The lack of internal consistency of the Lorentz transformation and its consequences for the concept of space-time mixing

Robert J. Buenker[1]

[1]*Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097 Wuppertal, Germany*

Abstract

The Lorentz transformation (LT) of Einstein's Special Theory of Relativity (STR) is analyzed with respect to its internal consistency. The LT leads to the prediction of time dilation and length contraction in moving rest frames. In addition, the relativistic velocity transformation (RVT) is derived from the LT by simply taking the ratios of its space and time coordinates, and this in turn guarantees satisfaction of Einstein's light-speed constancy postulate. On this basis, it is shown that the LT is not internally consistent since it can be used in different ways to obtain opposite answers for the same question, such as whether remote events occur simultaneously for two observers in relative motion. This characteristic eliminates the LT as a viable space-time transformation. Experimental results obtained with atomic clocks and other timing devices are shown to be in complete agreement with this conclusion. They invariably find that the rates of moving clocks are strictly proportional to one another (Universal Time-Dilation Law, UTDL), in disagreement with the space-time mixing expected on the basis of the LT. Finally, it is shown that a different transformation, referred to as the GPS-LT by virtue of its consistency with the

clock-rate adjustment procedure used on Global Positioning System satellites, also satisfies both of Einstein's postulates of relativity while nonetheless remaining consistent with the UTDL.

Keywords: Lorentz transformation (LT), relativistic velocity transformation (RVT), alternative Global Positioning System-Lorentz transformation (GPS-LT), absolute remote simultaneity, clock-rate proportionality, Universal Time-Dilation Law (UTDL), FitzGerald-Lorentz length contraction (FLC),isotropic length expansion

I. INTRODUCTION

The cornerstone of Einstein's Special Theory of Relativity (STR) [1] is the Lorentz transformation (LT). It is the basis for all of its predictions about the relationships between space and time. For example, it leads unequivocally to the conclusion that remote events generally do not occur simultaneously for two different observers who are moving relative to one another. Poincaré [2] had already discussed this possibility a few years prior to Einstein's landmark paper. FitzGerald-Lorentz length contraction (FLC) is also derived from the LT. It had first been suggested independently by FitzGerald [3] and Lorentz [4] in an attempt to explain the results of the Michelson-Morley experiment [5]. Einstein also used the LT to predict that the rates of clocks slow because of their motion, a phenomenon now referred to as time dilation. In both cases, he showed that the effect is symmetric, i.e. that a moving clock always appears to run slower than one that is stationary in the rest frame of the observer and that a moving object always appears to be contracted.

The LT is also used to derive the relativistic velocity transformation (RVT). The latter has had many successful applications, such as the aberration of starlight at the zenith [6], which is of critical importance in astronomical studies. The RVT also makes explicit Einstein's light-speed constancy postulate (LSP) [1], which states that the speed of light in free space is independent of the motion of both the light source and the observer.

The LT therefore makes separate predictions about all three of the quantities: distance, time and velocity. Since values for any two of the latter uniquely determine the third, it is essential that these predictions be self-consistent. It seems to have gone largely unnoticed, however, that this is not the case. In the following discussion, it will be shown that this characteristic of the LT causes it to give completely opposite conclusions for the same issue. Examples include the questions of whether remote events occur simultaneously for two observers in relative motion and if they actually agree on the value of the speed of light.

II. SPACE-TIME PREDICTIONS OF THE LORENTZ TRANSFORMATION

The STR conclusion of remote non-simultaneity is based directly on one of the four equations of the LT (see the discussion in Sect. III) in which space and time coordinates are mixed. A popular illustration of this effect involves a moving train that has been struck at opposite ends by two lightning flashes. Application of the LT shows that it is impossible for the light pulses caused by the lightning strikes to reach the midpoint of the train simultaneously for both an observer standing on the station platform and his counterpart riding with the train [7]. There is another way to analyze this problem using STR which leads to the opposite conclusion, however. The LT predicts that the rates of proper clocks in the two rest frames are related by a constant factor which depends on the speed v of the train relative to the platform (time dilation) [8]. Accordingly, each observer will find that the other's clock runs slower by a factor of $\gamma = \left(1 - v^2 c^{-2}\right)^{-0.5}$, where c is the speed of light in free space (2.99792458×10^8 ms^{-1}). For example, if the train observer finds that the elapsed time for the light pulse to travel between the midpoint and the front of the train is T_f, his counterpart on the platform will measure a corresponding elapsed time of γT_f. The same proportion holds for the corresponding elapsed time T_b for the light pulse to travel from the back of the train to the midpoint. Therefore, if the pulses arrive simultaneously for the train rider, i.e. $T_b = T_f$, it follows that they also will arrive simultaneously for the platform observer since his measured values of γT_f and γT_b will also be equal, i.e. $\gamma T_f = \gamma T_b$.

Another inconsistency in the LT predictions occurs for the distances traveled by each light pulse from the respective vantage points of the two observers. According to the FLC, the platform observer will find that the distances between the respective ends of the train and its midpoint will be contracted by a factor of γ in each case [8]. The train rider will find by virtue of the definition of light speed that the two distances can be obtained by multiplying his corresponding elapsed times by c, i.e. they will have values of $L_f = cT_f$ and $L_b = cT_b$. Because of the LSP, it is clear that the speed of the light pulses in both directions is equal to c for both observers. The same conclusion is obtained from the RVT. Therefore, the platform observer can obtain the values of the two distances from his perspective by again multiplying each of his measured elapsed times with c. He therefore finds that the two values are $c(\gamma T_f) = \gamma L_f$ and $c(\gamma T_b) = \gamma L_b$, respectively. Since $\gamma > 1$, however, this means that the platform observer finds that the distances on the train are *larger* than the respective values measured by the train rider, the opposite conclusion that one obtains from the FLC.

There is another clear inconsistency when the FLC is used by the platform observer to measure the speeds of the two light pulses on the train. By definition, he needs to divide the value of his measured distance between the midpoint of the train and one of the starting points for the light flashes by the corresponding elapsed time obtained using his stationary proper clock. The result for the light flash from the front of the train is $(L_f/\gamma)/\gamma T_f = (cT_f/T_f)\gamma^{-2} = \gamma^{-2}c$, which value is clearly in conflict with that expected from the LSP, namely c.

The above inconsistencies certainly raise the question as to which of two opposite predictions one should believe in a given case. The only definitive way to settle such issues is on the basis of unequivocal experimental data. However, there is another conclusion that can be drawn which does not require any additional information. This is because all of the theoretical arguments employed above are based on premises that are perfectly consistent with STR in general, and the LT in particular. These include the FLC and time dilation, both of which are obtained directly from the LT, as well as the LSP on which its derivation rests. Any theory which gives contradictory answers to the same question is clearly unacceptable. It is like having a weather forecasting model which tells us that tomorrow will be bright and sunny, but is equally likely to be cold with blizzard conditions. The inescapable conclusion is therefore that the LT is not a viable space-time transformation. It doesn't matter how many other successful predictions can be attributed to it. The challenge is to modify STR by eliminating the LT while still remaining consistent with the latter successes, and doing so hopefully without introducing any new assumptions that do not a have a firm experimental basis.

In order to approach this goal effectively, it is imperative that the open questions cited above be resolved satisfactorily. The simultaneity question will be treated first. Experiments with atomic clocks carried onboard circumnavigating aircraft [9] have shown that their rates decrease with their speed v relative to the earth's center of mass (ECM). Elapsed times were found to be inversely proportional to γ (v) in all cases. A gravitational effect was also noted. On this basis it was concluded that the rates of the clocks in different rest frames are *strictly proportional* to one another. This observation is quite important for the operation of the Global Positioning System (GPS). It has been shown that the rates of satellite clocks can be adjusted on this basis so that they are always equal to those of identical clocks located on the earth's surface. This observation makes it possible to obtain accurate values for the elapsed time required for a light pulse to pass between the satellite and the ground. This procedure would be useless were it not for the fact that the emission of a light pulse occurs at exactly the same time read from a satellite clock as that measured on its counterpart on the earth. The accuracy of GPS distance measurements is therefore irrefutable evidence for the remote simultaneity of events, in contradiction to what is expected on the basis of the space-time mixing assumed by the LT.

The question of whether the lengths of objects contract or expand when they are accelerated can also be settled unequivocally on the basis of experiment. Ives and Stilwell [10, 11] carried out a study of the transverse Doppler effect by accelerating a light source in the laboratory. After averaging out the first-order Doppler shift by observing the radiation in opposite directions, they found that the wavelength increased in direct proportion to γ (v). They concluded on the basis of the LSP that the frequency of the light was therefore lower in the accelerated rest frame, which is in accord with the STR prediction of time dilation. They neglected to mention that the wavelength increase directly measured in their experiment stands in contradiction with the FLC, however. Moreover, the fact that on the basis of the Relativity Principle (RP), Einstein's first postulate [1], the *standard* value for the wavelength is expected to be found in the accelerated rest frame indicates that the apparatus (diffraction grating) used to measure the wavelength there must also have increased by the same proportion in all directions. Ultimately, one is forced on the basis of the LSP to conclude that isotropic length expansion accompanies time dilation in a given rest frame, not the type of anisotropic length contraction predicted by the FLC and LT.

III. THE AMENDED VERSION OF THE LORENTZ TRANSFORMATION

The results of the experiments with atomic clocks [9] and accelerated light sources [10, 11] can be conveniently summarized into a single empirical formula referred to as the Universal Time-Dilation Law (UTDL) [12, 13]. It compares the measured elapsed times Δt_1 and Δt_2 for the same event obtained with identical clocks in two different rest frames:

$$\Delta t_1 \gamma(v_{10}) = \Delta t_2 \gamma(v_{20}) \quad (1)$$

The respective speeds v_{10} and v_{20} of the clocks are measured relative to a common rest frame known as the objective rest system (ORS). It is the ECM in the Hafele-Keating study [9] and the laboratory in the Ives-Stilwell transverse Doppler experiment [10, 11]. The formula also applies to the x-ray frequencies measured in the transverse Doppler study of Hay et al. [15-17] using high-speed rotors, in which case the rotor axis serves as the ORS. The UTDL is also used to adjust the satellite clocks in the GPS technology. Einstein's example [1] of an electron moving in a closed trajectory also is consistent with the UTDL, with the rest frame in which the force is applied to the particle serving as the ORS in this case.

The UTDL states that the relative rates of clocks are completely independent of their location in space. The only thing that matters is their respective speeds relative to the ORS (location does become important when gravitational effects need to be taken into account [9]). The LT states instead that space and time are irrevocably mixed. The proportionality of elapsed times on different clocks that is known as time dilation in STR supposedly only occurs under certain well-defined circumstances. Relativity theory must agree with experiment in all cases, however, so it is crucial to insure that any revised version of STR be consistent with clock-rate proportionality on a completely general basis, as the UTDL indicates must be the case

A natural starting point for pursuing this objective is the space-time transformation given below which was introduced by Lorentz [18, 19] prior to Einstein's 1905 paper:

$$\Delta t' = \gamma\varepsilon\left(\Delta t - v\Delta x c^{-2}\right) = \gamma\varepsilon\eta^{-1}\Delta t \quad (2a)$$

$$\Delta x' = \gamma\varepsilon\left(\Delta x - v\Delta t\right) \quad (2b)$$

$$\Delta y' = \varepsilon\Delta y \quad (2c)$$

$$\Delta z' = \varepsilon\Delta z\,, \quad (2d)$$

with η defined above as $(1\text{-}vc^{-2}\Delta x/\Delta t)^{-1}$. The transformation relates intervals of space and time Δt', Δx'... and Δt, Δx measured in two different inertial rest frames S and S' for the same pair of events. It is assumed that S' moves along the mutual x,x' axis with speed v relative to S. It is the most general transformation that satisfies the LSP and will therefore be referred to in the following as the General Lorentz transformation (GLT). Note that it contains a constant factor ε in each of its four equations, which can therefore be viewed as a normalization constant for the transformation.

Einstein obtained the same set of equations in his paper [1], but referred to the normalization constant as φ. He asserted that φ/ε is only a function of v. It is important to see that this amounts to a third postulate of relativity [20], although Einstein did not declare it as such. He went on to show on the basis of symmetry alone that the only allowed value for the normalization constant is ε=1. Substitution of this value in the GLT of eqs. (2a-d) then gives the LT [1]. Space-time mixing results for ε=1because of eq. (2a). It is clear, for example, that this value precludes simultaneity, i.e. Δt'=Δt, whenever both Δx and v are different than zero

The above choice is not consistent with the UTDL of eq. (1), however, and leads to the many inconsistencies discussed in Sect. II. One can introduce clock-rate proportionality, i.e. $\Delta t' = \Delta t/Q$, by making the following assignment for the GLT normalization constant: $\varepsilon = \eta(\gamma Q)^{-1}$. The constant Q is defined on the basis of the UTDL as

$$Q = \frac{\gamma(v_0')}{\gamma(v_0)} , \qquad (3)$$

where v_0' and v_0 are the respective speeds of S' and S relative to the ORS in a given case. It is especially noteworthy that the relative speed v of S and S' does not appear in the definition of Q, thereby removing any element of subjectivity from the revised theory. Substitution of this alternative value for the normalization constant ε then gives the following result [21-23]:

$$\Delta t' = \frac{\Delta t}{Q} \qquad (4a)$$

$$\Delta x' = \eta(\Delta x - v\Delta t) \qquad (4b)$$

$$\Delta y' = \frac{\eta \Delta y}{\gamma Q} \qquad (4c)$$

$$\Delta z' = \frac{\eta \Delta z}{\gamma Q} , \qquad (4d)$$

where the clock-rate proportionality relation appears explicitly in eq. (4a). This transformation is referred to as the GPS-LT because it is consistent with the clock adjustment procedure of the GPS technology. The GPS-LT clearly satisfies the LSP because of its relation to the GLT of eqs. (2a-d).

It is less obvious that the GPS-LT satisfies the RP. The latter requires that the inverse of the transformation be obtained by interchanging the primed and unprimed variables and reversing the sign of v. Consideration of the GLT equations shows that the condition for satisfying the RP is that the normalization constant ε' in the inverse transformation be the reciprocal of ε in the forward direction. That condition is obviously fulfilled by the LT since $\varepsilon' = 1$ by the above definition. Probably the main reason that physicists have insisted that the LT *uniquely* satisfies both of Einstein's postulates is because it seemed highly unlikely to them that any other choice for ε would be successful in achieving this objective. It is shown elsewhere [24], however, that the GPS-LT also satisfies the RP. The value of ε given above for this transformation, i.e. $\eta/\gamma Q$, requires for this purpose that $\eta\eta'=\gamma^2$, where $\eta'= (1+vc^{-2}\Delta x'/\Delta t')^{-1}$ according to the above specification for the inverse transformation. In addition, Q' must be the reciprocal of Q, as is evident from comparison of eq. (4a) with its inverse: $\Delta t=\Delta t'/Q'$.

Division of $\Delta x'$, $\Delta y'$, $\Delta z'$ in eqs. (4b-d) by $\Delta t'$ in eq. (4a) results in the RVT, the same transformation as Einstein obtained in his original work [1]:

$$u'_x = \left(1-vu_x c^{-2}\right)^{-1}\left(u_x - v\right) = \eta\left(u_x - v\right) \quad (5a)$$

$$u'_y = \gamma^{-1}\left(1-vu_x c^{-2}\right)^{-1} u_y = \eta\gamma^{-1}u_y \quad (5b)$$

$$u'_z = \gamma^{-1}\left(1-vu_x c^{-2}\right)^{-1} u_z = \eta\gamma^{-1}u_z, \quad (5c)$$

where $u_x'=\Delta x'/\Delta t'$, $u_x=\Delta x/\Delta t$, etc. The RVT also results when the same operations are performed on the GLT equations since the normalization constant ε is simply cancelled out in the process. This also obviously explains why the LT is consistent with the RVT. The most prominent confirmations of the LT are actually a direct consequence of its compatibility with the RVT, as, for example, aberration of starlight at the zenith [6]. It is therefore clear that the GPS-LT is also confirmed by all such results.

IV. Conclusion

The Lorentz transformation (LT) can be used to give opposite answers for the same question and this ambiguity disqualifies it as a viable physical theory. For example, Einstein's famous example of a train being struck by two lightning bolts has been claimed on the basis of the LT to be proof for the remote non-simultaneity of events. However, the *proportionality* in the LT's time-dilation formula requires that whenever two elapsed times are equal in one rest frame, they must also be equal in any other. Furthermore, it can be shown on the basis of the light-speed postulate, which is used to derive the LT, that the length of an object *increases* in a rest frame in which clocks undergo time dilation, and by the same fraction in all directions. Yet, the FLC, which is derived from the LT, predicts that lengths contract under these circumstances, and by varying amounts depending on orientation. It is important to see that these inconsistencies are inherent in the internal structure of the LT itself and do not require any experimental interpretation whatsoever to expose them.

One of the most emphasized features of the LT is its prediction of space-time mixing. Advocates of the concept of a single entity, "spacetime," are faced with the challenge of finding another coordinate transformation that satisfies both of Einstein's postulates while avoiding the inconsistencies of the LT. Clock-rate proportionality, which is the antithesis of space-time mixing, has considerable support from both theory and experiment. Newton's First Law of Kinematics (Law of Inertia) states, for example, that an inertial clock should continue moving indefinitely with constant speed and direction. By the same token, the rate of such a clock should also remain constant because of the absence of external forces. The ratio of the rates of any two such clocks must therefore also be constant in this view, which is by definition clock-rate proportionality. Experiment has also always come down on the side of Newton's Law. No violation of the UTDL of eq. (1) has ever been observed, and the adjustment of satellite clock rates in the GPS technology provides overwhelming support for its authenticity. Until such a violation is found, the concept of spacetime will always be suspect. All the available evidence to date supports Newton's thesis that space and time are completely distinct, whereas Einstein's claim to the contrary is based on a transformation that is characterized by numerous inconsistencies.

On the other hand, the GPS-LT of eqs. (2a-d) incorporates clock-rate proportionality as a third postulate of relativity, while still satisfying the RP and the constancy of light in free space. It does away with remote non-simultaneity, space-time mixing and the FLC. It also avoids Einstein's subjective view of the measurement process whereby two clocks can supposedly be running slower than each other at the same time. The GPS-LT is compatible with the relativistic velocity transformation (RVT) and is therefore in agreement with many successful predictions previously attributed to the LT. Satisfaction of the UTDL is also built into this version of relativity theory, with the result that the GPS-LT has been found to be in agreement with all relevant experimental observations as yet reported.

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