in the rest frame of the earth. However, to confirm the "ambiguous" character of time dilation for inertial systems, it is necessary to carry out "reverse" experiments in which, for example, clocks co-moving with the high-speed metastable particles in Refs. [15,16] are used to measure the lifetimes of the identical particles at rest on the earth's surface. In the past *it has been merely assumed* that an increase in lifetime over the proper value would be observed in this case as well, but this is precisely what needs to be proven experimentally in order to confirm this aspect of SR. This lack of experimental proof for the subjective theory of time dilation for inertial systems needs to be considered alongside the well-documented confirmation of the objective version of the theory for accelerated systems in the form of eqs. (2,3). *The possibility thus remains open that the symmetry ascribed to the time-dilation phenomenon by SR does not actually occur in practice.*

IV. Einstein's Undeclared Assumption

The preceding discussion has emphasized the failure of the symmetric Doppler formula of SR to anticipate the blue shift observed in the high-speed rotor experiments [8-10]. However, a potentially more interesting, and certainly more positive, fact emerges from a survey of the experimental data thus far obtained to study the effects of time dilation: *all results for the transverse Doppler effect [4-10] as well as for the lifetimes of metastable particles [15,16] do square perfectly with the objective theory embodied in eqs. (2-3).* The same can be said for the experiments with atomic clocks carried onboard circumnavigating airplanes [17]. In each instance it is only necessary to identify a unique reference system (ORS) from which to compute the velocities of the observer/receiver and the object of the timing measurements that are to be inserted in the appropriate formula. In both the case of the airplane experiments and the original transverse Doppler studies [4-7], the earth's non-rotating polar axis (or simply its midpoint) serves this function. In the Doppler investigations with ultracentrifuges, the ORS is the rotor axis. There are no known exceptions to this rule and, contrary to Sherwin's conclusion [15], there is still a good possibility that it will also hold for observers in uniform translation. One only has to assume that application of the small forces needed to bring a given detector/receiver into a state of uniform translation while making its measurements has only a qualitatively insignificant effect on clock rates (see the example at the end of Sect. III).

Given the fact that both the subjective (ambiguous) and the objective (unambiguous) formulations of time dilation were discussed and promoted in Einstein's original work [1], it is interesting to consider how the former came to be favored by conventional SR. The symmetric transverse Doppler formula of eq. (1) is based on the assumption of the invariance of the phase of plane waves under a Lorentz transformation (LT) [1,18]. A correspondingly symmetric formula for the variation of the lifetime τ of metastable particles with their speed v relative to a fixed observer is also derived in a straightforward manner from the LT [19]:

$$\tau = \tau_0(1 - v^2/c^2)^{-0.5} = \gamma\tau_0, \qquad (4)$$

where τ_0 is the proper lifetime of the particles. *It is therefore clear that the subjective theory of time dilation is intimately tied up with the LT.* It is not possible to eliminate one without rejecting the other as well, and this explains why there has been insistence upon retaining the symmetric form of time dilation for inertial systems even though its application for accelerated Mössbauer absorbers in the rotor experiments has been shown to lead to false predictions.

It is therefore instructive to take a close look at the derivation of the LT given in Einstein's paper [1], with special attention given to the question of how it ultimately leads to the predicted ambiguity in relative clock rates in SR. On p. 900 he arrives at a general form for the LT which contains a function φ that still needs to be defined in each of the four equations. He states that "φ is a temporarily unknown function of v," the relative speed of the two rest frames involved in the space-time transformation. It should be noted that he makes no attempt to justify this restriction on φ. He simply states as a matter of fact that such a function needs to be specified before the final form of the LT can be realized, and implies that there is no other choice but to assume that it can only depend on v. This assumption has been used in many subsequent discussions (see, for example, Refs. [3, 20-25]) of the foundations of SR, always accepting without question Einstein's assertion that φ must be at most a function of the relative speed v of the two rest frames. It should also be noted that the same function (denoted as ε) had been mentioned in the earlier work of Lorentz [26], although he did not make any restrictions with regard to its dependence on other variables at that time [23].

When one considers the possibility of choosing a different function to define the desired space-time transformation than Einstein did, two questions arise. First, on what other variables might the function φ reasonably depend than just the relative speed v? The answer to this question is best reserved until the second one is answered, namely what criterion should be used to distinguish between different choices for φ? In the latter case, it is clear that one should base such a decision on experimental data. This brings us back to eq. (2) and the objective formulation of time dilation. This equation gives us a relationship between clock readings t and t' that have been obtained in the two rest frames being considered. We assume that a reference frame (ORS [27]) has been designated from which to compute the speeds of the two clocks that are referred to in eq. (2). In order to make easy connection with Einstein's formulas in his original work, it is helpful to make the following definition:

$$t' = \gamma\,(v_0)t/\gamma\,(v_0') = Q^{-1}t, \qquad (5)$$

where v_0 and v_0' are the respective speeds of the two clocks relative to the ORS. At this point it is already clear that a different set of physical transformation equations must result than the LT because a simple proportionality is assumed between the two clock readings, with no involvement of any spatial coordinates. It should be emphasized that this relationship is not based on some theoretical assumption, but rather is taken over directly from the experimental data discussed in the previous sections from different time-dilation studies.

We can combine this equation with the corresponding expression from the LT in order to fix the value of Einstein's "unknown function" φ:

$$t' = \varphi\gamma(t - xv/c^2) = Q^{-1}t, \qquad (6)$$

where x is the position of the object of the measurement along the axis parallel to the relative velocity **v** of the two rest frames (the same definitions for coordinate axes are used as in Einstein's original work, but the notation is changed to conform to present-day conventions [18-20]; v>0 corresponds to motion of the primed rest frame in the +x direction while its unprimed counterpart remains stationary). Upon solving for φ, we obtain:

$$\varphi = \eta(Q\gamma)^{-1}, \quad (7)$$

where $\eta = (1- xv/c^2t)^{-1}$. According to the coordinate definitions, the ratio x/t is just u_x, the x-component of the velocity of the object of measurement. This is the answer to the first of the two questions above; in the present formulation, φ is a function of both the relative speed v of the two rest frames and also the *object's speed component u_x in the same direction*. The remaining three transformation equations are obtained by substituting the above value for φ in Einstein's other three general equations:

$$\begin{aligned} x' &= \eta Q^{-1}(x - vt) \\ y' &= \eta(Q\gamma)^{-1}y \quad (8) \\ z' &= \eta(Q\gamma)^{-1}z. \end{aligned}$$

The original LT [1] has much simpler relations for the coordinates in the transverse directions, namely y'=y and z'=z. This is because Einstein's assumption that φ can only be a function of v leads in a straightforward manner to the conclusion that φ = 1. Goldstein [28] has argued that such a simple form for the equations for these coordinates is essential because they correspond to directions perpendicular to the relative velocity, but again there is no *a priori* justification for such a conclusion, just an intuitive feeling that this should be so. The corresponding relations for these coordinates in eq. (8) indicate instead that there can be a difference in normalization between the respective primed and unprimed variables, but at least that there is no mixing between the two perpendicular directions.

Division of eq. (8) by the time variables in eq. (6) leads directly to the same relativistic velocity transformation (VT) as Einstein obtained in his original work [1]:

$$\begin{aligned} u_x' &= (1 - vu_x/c^2)^{-1}(u_x - v) = \eta(u_x - v) \\ u_y' &= (1 - vu_x/c^2)^{-1}\, u_y/\gamma = \eta\gamma^{-1}u_y \quad (9) \end{aligned}$$

$$u_z' = (1 - vu_x/c^2)^{-1}\, u_z/\gamma = \eta\gamma^{-1}u_z\ .$$

In these equations the analogous definitions for the various velocity components of the object of the measurement are used as for u_x above: $u_x' = x'/t'$, etc. Indeed, any choice for φ in the general LT must lead to the VT because this function appears on the right-hand side of each of the former equations and thus is cancelled out when the relations for x', y' and z' are divided by the corresponding equation for t'. This is a quite significant result *because it is the VT that is actually verified in the Fresnel light drag and aberration of starlight from the zenith observations [29, 30], and also Thomas spin precession [31]*. It is sometimes claimed [32] that the latter effects are direct confirmations of Einstein's LT, but this view overlooks the fact that any choice for φ in the generalized LT, including most specifically that of eq. (7), accomplishes the same result.

The underlying reason for this state of affairs is that the only information that is used by Einstein [1] to that point in his derivation is his second postulate, the constancy of the speed of light in free space. It is only when more than merely relationships between the object's velocity components is required that one even has to start thinking about the extra degree of freedom that the function φ (or ε in Lorentz's original work [26]) represents. It also should be clear that the only definitive means of determining a unique value for this "unknown" function is on the basis of experimental data. In retrospect, it is understandable that Einstein decided that the way out of this dilemma was to make an additional assumption about the functional dependence that could reasonably be expected for φ. Since he had already decided that a clock at the Equator must run slower than its counterpart at one of the Poles [1], it is nonetheless surprising that he didn't call upon this conclusion in reaching his goal of a space-time transformation with no free parameters. The result would have been an objective theory of measurement in general, and of time dilation in particular, that would have quantitatively anticipated the results of the rotor experiments that were carried out over a half-century later.

The VT of eq. (9) involves limiting values of the ratios of infinitesimal quantities. This suggests a different approach to the definition of the origins of coordinate systems than Einstein used. One can define a differential form of eq. (7), namely

$$dt' = Q^{-1}dt, \quad (10)$$

which can be used on an instantaneous basis, and then multiply the VT with it to obtain a differential form for the corresponding spatial coordinates analogous to eq. (8):

$$dx' = \eta Q^{-1}(dx - vdt)$$
$$dy' = \eta(Q\gamma)^{-1}dy \quad (11)$$
$$dz' = \eta(Q\gamma)^{-1}dz.$$

There is no need to assume that the spatial axes for the primed and unprimed systems coincide exactly when t = t' = 0, contrary to what is conventionally done with the LT. The quantities γ and η as well as the relative speed v are all defined for a particular point of time in eq. (11). They do not have to be constant at the time application is to be made. There is no need for the clocks in the two rest frames to be synchronized, only that their rates adhere to the proportionality specified by the parameter Q at the time of measurement. Once two clocks are running at strictly proportional rates, it is always possible to adjust their readings so that they coincide exactly so long as this ratio does not change. Relativity enters into the process of synchronization only when the speed of light needs to be used to compute differences in the time of the actual event and the respective arrival times of the corresponding information to the two observers and their clocks. The salient point is that quantities such as frequencies and lifetimes involve differences in clock readings and are completely independent of the time of day in which they are made.

There are significant distinctions between the two Lorentz space-time transformations discussed above, despite the fact that they both satisfy Einstein's second postulate of the constancy of the speed of light in free space. The most important of these is the objective character of eqs. (6,8) and (10,11) as opposed to the well-known subjective approach to measurement that is inherent in Einstein's LT. According to the latter, there are circumstances in which observers can legitimately disagree as to which of two clock rates is slower. This possibility is excluded in the former version, which will be referred to as the *objective Lorentz transformation* (OLT) in the following discussion. This difference shows up in the way time dilation is described in the two theories, namely in the symmetric form of eq. (4) derived from the LT as opposed to the inverse proportionality of eq. (2) in the OLT. There is also an operational distinction in the way they are used. The LT version only requires knowledge of the relative speed v of the two rest frames and thus can be applied without having any further information concerning the whereabouts of the two clocks in the universe. By contrast, the OLT formula given in eq. (2) needs a specific reference frame [27] from which to compute the speeds that are to be inserted in the two γ quantities before it can be successfully applied. The symmetric time-dilation formulas of eqs. (1,4) can be shown to be simply special cases of eqs. (2,3) once this is realized. We will return to this point in the next section.

Another key distinction lies in the fact that, as already mentioned, the space-time mixing in the LT that has been the subject of so much discussion is completely absent in eqs. (6, 10) of the OLT. Instead, a strict proportionality between the timing measurements made with clocks in different rest frames is assumed. This relationship is of great practical consequence in the design of the Global Positioning System (GPS). The frequency of the atomic clock carried on the satellite in a circular orbit is "pre-corrected" so that it will be the same as that of an identical clock on the earth's surface. The formula used to determine the amount of the correction is based on eq. (2), using the speed of the satellite relative to the earth's center to compute the required γ factor.

Another major distinction shows up in the Lorentz invariance equation:

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = \varphi^2(x^2 + y^2 + z^2 - c^2t^2). \qquad (12)$$

The LT simply has $\varphi = 1$ and thus the invariance equation exhibits perfect symmetry between the primed and unprimed variables. The analogous expression for the OLT on the other hand has a distinctly asymmetric appearance. In the general case symmetry merely requires that the analogous inverse relationship be satisfied:

$$x^2 + y^2 + z^2 - c^2t^2 = \varphi'^2 (x'^2 + y'^2 + z'^2 - c^2t'^2). \qquad (13)$$

In this case φ' is obtained by interchanging the primed and unprimed variables in φ and changing the sign of the relative speed v therein (the same operation leads to the inverse equations for the OLT, also as for the LT). In the usual way, by carrying out the transformation successively in the forward and reverse directions, one is the led to the condition: $\varphi\varphi' = 1$. The quantity η' is defined accordingly as $(1+ x'v/c^2t')^{-1} = (1+ u_x'v/c^2)^{-1}$. The above condition therefore becomes

$$\varphi\varphi' = \eta\eta'/QQ'\gamma^2 = 1. \qquad (14)$$

Substitution of the definitions of η and η' shows that their product is equal to γ^2. and QQ'=1 *because of the requirement that the two conversion factors must be the reciprocal of one another in an objective formulation*. This point will also be discussed in more detail in the next section. As with the space-time mixing characteristic of the LT, there has never been an experimental confirmation of Lorentz invariance, so its violation in the OLT does not conflict in any way with the latter's use as the relativistic space-time transformation.

It is also interesting to consider Einstein's argument against *absolute remote simultaneity* [1] by comparing with the objective formulation based on eq. (2). The basic idea can be simply formulated in terms of light pulses on an airplane that traverse equal distances (D) but in opposite directions (+x and –x). Let us assume that the airplane is moving at speed v along the + x axis relative to an observer on the ground. Einstein's point was that the distance travelled would also be D but that the speed of the pulses relative to the observer would be different, unlike the case for a local observer on the airplane. He therefore concluded that the pulses would arrive simultaneously for the latter, but at different times for the former. This conclusion contradicts the VT, however, according to which both observers must measure the light speed to have the same value c in both directions [33]. That is of course the great mystery of his second postulate, that "c+v" somehow is the same as "c-v," but that is what is found when these values are substituted in eq. (9). If the clocks on the airplane run Q times slower than those on the ground, as assumed in eqs. (5,6) of the OLT, that only changes the values for the elapsed times of the light pulse traversals for the two observers. Each one still finds that the times for the forward and backward pulses are equal and hence that the events are simultaneous for both [33]. The LT leads to a different conclusion because it assumes that the spatial and temporal variables are mixed [i.e., by setting $\varphi = 1$ in eq. (6)], so that the times for the forward and backward pulses would not be equal in this case.

An even easier way to see that the LT is not a physically valid transformation is to compare its two predictions of remote non-simultaneity and time dilation. On the one hand, it is concluded that time differences ΔT and $\Delta T'$ measured for the same two events on clocks in two different rest frames must always be strictly proportional to one another, i.e. $\Delta T' = X\Delta T$. On the other hand, because of the prediction of remote non-simultaneity, it must follow that $\Delta T = 0$ (condition of simultaneity) on one clock and yet $\Delta T' \neq 0$ on the other. It is evident that the *proportionality* inherent in the time-dilation relationship rules out the existence of remote non-simultaneity, in clear contradiction to the expectations based on the LT.

It can also be argued that absolute remote simultaneity plays a critical role in the GPS procedure [34,35]. After all, it would be pointless to compare clock readings on the satellite with those on the ground (even after the rate on the satellite had been corrected to be the same as for clocks on the ground) if the time of emission of a radar signal were not simultaneous in the two rest frames. One can argue that the accuracy of the GPS procedure is not sufficient to distinguish between the OLT and LT in this respect, but at least this discussion makes clear that there is empirical evidence in favor of absolute remote simultaneity and that the OLT provides theoretical justification for it. The OLT has also been referred to as the GPS-LT because of its relevance to the operation of the Global Positioning System [36,37].

V. Rational Units and the Relativity Principle

The use of a rational system of units in physical theories is based on the principle that different observers must always agree on the relative amounts of any given quantity. It is assumed that the only way two observers can legitimately disagree on the numerical value of a measurement is because they use different units to express their results. In that case it is always possible to use a "conversion factor" that allows one to change from one unit to another. The symmetry principle of SR precludes such an arrangement because it holds that observers in relative motion will each find that it is the other's standard clock that is running slower or the other's meter stick that is shorter. As discussed in Sect.

II, experimental confirmation of this prediction is surprisingly non-existent, however, despite frequent assertions to the contrary [14]. Indeed, every observation of time-dilation phenomena as yet reported can be quantitatively explained by assuming that there is complete objectivity in all physical measurements, specifically in terms of eqs. (2, 6).

It is therefore important to carefully consider the feasibility of employing a system of rational units in describing the results of measurements for which the observer and the object of the investigation are moving at high speed relative to one another. The Ives-Stillwell experiment [4] provides a useful example. There are two rest frames of interest, the laboratory L in which the spectrometer is at rest, and that of the rapidly moving hydrogen atoms H emitting the radiation. A red shift is observed due to the time-dilation effect in the rest frame H. A simple way to express this result is *to assume that the unit of time in H is larger than in L by a factor of γ*, i.e. it is γ s in H and 1 s in L [38]. The observer at rest in H measures the period of the (standard) radiation to be T in his units (γ s). Since the observer in L employs a smaller time unit, he must use a "conversion factor" of γ to obtain the corresponding value of the period measured in his rest frame. He therefore multiplies T with γ to make his prediction and obtains the observed result of γ T in his units (s). The two numerical values are different as a result, even though the actual period of the radiation is the same for both. Moreover, the same factor can be used for any measured elapsed time in H to convert the result to the corresponding value observed in L.

The problem with a subjective theory of measurement arises when one attempts to reverse the process, i.e. to predict the elapsed time observed in H for an event that has occurred in L. The reason is because the conversion factor *in this direction is assumed to be the same as in the original case*. The symmetry principle of SR demands this. Thus if L reports a period of γ T in his units (s), the same as above, H must multiply this value by γ because SR holds that he will observe L's clock to run slower by this amount. He therefore obtains a result of γ^2 T in his units in this case, not the original value of T, even though we are dealing with exactly the same amount of time in L in both cases.

It is obvious from the above example that any attempt to introduce the concept of a rational set of units into SR ultimately leads to nonsense. This result is inescapable because of the subjective character of the theory. It's as if two observers decide to use km (A) and m (B) to express their respective length measurements, but to use *the same conversion factor* in comparing their results. Observer A reports a value of 1 km and B converts it to 1000 m in his units. But then A takes B's result of 1000 m and converts it to 1000000 km to convert it back to his units. The absurdity of this approach is self-evident and this explains why one rarely sees the question of conversions between different units even discussed in SR, much less analyzed in a comprehensive manner. The argument that measurement does not have to be objective in an *a priori* sense is certainly well taken as a general premise, but there should at least be some concrete experimental evidence that definitively rules out this possibility before the idea is rejected on a definitive basis.

There is a source of confusion that needs to be dealt with when attempting to reduce the discussion of comparative measurement to easily intuitive concepts, however. In the Ives-Stillwell example discussed above, it needs to be emphasized that both observers are under the distinct impression that they are expressing their measurements in the same standard unit (1 s). This is because there is no way that they can deduce any change in the rates of local clocks in their respective rest frames since they all change in a completely uniform manner as they are accelerated. Any observation to the contrary would be inconsistent with the Relativity Principle. One has to make comparisons with the rates of clocks that are moving with respect to them in order to detect any changes, and then the obvious assumption is that it is the clocks in the "other" rest frames whose rates have changed, not those in their own rest frame.

Nevertheless, this state of affairs does not preclude the use of standard units in an *objective* version of relativity theory. In the case of the Ives-Stilwell experiment discussed above, one must begin by assuming that the observers in rest frames L and H each think their standard unit is 1 s. In order to compare their respective elapsed-time measurements, it is simply necessary that they use the appropriate conversion factor in each case. The relation in eq. (2), or more directly its simpler form in eq. (6), is the basis for making this determination. The conversion factor is Q in one direction, but application of simple algebra shows that it is the reciprocal, Q' = 1/Q, in the other.

In the Ives-Stilwell experiment, both observers measure the period of the radiation in their own rest frame to have the same value of T s. One knows, however, that clocks in H run γ times slower than their identical counterparts in L. The conversion factor for L is therefore equal to γ, whereas the corresponding value for the observer in H is $1/\gamma$. This explains why the laboratory observer records a red shift for radiation emitted from H. He finds that the period of the incoming radiation is γ T s in his units, i.e. longer than the standard value. When he sends radiation from an identical source to H, the observer there measures a shorter value (blue shift) of T/γ s in his system of units. There is no disagreement between the two observers as to which period of radiation is shorter, namely that emitted from the hydrogen atoms at rest in H. They simply disagree with regard to the respective numerical values in their own system of units. Accordingly, the observer in L finds that the radiation coming from H has a period of γ T s as compared to the locally measured value from the identical source of T s. At the same time, H finds that the period of the standard radiation in his rest frame is T s, whereas that coming from L has a period of T/γ s in his units. The ratio of the two measured values is clearly the same in both rest frames (γ T vs. T in L and T vs. T/γ in H), so the objectivity principle is satisfied quantitatively. More to the point, the *absolute* values of the two frequencies/periods of the radiation are the same for both observers once the distinction in their respective units is taken into account. As a consequence, the observer in H sees a blue shift for radiation emitted from the rest frame of L, while his counterpart in L necessarily observes a red shift for the reverse process. The latter experiment was actually carried out by Ives and Stilwell [4,5] and others [6,7], *but the other, in which the observer is co-moving with the fast-moving hydrogen atoms in the laboratory, has not.*

On the other hand, the empirical formula reported for the frequency shifts in the rotor experiments [8-10] indicates explicitly that a blue shift is measured by one observer (the one located at the rim of the ultracentrifuge and thus moving faster) and a red one by the other who is located closer to the rotor axis. The measurement process is clearly objective in this case and its results can be described in a transparent manner in terms of the difference in units employed by the respective observers.

The above analysis can easily be extended to other physical properties. The early experiments of Bücherer [39] confirmed Einstein's prediction [1] that the inertial mass m_i of a particle increases in direct proportion to γ (v), that is, with the same functional dependence as the lifetimes of muons [15,16]. The measurements were always made from the vantage point of a laboratory observer, however, so there is again no confirmation that the symmetry principle of SR holds for this property either. The experience with the Doppler measurements with ultracentrifuges [8-10] indicates instead that there is no ambiguity [14] concerning the relative masses of two particles. As a consequence, one can anticipate that an analogous relation to eq. (2) holds for the measured values of inertial masses, namely:

$$m_{i1}\,\gamma(v_{10}) = m_{i2}\,\gamma(v_{20}). \qquad (15)$$

Qualitatively, this means that a mass at the rim of the ultracentrifuge is greater than one closer to the axis. As a consequence, an observer at the rim would find that the inertial mass of an identical particle near the axis is *smaller* than that (the so-called *proper mass*) measured in his rest frame. This conclusion follows directly from the general assumption that measurement is always an objective process. There is never any ambiguity about which quantity is larger or smaller in a direct comparison according to this view, which finds experimental verification in the aforementioned experiments with ultracentrifuges. One simply needs a conversion factor to go from the results obtained by one observer to those of another. To help simplify matters, this factor (Q) is exactly the same as defined above for elapsed times in eq. (6), namely as a ratio of the two γ factors in eq. (2), whereby the same reference system is to be used to compute the relative speeds v_{10} and v_{20} as before. In short, in the notation of eq. (6),

$$m_i' = Q^{-1}\, m_i. \qquad (16)$$

Not all physical quantities have different values for observers in different rest frames, however. The prime example is the velocity **u**. Einstein's second postulate demands that observers all use the same unit of velocity. Otherwise, they would measure different numerical values for the light speed c even though its absolute value is the same for everyone. One consequence of this fact is that the measured values of both energy E and momentum **p** also have the same conversion factor for different observers as for inertial mass in eq. (16) since $E = m_i c^2$ and $\mathbf{p} = m_i \mathbf{u}$.

The conversion factor for distance measurements **r** can also be deduced from the definition of velocity:

$$\mathbf{u}' = d\mathbf{r}'/dt' = d\mathbf{r}/dt = \mathbf{u}. \qquad (17)$$

Because of eq. (6) it therefore follows that

$$d\mathbf{r}' = Q^{-1}\, d\mathbf{r}, \qquad (18)$$

i.e., distance measurements have the same conversion factor as elapsed times in the objective version of relativity theory. This relation is clearly consistent with the modern definition of the meter as the distance travelled by a light pulse in 1/c s. If the unit of time increases because of time dilation in a given rest frame, it follows that the distance travelled by the light in the above elapsed time must also increase by the same factor. The increase is the same in all directions because of the vector relationship in eq. (18), and thus the conclusion in the objective theory is that *time dilation is accompanied by isotropic length expansion*. This result again stands in clear contradiction to the corresponding prediction of SR, which states that *length contraction* generally accompanies time dilation. Moreover, according to SR the effect is anisotropic, having its maximum along the direction of the relative velocity of the two inertial frames but showing no difference in a perpendicular direction (Fitzgerald-Lorentz contraction [1]).

The conclusion of isotropic length expansion follows in a straightforward manner from a Gedanken experiment. First, assume two observers are initially at rest in the same inertial system. They measure the length of a metal rod at rest in this rest frame to have a value of D m. This is done in each case by determining that the elapsed time required by a light pulse to traverse the length of the rod to be D/c s. The metal rod and one of the observers (O_2) are then accelerated until they reach a state of uniform translation with speed v relative to the other observer (O_1) who is left behind. Observer O_2 again measures the length of the rod and, consistent with the Relativity Principle, finds the same results as before (D m and D/c s). Because of time dilation, however, let us further assume that O_1's clock runs $Q=\gamma$ times faster than O_2's. He therefore measures the elapsed time for the light pulse to traverse the metal rod to have a larger value than before (γ D/c s) when it was not moving relative to him. The value of the speed of light is still c for O_1, however, and so according to the definition of the meter he now measures the length of the metal rod to also be larger, namely γ D m. How does one explain the difference in O_1's measured values? He has not changed his velocity and thus his clock should still be running at the same rate during both measurements. Einstein's light speed law continues to hold for him as well. The only possibility left open is that *the length of the rod has actually increased as it was transported to a different inertial system*. The measurement process is entirely independent of the orientation of the metal rod to O_1, so it is also clear that his value for the length of the rod in the same in all directions. In short, he has measured isotropic length expansion of the metal rod at the same time as he notes that the rate of O_2's clock has decreased as a result of the time dilation in its new rest frame.

The theory can be formulated quite simply [40] in terms of the three units of the mks system, the meter, the kilogram and the second. As discussed above, the magnitude of each of them increases by the same factor Q as the various standards of measurement move between inertial systems. The variation in the units of all other physical quantities can be deduced in terms of their respective composition in terms of these three. For example, the units of velocity and force $\mathbf{F} = d\mathbf{p}/dt$ do not change at all (Q^0). The unit of angular momentum is Js or m^2kg/s and thus varies as Q^2. The latter result leads to a prediction regarding the quantum mechanical energy-frequency relationship since Planck's constant h has units of angular momentum. Since energy varies as Q but frequency (as the inverse of the radiation period) as Q^{-1}, it follows that their ratio must vary as Q^2, consistent with the above result. Thus, a study of the photoelectric effect based on the hydrogen-atom radiation in the Ives-Stillwell experiment is predicted to give a value for the energy/frequency ratio of γ^2h Js, and hence with slightly greater kinetic energy for the ejected electrons than is the case when the radiation is emitted from the same source at rest in the laboratory.

The role of physical units in an objective theory leads to a somewhat different formulation of the Relativity Principle than is commonly given in SR: *The laws of physics are the same in every inertial system, but the standard units in which they are expressed generally differ. This is because of the effects of acceleration on these quantities when the observer's state of motion changes.* Since the laws in question are equations involving various physical quantities, it is obvious that their validity is not affected by any consistent change in units. Observers in different rest frames will generally disagree on the values of the individual quantities that appear in these equations, but the laws themselves are satisfied for each of them.

As discussed above, such a simple approach to relativistic invariance cannot be used in SR because of the subjective character of the theory. Einstein therefore took a different approach by demanding that the laws of physics be invariant to the LT connecting any pair of inertial systems [1]. The result is an aesthetically pleasing mathematical structure in which terms such as covariance, four-vectors, Minkowski diagrams and Lorentz groups play an essential role. Nonetheless, the ultimate criterion for judging the validity of a theory is its capacity to predict and otherwise explain the results of experiment. The requirement of Lorentz invariance leads to quite specific relationships between measureable quantities that are often difficult to confirm empirically. This is why the transverse Doppler investigations that have been carried out over the years are of such great importance. In this case Einstein based his predictions on the condition that the phase of an electromagnetic wave must be invariant to an LT. The result for observation of radiation in the transverse direction is eq. (1), with its unequivocal prediction that the ratio of the receiver frequency to that emitted by the light source must always be equal to $1/\gamma$ (v), independent of whether the former is moving faster or slower in the laboratory. The results of the ultracentrifuge experiments are not consistent with this prediction, however, *since they show that the above frequency ratio can have any value.* In particular, as Sherwin [14] clearly pointed out, there is no question that it is the clock that is moving faster in the laboratory that runs with a slower rate, contrary to what is claimed in eq. (1).

The derivation of the general Doppler formula in the objective formulation of the theory starts out by recognizing that the period T of the radiation at the light source is measured in local units. The conversion factor Q given by eq. (6) is thus needed to change over to the unit of time employed by the clocks in the laboratory rest frame. Aside from this, one must recognize that the measured frequency also depends on the traditional (non-relativistic) Doppler effect that takes account of the motion of the light source either toward or way from the observer. The pertinent factor for the corresponding period of the radiation is $(1 – v \cos \chi /c)$ [41], where v is the speed of the source relative to the observer and χ is the angle of approach ($\chi = 0$ is for motion directly toward the observer). Note that the above factor is symmetric with respect to exchanging the rest frames of the light source and observer. The combined formula including the effect of the different clock rates is not, however, because of the reciprocal relationship between the respective conversion factors (QQ'=1):

$$T' = Q^{-1} T (1 – v \cos \chi /c), \qquad (19)$$

where T' is the value measured by the observer.

The corresponding formula for wavelengths λ and λ' is obtained by using the formula for the phase velocity of light in free space ($\lambda T^{-1} = \lambda' T'^{-1} = c$). The resulting equation agrees with the results of both the Ives-Stilwell and the ultracentrifuge transverse ($\chi = \pi/2$) Doppler experiments, unlike the case for eq. (1) of SR. It is also possible to generalize the above relation to include the effect of the gravitational red shift when the light source and observer are located at different altitudes. This change only requires a second conversion factor which then needs to be multiplied with Q. More details about the computation of the gravitational scale factors for the units of physical properties in the general case may be found in Refs. [40,42].

VI. Conclusion

One of the most striking aspects of Einstein's SR theory is its prediction that time dilation is a symmetric phenomenon, whereby two observers in relative motion will each find that it is the other's clock that has a reduced rate. However, a survey of the literature reveals that this claim has never been confirmed experimentally, despite frequent assertions to the contrary. For this purpose it is clearly necessary to carry out a *two-way* experiment. For example, the transverse Doppler studies of Ives and Stilwell were carried out with a fast-moving light source and a spectrometer that is at rest in the laboratory, but not the other way around. The analogous situation holds for the lifetime measurements of meta-stable particles. The possibility thus remains open that if the observer were moving faster with respect to the laboratory than the radiation source, a frequency shift in the opposite direction would be measured in the transverse Doppler experiment; similarly, the lifetimes of particles on the earth's surface might be found to be shorter than their proper values if the measurements were carried out by an observer in a rocket ship.

An attempt to fill this information gap was made in 1960 in the form of transverse Doppler experiments in which a Mössbauer x-ray source and absorber were mounted on an ultracentrifuge. The actual arrangement used by Hay et al. has the absorber near the rim of the rotor axis and thus moving faster in the laboratory than the light source. The results were in direct contradiction to the predictions of SR. The empirical formula obtained shows unequivocally that the measurement process is objective and that the sign of the Doppler shift does change when the positions of the absorber and source are reversed on the ultracentrifuge. The rate of a given clock decreases with its speed relative to the rotor axis. It is not a matter of the perspective of the observer as to which clock is running slower. Sherwin theorized that the problem with SR is that it is only valid for uniform translation and thus is not applicable to the ultracentrifuge experiment because the absorber was under constant acceleration during the measurement process. This argument became considerably less convincing after the timing results for atomic clocks on circumnavigating airplanes became known ten years later. Exactly the same relationship between the rates of the clocks and their speed relative to a reference system (the earth's midpoint) was found as in the ultracentrifuge experiments. In this case the degree of acceleration for the onboard clocks is not significantly greater than for laboratory clocks at rest on the earth's surface. The question thus arises as to whether it is reasonable to expect that the symmetric/subjective character of time dilation predicted by SR suddenly becomes inoperative because of the application of a small force to the clocks being compared.

In the present work another approach has been taken to resolving the conflict between theory and experiment with regard to time dilation. It is pointed out that Einstein made a critical unsubstantiated assumption in his derivation of the LT, namely by claiming that a scaling parameter could only be a function of the relative speed v of the two rest frames involved. This condition is easily shown to be responsible for the subjective character of SR. The empirical data from time-dilation studies indicate that the rates of clocks in different rest frames are strictly proportional to one another, and that their ratios can be evaluated quantitatively on the basis of their respective speeds relative to a specific reference frame. Einstein's two postulates regarding the Relativity Principle and the speed of light are still satisfied upon making this change in the derivation, and the result is a different version of relativity theory in which the fundamental objectivity of the measurement process is directly incorporated. Differences in the numerical values of physical properties obtained by two observers in relative motion are quantitatively explained on the basis of their use of a different system of units. The laws of physics are the same in all inertial rest frames, in accordance with the Relativity Principle, and the conversion factors for the units of all other physical properties are seen to be directly related to the aforementioned clock-rate ratios. The latter can be computed on the basis of the general time-dilation relationship in eq. (2) or by direct measurement on the basis of the transverse Doppler effect or the lifetimes of meta-stable particles. Perhaps the best evidence for the viability of the objective version of relativity theory discussed in the present work is the fact that the above principles find everyday application in the GPS methodology, specifically through the proportionality assumed between the rates of clocks on the earth's surface and those located on the participating satellites. The resulting theory [43] is consistent with the principle of absolute remote simultaneity, which, just as the objectivity of measurement, has never been contradicted in actual experiments. It also does not have to attribute special significance to inertial systems and is instead applicable on an instantaneous basis to rest frames of all kinds, thereby greatly extending its range of applicability over the subjective version of relativity theory Einstein introduced over a century ago.

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