New Synthesis of

Elementary Particle Physics

A Theory of Elemental Balance in Physical Transformations

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I. INTRODUCTION

After the discovery of quantum mechanics, nothing quite so excited the imagination of physicists as Dirac's prediction[1] of the existence of antimatter. Within a few years Anderson had demonstrated[2] the existence of the positron as the antiparticle of the electron, and within decades the antiproton[3] had also been found. Although Dirac's line of reasoning involved first and foremost his treatment[1] of the fine structure in the spectra of hydrogenic atoms, the ultimate theoretical basis for his prediction can be traced back years before to Einstein and the special theory of relativity (STR).[4] The famous mass-energy equivalence relation, $E=mc^2$, was given a more concrete interpretation by Dirac, namely that two particles of equal rest mass but opposite electric charge could be converted entirely into energy. In the case of e^+ and e^-, the energy appears as electromagnetic radiation or photons, whose frequency ν is related to the released energy by the Planck relation,[5] $E=h\nu$. A key element in this interpretation is the series of mechanical laws of classical physics, particularly the conservation of energy and each of the components of linear and angular momentum, which were enunciated in the seventeenth century by Newton[6] and Galileo.[7]

The creation and annihilation of matter, as this phenomenon is generally called, is central to the theory of modern physics, ranging from the sub-microscopic regime of atoms, electrons, photons and more exotic elementary particles to the super-macroscopic world comprising astronomy and cosmology. Indeed, one uses these terms to discuss processes which are commonplace in our everyday experience, such as the absorption and emission of light. Energy released or gained upon electronic transitions in atoms or molecules is said to cause the creation or destruction of a photon, again in accordance with the Planck frequency relation.

As with other fundamental theoretical concepts of natural science, the principle of creation and annihilation of matter forced a rethinking of old themes from the realm of philosophy and metaphysics. For example, a new interpretation could be given to the famous statement[8] of Lucretius in the first century B.C. in *de Rerum Natura:* "Nothing can be created from nothing," which is paraphrased by Shakespeare[9] in *King Lear* as "Nothing will come of nothing." In other words, energy in the form of photons of a given frequency is to be

distinguished from nothing and therefore the production of electrons and positrons from it is thus not inconsistent with Lucretius's rule.

Yet it is clear that the creation-annihilation concept still represents a revolutionary departure from ancient precepts, particularly certain aspects of the atomic theory of matter dating back to the work of Democritus.[10] According to this traditional view, the elements from which all matter is assumed to be constructed possess very definite characteristics. Again in the words of Lucretius:[11] "Material objects are of two kinds, atoms and compounds of atoms. The atoms themselves cannot be swamped by any force, for they are preserved indefinitely by their absolute solidity." Suffice it to say that the annihilation of an electron and positron to produce pure energy is not consistent with this statement. The present investigation concerns itself with the theory underlying this revolutionary process in which material particles interact with one another with such force that they are claimed to lose the very identity that Lucretius imagined was their inherent property.

II. POSITRONIUM DECAY AND THE CREATION-ANNIHILATION HYPOTHESIS

In a book[12] published in 1661, entitled *The Skeptical Chymist,* Robert Boyle originated the modern concept of chemical elements. Simply stated, he suggested that the elements can be distinguished from all other substances by virtue of the fact that they cannot be split up by chemical reactions into simpler substances. The relationship between his ideas and the atomic theory of Democritus and continuing on to Lucretius is unmistakable. The concepts were made more concrete in Dalton's atomic theory,[13] proposed in 1803, which provided a sound basis for interpreting known facts of chemistry. Thus when one observed two different gases, oxygen and hydrogen, coming together to react explosively to form steam, one could speak of the process as involving two elements which were simply joined together in different ways before and after the reaction.

Until the advent of Einstein's STR[4] it was believed that each element had a fixed mass and that the sum of the masses of the reactants was exactly equal to that of the products. Even today one adheres to the underlying principle that the numbers of each type of element are unchanged in the course of chemical reactions, and thus the concept of a balanced equation remains an integral part of the teaching of fundamental chemistry to the present time. Special relativity merely rejects the assumption of fixed masses for the elements and replaces it with the principle that energy absorbed or emitted in the course of a physical transformation must be taken into account by means of the $E=mc^2$ relation in order to predict the combined mass of the products from that of the reactant species.

The concept of elemental balance in chemical reactions is in no way disturbed by this adjustment, and with its help it was possible to carry over the ideas of the early chemists to the interpretation of nuclear reactions involving much larger energy changes. For example, when two deuterium atoms combine to form an alpha particle the energy released is great enough to allow for direct measurement of the corresponding loss of mass of the product system relative to that of the reactants. The discovery of the neutron by Chadwick[14] in 1932 made it possible to specify more precisely what the elements are in this process. The deuteron is thus a compound consisting of a single proton and neutron, whereas the alpha particle is the ^{4}He nucleus composed of pairs of each of these elements.

Nonetheless, one of the casualties of the creation-annihilation concept is the principle of elemental balance in all physical transformations. The above example involving a nuclear

process is more the exception than the rule in the theory of modern physics, particularly as one makes the transition into the field of elementary particles. If particles can simply be converted into pure energy, there is no longer any basis for demanding that the same number and type of elemental species is present before and after a reaction has occurred. One need only try to imagine how the development of the theory of chemical transformations would have been affected *if the principle of a balanced reaction had not been enunciated* in order to appreciate the consequences of rejecting the idea in other branches of the natural sciences.

A. ELECTRON-POSITRON INTERACTION

With this background it is interesting to analyze in some detail the process which first led to the postulation of the creation and annihilation of matter, namely the interaction of an electron with its antiparticle, the positron. For this purpose it is important to consider the experimental data with reference to familiar theoretical models without accepting their conclusions *a priori,* recalling Newton's prescription[15] that "these laws must be considered as resting on convictions drawn from observation and experiment, not on intuitive perception." Assuming the positron and the electron to be initially at rest, the first observation is that they form a weakly bound complex known as positronium. After a short lifetime (ca. 10^{-10} s) something much more dramatic happens to the system, however. In the most commonly observed process, the decay of positronium leads to the production of two high-energy photons which fly away in opposite directions to one another. In another branch of this reaction which occurs much less frequently, three photons appear. In the primary decay process the two photons are always found to show opposite polarization, whether circular, plane or elliptical, and have equal energy (frequency).

The existing theory for these various observations can be summarized as follows. First, a typical low-energy phenomenon occurs, corresponding to the binding of the electron and positron together. This process can be described accurately in close analogy to the treatment of the hydrogen atom by means of the non-relativistic Schrödinger equation.   An even more thorough description in terms of quantum electrodynamics is also possible.[16]   One knows that the ionization potential of the hydrogen atom is 13.605 eV (0.5 hartree) and the Bohr theory of 1912 had shown that the amount of binding is proportional to the reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$ of the proton-electron system. The equality of the rest masses of the electron and positron leads to the conclusion that μ for positronium is only about half that of the hydrogen

atom. Hence, the binding energy in this case is one-half as large (6.80 eV). However, the *instability of the positronium complex stands in sharp contrast to the known characteristics of the corresponding H atom,* which is quite stable and appears to exist in this state for indefinite periods in the absence of any outside influence.[17]

Were it not for the spontaneous decay of positronium, one would have no difficulty relating these observations to Dalton's atomic theory.[13] In this case, the elements are one electron and one positron. At the beginning of the reaction they are separated (existing in elemental form, to use Boyle's terminology[12]), whereas afterwards the compound positronium is formed in which the two elements are bound together. *Why then must we give up the atomic theory of elements when it comes to the decay of positronium?* The conventional answer to this question is that the elements with which we started, namely an electron and a positron, are no longer present at the conclusion of the process. Instead, one is left with a pair of photons in the most commonly occurring case.

Yet in a strict sense the creation-annihilation explanation for positronium decay violates Newton's prescription about always basing theory firmly on observation. *How can one truly observe that something disappears?* If natural science is restricted to the domain of observation, how does one fit the phenomenon of creation and destruction into the picture? By definition these processes involve material transformations either to or from nothing, and therefore can never be observed direct1y in their entirety. There is clearly something about nothingness which defies observation. By the same token, it is impossible to prove that things do not disappear. It only seems prudent to recognize that the inability to observe an object is not an unambiguous sign that it has ceased to exist.

Especially since the natural sciences underwent a long and successful development without having to yield on the ancient view that all material objects are synthesized from impermeable elements, it is important to probe the creation-annihilation hypothesis with utmost scrutiny. To this end let us follow the tried-and-true principle of mathematics employed whenever a theorem is to be proven, namely to assume the opposite and examine whether a contradiction can be derived as a result. In the physical sciences the definitions of initial assumptions cannot always be as clearly drawn as in mathematics, however, so it is difficult to be certain that the list of alternative hypotheses has been exhausted. In the present discussion of particle-antiparticle interactions the finding of an incontrovertible hypothesis which does *not*

require that matter is created or destroyed under any circumstances would have its merit, particularly from the point of view of the proponents of the classical atomic theory. To paraphrase another author with less direct involvement with the physical sciences who put these words in the mouth of Sherlock Holmes:[18] "When you have eliminated the impossible, whatever remains, *however improbable,* must be the truth."

B. IS THERE A NON-HYDROGENIC STATE OF POSITRONIUM?

In the first stage of the electron-positron interaction there is a close analogy to what is observed in the hydrogen atom formation. However, the fact that the 1s state of positronium undergoes spontaneous decay clearly distinguishes it from the proton-electron combination. In a broader sense, however, the positronium decay is similar to emission processes occurring in excited hydrogenic states such as the $^2P_{1/2}$ species, for example. After a short lifetime a decay photon is observed as the hydrogen atom returns to its ground state. Again quantum electrodynamics is able to describe this process with extremely high accuracy. The fact that no subsequent emission has ever been observed from the hydrogen $^2S_{1/2}$ state is why one refers to it as the ground state of this system. *But does this mean that the analogous 1s state of positronium is its ground state?* The close similarity between the hydrogenic and postitronium spectra predicted by quantum mechanical theories ranging from the non-relativistic Schrödinger equation to quantum electrodynamics is the primary justification for answering this question in the affirmative, but the observed positronium decay from its 1s state raises at least the possibility that this conclusion is incorrect. If there is a state of positronium below that of the 1s species, the observed photon appearance can be explained in a different manner.

To explore this possibility let us examine the mechanics of the $P_{1/2}$–$S_{1/2}$ emission process in more detail for the hydrogen atom. The initial system is clearly the atom in one of its excited states, whereas the final system consists of the same atom in its ground state along with a photon associated with a characteristic frequency. With reference to what has been said previously, it can be noted that this theoretical description does not represent a balanced reaction in the traditional sense. Nonetheless, there are clear similarities between this process and the positronium decay from its 1s state. This can be seen by considering the distribution of energy and momentum among the partners of the transition. Because of the relatively large mass of the H atom, conservation of energy and linear momentum requires that the photon carry away most

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of the energy accompanying the transition, but not all of it. In order that momentum also is conserved in the process, it is necessary that the H atom recoil slightly relative to its initial position. The momentum of the atom is thus given by the de Broglie relation[19] as $p = h/\lambda$, where λ is the wavelength of the emitted radiation and h is Planck's constant. Consequently, some of the energy lost in the transition is also carried away by the H atom, specifically an amount equal to $p^2/2M_H$, where M_H is the total mass of the atom, i.e., electron plus proton. For higher-energy transitions, such as gamma decays in nuclear processes, the recoil energy can be so large that the energy of the emitted photon differs considerably from the internal energy difference of the pair of nuclear levels involved (Mössbauer effect[20]).

The decay of positronium can be viewed as similar in nature to the above emission processes, differing from them only in quantitative detail, provided one makes a crucial assumption, as outlined in what follows. It is generally accepted, for example, that the reason at least two photons are always observed after positronium decay is because of the need to conserve energy and momentum in the process. The rest masses of the particles present after positronium decay are equal, however, whereas in the other case the hydrogen atom is far more massive than the emitted photon. Consequently in the two-photon positronium decay the available energy and momenta are equally distributed (at least as viewed from the original center of mass of positronium), whereas a much less equal distribution is found among the H atom emission products. The assumption of a low-energy state of positronium below the 1s entity clearly would give more substance to this analogy, but there still remain difficulties as to how best to correlate the various product and reactant systems in the two processes. To begin with, it seems straightforward to associate the emission quantum in the H-atom case to *one* of' the photons observed in positronium decay. The energies of the H atom and positronium photons are different to be sure, but so as to perfectly satisfy the pertinent conservation laws in each case. In order to make the analogy even closer, however, one might correlate the second photon observed in the positronium decay to the H-atom $1s_{1/2}$ product itself in the other example. The latter association is tantamount to saying that this photon continues to have the same e^+e^- structure as the initial complex, simply existing in a lower-energy state than the 1s species (see Fig. 1). *Is something of this nature not a possibility?*

Pursuing this supposition further, it is important not to forget that both photons accompanying positronium decay (again in the most frequently occurring process) appear to be

identical in every way, including with regard to their energies and absolute values of linear and angular momenta; only the directions of the latter two vector quantities are different, i.e., opposing. There is thus no justification from experiment to attribute a different composition to one of the decay photons than to the other. Before one can claim that particle balance has been achieved by the above assumption, however, it is necessary to face up to the fact that in this model there is apparently (at least) one more photon (even if its proposed e^+e^- structure is correct) present after the positronium decay than before it. At least there is comfort in realizing that the same state of affairs exists in the H-atom emission process, and therefore that the analogy under consideration is not weakened on this basis.

In any atomic, molecular, nuclear or other radiative emission process, the conventional view holds that while the initial system consists of a single substance in an excited state, the final system consists of the same substance in a more stable state plus a photon of well-defined energy and momentum. Despite the universality of such processes, however, there is a way to describe them consistently, positronium decay included, without giving up the concept of complete particle balance. It is simply necessary to assume quite generally that the observed emission photons are also present *prior* to such transitions, *but that their energy and momenta are exactly zero in these initial states.*
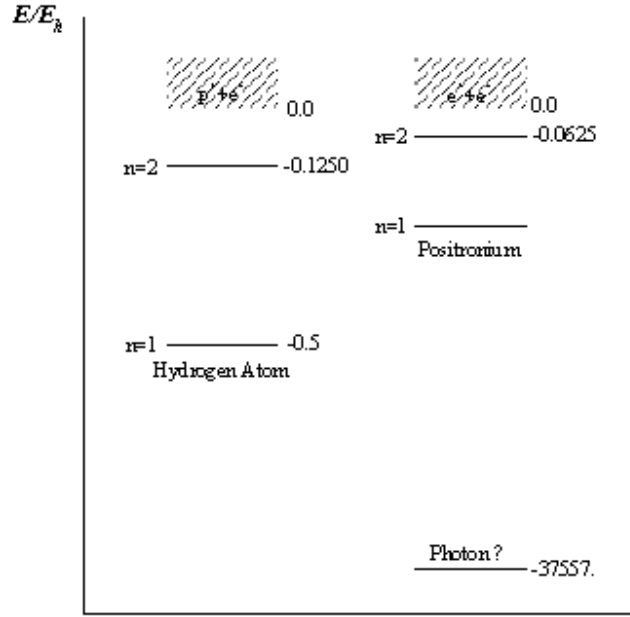
FIG. 1. Energy level diagram comparing the hydrogen atom and positronium. In the standard quantum mechanical theory the lowest (1s) levels of each system occur at -0.5000 and -0.2500 hartree, respectively. Corresponding ionization energies for the excited states of these two systems always differ by a factor of two as well, reflecting the different reduced masses of the electron in the two cases. The hydrogenic ls state is known to be stable, however, whereas the corresponding e^+e^- state has a short lifetime and decays radiatively. This suggests that the true lowest state of the e^+e^- system actually lies far below the n = l state of positronium, and as such corresponds to the massless state of the photon itself, with an "ionization" energy equal to $2m_{oe}c^2$ or 37557.7306 hartree.

We come then to the crux of the creation-annihilation hypothesis. To deny this proposition is to insist at the very least that photons with zero energy and momentum exist in their own right, despite the fact that according to the theory of special relativity they must be in a massless state under these conditions. Furthermore, the fact that radiative emission is observed whenever a system populates an excited internal state inevitably forces a new assumption, namely that such massless particles can be found in sufficient numbers anywhere throughout the universe at all times.

C. RELATIVISTIC CONCLUSIONS ABOUT PHOTONS OF ZERO ENERGY

The concept of photons other than the real variety encountered in everyday experience is by no means foreign to physical theory. So-called virtual photons[21] play a key role in relativistic quantum electrodynamics and are invoked to explain details of the interaction of radiation with matter wherever it exists. Care is generally taken to exclude the possibility that such entities have any but a theoretical existence, however. One prefers instead to speak of them as a field quantity, with the non-localized properties of a wave-like substance, rather than simply as particles in the Newtonian sense. Even in classical electrodynamics it has long been known that there is a need to attribute non-zero energy and momenta to an electromagnetic field[22]. It is easy to find situations where failure to do this is tantamount to assuming that neither quantity is conserved in such processes. This line of approach is at least a clear indication that the *properties* of photons must be assumed to be present on a large scale everywhere in the universe, even if it is insisted that the particles as such are non-existent.

But why not actual particles? A typical argument is drawn from STR, asserting that once a particle with zero rest mass (as one assumes for a photon *in vacuo)* does not move with the speed of light c, it ceases to exist. The justification comes from the law of mass dilation[4]: $m = m_o (1 - v^2/c^2)^{-1/2} = m_o\gamma$, where m_o is the rest mass and m is the relativistic mass of the particle moving relative to the observer with speed v. Accordingly, if $m_o = 0$ and $v < c$, then γ is finite and $m = 0$. On this basis it is generally concluded that the corresponding particle cannot exist.

Yet examination of this argument shows that the supposed non-existence of the zero-mass photon is really just an assumption. All one learns with certainty from the above formula is that $0 < v < c$ is a condition under which the relativistic mass of such a particle vanishes. *It does not say that the particle itself necessarily ceases to exist* as a result, however. The equation of non-zero mass with the possibility of a particle's existence cannot be said to be a logical consequence of the theory of special relativity. Instead, it constitutes an additional assumption which has had enormously broad consequences over the length and breadth of modern physical theory. At the very least it seems only prudent to give careful consideration to alternative interpretations of the possible meaning of a massless state of a system, as will be done below.

To begin with, it is possible to give a simple continuity argument which makes the existence of zero-mass photons plausible. Combining the mass-energy equivalence with the Bohr frequency relation gives: $E = h\nu = mc^2$. If $m = 0$, it follows that the corresponding frequency of the radiation is also of vanishing magnitude, whereas the wavelength $\lambda = \nu/c$ is infinitely large. A photon field with infinite wavelength is inaccessible to experimental detection and thus is not observable in the traditional sense. However, there is a clear distinction to be drawn between non-observability and non-existence. When an infinitesimal amount of energy is added to the same system, the above equation indicates that the photons now correspond to a non-zero frequency and a finite wavelength, which at least in principle can be measured. While there are definite limits as to how long a wavelength or how short a frequency can be measured in practice, it seems arbitrary to insist that beyond this point we dare not think of the particles as continuing to exist. If one can systematically withdraw energy from a single photon, at what point can it be safely assumed that it has been annihilated? The point is surely not that one can prove that a photon with exactly zero energy exists, but rather that *the converse cannot be proven* by this or any other means either. There is even a certain logic in dealing with the massless photon simply as the limiting case in the above experiment as the wavelength of the radiation becomes infinitely large.

Another mathematical point relating to the mass dilation formula should also be considered in the present context. If $m = 0$ for a particle of zero rest mass, must it continue to move at the speed of light? The answer based on the formula alone is clearly

negative. Any value of the speed v up to and including c is consistent with zero relativistic mass for a system of zero rest mass. Even v > c is not inconsistent, which fact, if nothing else, demonstrates that the mass dilation formula is really not at all restrictive on this point. The possibility exists in particular that a massless photon is *at rest* in a given rest frame, unlike photons with non-zero relativistic mass. There is no contradiction in this with regard to the postulates of STR, since such photons defy observation in any and all inertial systems. The photon in a massless state actually corresponds to the null world-vector of energy, which means that application of any Lorentz transformation with v < c leaves it unchanged. This characteristic thus precludes the possibility that observers in inertial systems moving relative to one another would ever come to different conclusions about whether a given photon's mass is zero or not. A zero-energy photon thus remains undetectable to all observers regardless of their velocities relative to one another. The same conclusion is reached by consideration of the relativistic Doppler effect,[23] which holds that the frequency of light as measured by an observer moving relative to the source with a speed v < c is a finite multiple of the frequency measured in the inertial system of the source itself. With reference to the *Gedanken* experiment of the last paragraph, a photon with decreasing energy continues to move at the speed of light as long as its mass exceeds null. In the limit of zero energy it becomes free to change its speed over a continuous range, although it must not do so. As a massless system, its momentum is unaffected by a change in velocity, so that a gradual reduction to zero velocity in any given inertial system is possible without altering its energy.

D. STATISTICAL MECHANICS OF MASSLESS PARTICLES

In order to obtain a satisfactory explanation for spontaneous radiative emission which avoids the creation-annihilation hypothesis, it is not only necessary to assume that photons can exist with zero energy and momentum, but also that they exist in great numbers everywhere in the universe. One can approach this aspect of the problem on two levels. The first simply relies on the arguments of the last section and takes them a step further, namely if it is not possible to observe a single photon in this state, then it is also not possible to contradict the view that there are great numbers of such systems. However, it is also possible to find more positive indications regarding this point by

considering the phenomenon of blackbody radiation. Quantum mechanics originated with Planck's discovery[24] that the observed intensity distribution in a perfect absorber can be quantitatively described within the framework of Maxwell- Boltzmann statistics, provided one assumes that only certain energy values are available for radiation of a given frequency. Specifically, Einstein showed that the mean value of the energy is obtained as

$$\langle E \rangle_v = \sum_{n=0} nh\nu \ \exp(-nh\nu/kT) / \sum_{n=0} \exp(-nh\nu/kT), \qquad \text{(II.1)}$$

rather than as a ratio of integrals in which n is treated as a continuous variable with non-integer values. The key point of interest in the present context is that *the n = 0 term in the above sums must be retained* to provide for an accurate representation of the observed spectral intensity distribution. This term does not alter the sum in the numerator, but it makes a decisive contribution to that in the denominator (partition function).

According to the theory of statistical mechanics, each term in the above sums corresponds to an allowed state for the system, in this case a collection of oscillators or photons with energy $E_n = nh\nu$. The zero-energy (n=0) photon is thus an ingredient in Planck's long-accepted solution to the blackbody problem. Moreover, as the lowest-energy state available to a photon associated with a given frequency ν, it is also the most frequently populated according to the Boltzmann exponential law, and this at any temperature T. In order to obtain the total intensity distribution it is necessary to integrate over all frequencies from null upwards. It is important to note, however, that zero-energy photon states are present in the distribution for *each value of v*. This situation is illustrated in Fig. 2, in which the various frequencies are represented by the spokes of a wheel. The allowed states for a given v can be thought of as being plotted as points along the corresponding spoke at a distance from the center of the wheel which is proportional to their energy. Especially if the Boltzmann populations are taken into account, it is found that by far the largest concentration of photons is at the center of the wheel, *i.e.* with exactly zero energy and momentum.

Allowed Photon Energies
in Blackbody Radiation
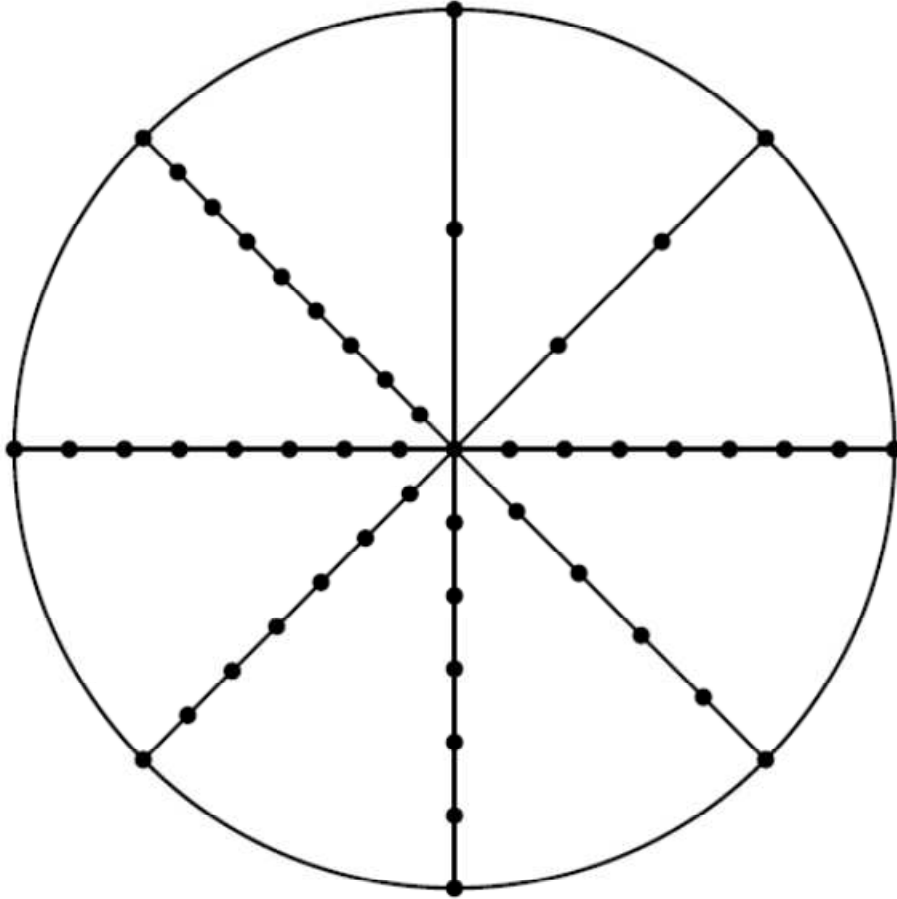
FIG. 2. Schematic diagram representing the distribution of allowed energy levels in the theory of blackbody radiation. Each spoke of the wheel corresponds to a fundamental frequency ν whose energy quantum $E = h\nu$ is always proportional to the distance between adjacent points on such a radius. Each such (equally spaced) point thus corresponds to an allowed energy level, one of which is always found at the hub of the wheel for each spoke, i.e. $E = 0$ is allowed for every value of ν. The magnitude of the energy quantum is shown to decrease monotonically as one proceeds in a clockwise fashion from the twelve o'clock position. The partition function used in Einstein's explanation of the blackbody radiation phenomenon must include the $E = 0$ levels explicitly for each fundamental frequency in order to obtain results which agree with experiment. It is thus clear from the diagram that the highest concentration of allowed energy levels by far is located at the hub of the wheel, and the form of the Boltzmann exponential factors $\exp(-E/kT)$ insures that the highest photon population always occurs for this energy value.

Since a blackbody of a given temperature displays the same intensity distribution regardless of its location, it must be assumed that this state of affairs exists everywhere. The relatively high density of zero-energy photons is a theoretical assumption apparently needed to explain observed phenomena. This circumstance does not constitute a proof of the hypothesis in the mathematical sense, but at least it can be said that the idea does not lead to a contradiction either. Put the other way around, it would be a very damaging piece of evidence to the massless photon concept if states of zero energy had to be *excluded* from the partition function in order to achieve a satisfactory representation of the experimental observations. Quite to the contrary, Einstein made the opposite assumption of a high density of zero-energy photons. Seemingly the most natural interpretation for this theoretical approach is to conclude that the populations of *all* the various photon states are given correctly by the Boltzmann exponential factors in eq. (II.1), not only for those corresponding to non-zero quantum numbers.

E. SEARCHING FOR A QUANTUM MECHANICAL REPRESENTATION OF THE ZERO-ENERGY STATE OF THE PHOTON

It is interesting to shift the discussion from the general topic of radiative emission back to the original subject of positronium decay. By comparison with details of the hydrogen atom emission process, it was concluded that the appearance of only photons after positronium decay does not rule out the possibility that the internal structure of both the initial and final participants in this reaction is exactly the same. At least the creation-annihilation hypothesis can be avoided by means of such an alternative assumption. In other words, it is proposed that the photon may also have an $e^+ e^-$ elemental composition, simply existing in a state of lower energy than the 1s species associated with positronium itself. The observed frequencies of the decay photons dictate that the energy of such an e^+e^- state must be $-2 m_e c^2$ or -37557 hartree relative to that of the separated electron and positron, i.e., the negative of what one conventionally refers to as the annihilation energy. By comparison the energy of the positronium 1s state is -0.25 hartree (Fig. 1).

Qualitatively one can imagine an attractive potential which binds the electron and positron so tightly together that there is a mass reduction similar to that known to occur in

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nuclear reactions. The difference in this case is that there is a total loss of mass, and not just a few parts in a thousand. Moreover, such a tightly bound e^+e^- state can have no counterpart in the hydrogen atom spectrum. It is clear that quantum electrodynamics provides no such attractive potential or corresponding internal state, but one also knows that the range of validity for this theory is limited to electromagnetic interactions. Nonetheless, any conceivable extension of this theory to include a tight-binding e^+e^- state of the type required must remain consistent with the latter theory in its description of conventional electromagnetic phenomena. Before looking for a more quantitative model to describe such a positronium state, however, it is well to remain on the phenomenological level in considering the consequences of avoiding the creation- annihilation hypothesis on the interpretation of other experimental observations in the field of modern physics.

III. A SURVEY OF OTHER EXPERIMENTS INVOLVING PHOTONS

The discussion in the preceding chapter demonstrates first and foremost that there is no compelling proof that particles pass to and from existence in the decay of positronium. It is impossible to distinguish between objects which have gone out of existence from those which are simply lost from view for a period of time. The alternative assumption to the creation and annihilation of matter is thus that particles can exist in great abundance in a massless (zero-energy) state without being directly observable. Put more descriptively, this amounts to saying: *"We live in a sea of photons."* The question to be explored in the present chapter is how these concepts can be used to explain other fundamental observations in modern physics.

A. PROPERTIES OF THE PHOTON

The interpretation of positronium decay as an emission process involving different states of the same physical system has been seen to suggest that the photon itself is a compound of a single electron and positron. It is therefore interesting to compare the properties expected for such an e^+e^- structure with those known experimentally for the photon. To begin with, it can be noted that a system containing two fermions in a highly bound state would be expected to obey the Bose-Einstein statistics observed for photons. The spin of the combined system must be integral, just as for positronium in any of its

hydrogenic states. Whether a system consisting of an even number of fermions behaves as a boson or not is known to depend on the strength of the interactions holding the individual particles together[25]. The ^{3}He isotope, for example, is fermionic and non-superconducting, but combining it with another fermion (the neutron) produces ^{4}He, which behaves as a boson.

Otherwise, what we know of photons is that they have zero rest mass and no charge, the latter property being clearly consistent with an electron-positron composition. The fact that photons of a given energy are characterized by a definite frequency and wavelength does not distinguish them from other particles, as emphasized by the de Broglie relation,[19] p = h/λ, and the Bohr frequency law,[5] E = hν, and demonstrated explicitly for electrons by Davisson and Germer.[26] For photons there is the additional feature of oscillating electric and magnetic fields being involved explicitly in the wave motion. However, especially for optical photons, the frequency of the oscillations is too large to enable a direct measurement of the individual electric or magnetic fields.[27] The oscillating properties of photons/light are actually deduced from theoretical considerations, namely the solution of Maxwell's classical equations of electromagnetism.[28] In quantum mechanics photons have traditionally been treated as oscillators,[24] without giving a detailed description of the internal structure which is ultimately responsible for such characteristics. All that can be said in the present context is that an e^+e^- composition for the photon is at least consistent with electromagnetic phenomena. The dipolar nature of such a binary system meshes qualitatively with the photon's capacity for interacting with charged particles, especially when the photon is in relative motion to the latter. One would have to have much more detailed information concerning the wave function of the e^+e^- system in a given state of translation to make more specific comparisons with real photons. Similarly, since the speed of the photons is a consequence of their zero rest mass, this is again a conceivable property for a system with such a dipolar composition, one whose verification would require a more quantitative theoretical treatment.

The polarization of light has been one of its most intriguing properties. It has been interpreted by Wigner[29] to result from the fact that the photon possesses non-zero angular momentum **J**. The "twoness" of the photon's polarization is thereby explained as a relativistic requirement according to which a particle moving with the speed of light must

have **J** oriented either parallel or anti-parallel to its line of motion. Quantum mechanically this means that only $M_J = \pm 1$ is allowed for photons, despite the requirement of symmetry that components with $M_J = 0$ also must exist. Circularly polarized light corresponds to an eigenfunction of J_z, while plane-polarized implies a 50-50 mixture of both allowed M_J values and elliptically polarized light is any combination in between, all of which is consistent with the existence of an effective two-fold degeneracy. Careful experiments[30,31] have demonstrated that the magnitude of a circularly polarized photon's spin component is $\hbar$, corresponding to $|J| = 1$, which is consistent with the Wigner interpretation[29] but also with a possible e^+e^- constitution for the photon itself.

Altogether it should be recalled that despite intense investigation over centuries, going back at least to the work of Newton[32] and Huygens,[33] there is very little consensus about the structure of the photon itself, or indeed whether it has any internal structure at all. Einstein remarked [34] in 1951 that, despite his efforts of the preceding half-century, he did not feel that he had come any closer to answering the question of what a light quantum is. He went on to say that apparently many people[35] did think they understood the matter, but that they were only deceiving themselves. At the very least his comments would seem to allow considerable latitude for further research into this question.

B. **PRODUCTION OF PARTICLE-ANTIPARTICLE PAIRS FROM PHOTON COLLISIONS**

The reverse process to positronium decay, in which an electron and positron are produced with the aid of high-energy photons, also needs to be considered in the present context. The assumption of an e^+e^- structure for each photon is obviously consistent with this result, but a few details require special attention. When a photon with energy equal to $2\,m_ec^2$ collides with a massless photon, no electrons are produced unless a heavy nucleus is also present. By contrast, if two photons collide head-on, and each has m_ec^2 energy, electron production is possible in free space. The distinction can be understood from relativity theory.

A collision between such a massless photon and one with $E = 2m_ec^2$ is characterized by a total momentum of $p = E/c = 2m_ec$. If one of the photons were to dissociate into its elements e^+ and e^-, all the available energy would be used up for this purpose, so that the

translational energy of the two electrons produced would have to be null. The latter condition makes conservation of linear momentum in such a process impossible, however. By contrast, if both photons have $E = m_e c^2$ and collide head-on so that the momentum sum $\Sigma p_i = 0$, it follows that the electron and positron can be set free, but must remain at rest in the original inertial system (the one in which the photons are observed to have equal energy). The latter process is seen to be simply the reverse of the positronium decay process, or more precisely the reverse of the interaction of a free electron and positron which are initially at rest in a given rest frame.

More generally, it needs to be recognized that for a given energy E, the momentum of the photon (E/c) is always greater than for any particle with rest mass $m_A > 0$, for which $p_A = (E^2/c^2 - m_A^2 c^4)^{1/2}$. This fact prevents a single photon of any energy from causing a zero-energy photon to dissociate, because no matter what energy might be transferred, there is a disparity in the corresponding photon momentum lost and that which could be theoretica11y given to each of the electron products. The presence of a third body has the potential of removing this restriction, as is well known, but the point to emphasize in the present discussion is that the same result is found whether free space is thought to be involved, as foreseen in the creation-annihilation hypothesis, or if a massless *but existing* photon of $e^+ e^-$ structure is assumed instead.

To make this point more clearly, it is interesting to consider the effect of relative motion of the observer on the outcome of such experiments. The relativistic Doppler effect[23] tells us that the energy (frequency) of the photons in the above examples is dependent on the relative speed of the inertial system from which these quantities are measured. There is a clear exception to this rule, however, namely if the energy of the photon is zero in one inertial system, it must remain zero in any other. Thus it is not possible to make the transition between the above two cases simply by changing the relative speed of the observer. As noted in Sect. II.C, a massless photon corresponds to a null vector in Minkowski space,[36] and as such is unaffected by any Lorentz transformation. At the same time, a photon with non-zero mass can have its energy changed to any conceivable value other than zero by virtue of such a transformation. The consequences of these relationships are crucial in the present case, with electron-positron production in "free space" occurring only if both photons have non-zero energy, just as is observed experimentally.

With much higher energies it is also possible to generate proton-antiproton pairs,[3] again as predicted by the Dirac theory.[1] It is clear that this result cannot be entirely explained by assuming an e^+e^- structure for the photon. Nonetheless, it cannot be said that such observations are inconsistent with what has been assumed so far. Rather, they force an additional assumption, namely that other types of massless particle- antiparticle binaries exist as well. There is of course a natural tendency to avoid introducing new types of particles into any theoretical framework, however. At the very least one hopes to keep their number to an absolute minimum.

As long as its rest mass is exactly zero, the mechanical properties already mentioned for e^+e^-, such as $v=0$, $\lambda = \infty$ and the like, could also apply to p^+p^- or related entities. One can only speculate that a p^+p^- system of zero rest mass will exhibit different properties under translation than do the corresponding e^+e^- species. Clearly, the dissociation energy of p^+p^- must be 1836 times greater than for e^+e^-, which condition already constitutes a major distinction. By the same token, the fact that neutron decay produces neutrinos[37], whose rest mass is already close to or equal to zero, implies that there must be $v\,\bar{v}$ binaries as well, with extremely small to vanishing dissociation energies. The real challenge presented by these observations is to construct a quantitative theory, requiring as input at most such quantities as the rest mass, charge and perhaps magnetic moment of the interacting species, *which leads to binding energies of the above particle-antiparticle pairs which are equal to $2c^2$ times the rest mass of each of the respective constituents.*

Since the charge-to-mass ratio is much smaller for the proton than the electron, it seems clear that a p^+p^- binary would show much weaker electromagnetic effects than its e^+e^- counterpart. On this basis, it seems plausible that the traditional properties of a photon *i.e.* oscillating electromagnetic field which is involved even in low-energy emission and absorption processes, are exhibited exclusively by the electron-positron massless binary systems. The statistical arguments given above in conjunction with the discussion of blackbody radiation (Sect. II. D) speak equally well for a high density of other systems of zero rest mass. At least one knows that protons and antiprotons can be produced together wherever the appropriate energy and momentum conditions are fulfilled.

C. QUANTUM CONDITIONS OF PHOTON INTERACTIONS

The quantum jumps associated with photon interactions provided an important clue regarding the particle nature of light. In his explanation of the photoelectric effect,[38] Einstein reversed a trend away from the Newtonian view[32] of light as "corpuscles". He showed that surface ionization of metals could be most consistently explained by assuming that a single quantum of light gives up all its energy to a single electron. He used the word "heuristic" in describing his ideas because the (exclusively) wave theory of electromagnetic radiation was widely accepted by the physics community at that time. While there can be general agreement that the photoelectric effect is inconsistent with a totally wave-like nature for light, it still must be regarded as extraordinary that any particle would transmit *all* its translational energy to a single electron in given interaction.   Such a property of photons is consistent with the concept of annihilation, because it is reasonable to assume that a particle which has gone out of existence does so by leaving behind all its energy and momentum.   However, f it is assumed instead that the photon retains its existence after photoionization has occurred and simply assumes a massless state which defies direct experimental observation, it is necessary to look more closely at the dynamics of this process to better understand the nature of the quantization phenomenon.

To this end it is instructive to apply the laws of energy and momentum conservation to the absorption process, as depicted in Fig. 3. If the photon γ were to give off an arbitrary amount ΔE of its energy to an atom A with mass M_A, its momentum would decrease by $\Delta p_\gamma = \Delta E/c$. If the atom were to remain in the same internal state, this amount would appear in the form of translational energy, which means that the momentum of the atom would change by $\Delta p_A = (2M_A\Delta E)^{1/2}$. Conservation of momentum requires that Δp_A and Δp_γ be equal. For small ΔE this can never be the case, however, in view of the large mass of A. Setting Δp_A equal to Δp_γ shows that ΔE would have to be equal to twice the rest energy of A or $2M_Ac^2$, which corresponds to the GeV range[39]. There is a solution to this dilemma, however, namely to have a part of the photon's energy be added to the internal energy of the atom, *i.e.* that another electronic state of the more massive system be reached. If the excited electronic state differs by $h\nu$ in energy from that of the initial state, conservation of momentum requires that

$$\Delta p_\gamma = \Delta E/c = \Delta p_A = 2M_A(\Delta E - h\nu)^{1/2} \qquad\qquad \text{III.1}$$

which is possible provided $h\nu$ is only slightly smaller than ΔE, again by virtue of the relatively large mass of A as well as the magnitude of c.

It is important to distinguish between two aspects of the absorption process in the foregoing discussion. First, the quantized nature of the atomic spectrum is seen to be directly connected with the large disparity between the respective masses of the atom and the photon. When one considers the translational motion of the atom, it is recognized that the energy levels available to it are actually continuous. It is the requirement of momentum conservation which restricts the possible transitions between different states of the same atom and thereby produces the quantization phenomenon. On the other hand, on the basis of these arguments by themselves there is no restriction put on the magnitude ΔE of the energy lost by a photon in the absorption process, save that it be less than its total energy, $E = m_\gamma c^2$. Indeed, the analogous excitation brought about by electron impact is well known[40]. One is thus still left with the conclusion that there is something special about a zero-energy, zero-momentum state of the photon, even though many aspects of the absorption phenomenon can be understood by just assuming that the photon is a particle of relatively small mass compared to the system with which it interacts.
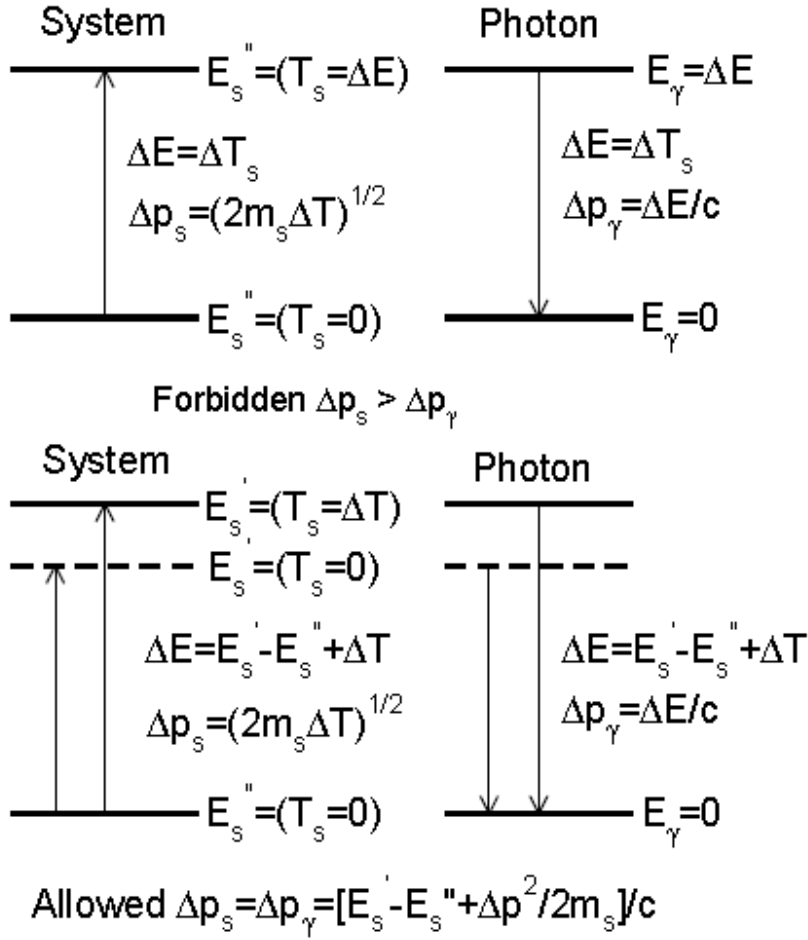
FIG. 3. Energy level diagram detailing the role of conservation laws in determining whether a given radiative absorption process is allowed or not. At the top of the diagram, the system is to retain the same internal energy E_s" in the transition, *i.e.* the two levels shown differ only in translational energy $\Delta T_s = \Delta p_s^2 /2m_s$ (non-relativistic theory), where m_s is the mass of the system and Δp_s is the corresponding change in the momentum of its center of mass. Such a radiative process is always forbidden by the law of conservation of linear momentum, because the rest mass of the photon is so much smaller than that of the system ($\Delta p_s > \Delta p_\gamma$). The only way for radiative absorption to occur is if the system changes its internal (from E_s" to E_s') as well as its translational energy, as depicted in the lower part of the diagram. Under these circumstances the momentum conservation law can be satisfied for a particular value of Δp_s, namely one that is equal to (E_s' - E_s" + $\Delta p_s^2 /2m_s$)/c, where c is the speed of light. This condition rules out the occurrence of a radiative absorption process in which the system's translational energy does not change at all, also as indicated. Thus the "quantized" nature of radiative transitions is seen to be intimately connected with the photon's vanishing rest mass.

The fact that the energy transferred in the above process is exactly equal to $E_\gamma = m_\gamma c^2$ is thus seen to be a separate issue from the photo-ionization phenomenon itself. In other words, why doesn't the photon give off only part of its energy in inducing a transition in another system? Dirac used time-dependent perturbation theory[41] to answer this question, arguing that the incident radiation introduces a frequency-dependent term in the Hamiltonian of the atomic system. A resonance condition results according to which the energy of the most probable atomic transition, $h\nu = E_i - E_f$, must be the same as the energy of the incident photon, $E_\gamma = m_\gamma c^2$. The prospect of a massless photon being formed as a result of this energy exchange (rather than that the original photon is annihilated in the process) suggests a somewhat different interpretation for this phenomenon, however, one which does not rely on the *ad hoc* assumption of wavelike properties for the incident radiation. If one simply looks upon the process as a collision between an atom and a photon moving with speed c, it seems plausible to demand that the observed energy exchange take place over a relatively small but finite period of time. As a consequence, the temporal requirements of the interaction are more readily fulfilled by an outgoing system whose velocity has been considerably reduced below the speed of light in a vacuum. As long as the departing photon possesses a non-zero amount of energy, this condition can never be fulfilled, but as has been pointed out in Sect. II.C, a *massless* photon is free of any such restriction, and thus can move at any speed less than c, including zero. In this view, the only practical means available to a photon to reduce its energy by virtue of an atomic collision is to assume a massless state, so that its relative speed compared to the system with which it interacts can be made as close to zero as possible. Accordingly, this interaction mode represents the only inelastic collision process available to a system of zero rest mass, since it is otherwise forced to move with the speed of light as long as it possesses any non-zero amount of translational energy.

By combining this result with the conservation of energy and momentum arguments first discussed it is seen that the quantum characteristic associated with radiative absorption (and emission[42]) can be deduced exclusively on the basis of the rest mass values of the photon and the interacting system, respectively. There is no need to postulate any wave characteristics for the field inducing the transition. Rather, one is led to conclude from knowledge of the internal energy states of the interacting system and the magnitude of its rest mass exactly which photon energy is required to induce maximum transition probability. The magnitude of this transition probability itself cannot be determined quantitatively on the basis of the above information

alone, and thus for this purpose one does have to introduce some additional information about the nature of the perturbing Hamiltonian, which itself is ultimately based on other experimental observations. This state of affairs does not affect the main conclusion in the present discussion, however, namely that the properties expected on the basis of relativity theory for a massless but nonetheless existent system are sufficient in themselves to allow for a suitable explanation of the observed tendency of photons to give up all their translational energy upon interacting with other particles.

These observations are also relevant to the positronium decay process discussed in Chapter II. In order for the de-excitation process to occur from the positronium 1s state to the proposed tightly-bound e^+e^- photon state (as depicted in Fig. 1), it again seems highly desirable that there be a minimum of relative motion between its initial and final systems. This condition cannot be said to be satisfactorily fulfilled when the product photon carries translational energy, because it must then move away from the original point of interaction with the speed of light. That would be something akin to a business transaction carried out between two people, one of which is riding on a speeding train while the other is standing on the station platform. In its massless state the photon can move with exactly the same velocity as the initial positronium system, thereby greatly improving the chances for such a transition. In this way, momentum can be conserved in the process, but the energy lost by the positronium complex still has to be carried away. As shown in Fig. 4, the simplest way to accomplish this objective is to have the released energy divided up equally between two other photons which are in the neighborhood of the interaction locale, which again means they must initially possess zero translational energy. The conservation laws can then be satisfied by dividing the emitted energy equally among the two departing photons and having them move with exactly opposed momenta. There are also angular momentum conditions to be satisfied, which is why the number of emitted photons is different depending on the multiplicity of the positronium state prior to its decay. This point will be considered more closely in Chapter VI. To summarize. it is possible in this way to look upon the most commonly occurring positronium decay process (Fig. 4) as involving three distinct photons, each of which exists in its massless state at some point in the interaction. One of them is formed as a result of the de-excitation of the (singlet) positronium 1s state (Figs. 1 and 4), thus eluding detection by virtue of its null frequency. The other two are already present at the start of the reaction and are also unobservable as a consequence of their masslessness. Upon taking up their

share of the energy released in the decay process they are detected, however, giving rise to the "two-photon" classification commonly ascribed to this interaction.

Another important aspect of this topic arises in the treatment of the blackbody radiation phenomenon. As discussed in the preceding chapter, Einstein's quantum assumption assigns allowed states to a given photon with only integral multiples of its frequency ν. Since a blackbody is a perfect absorber, each frequency is present, at least in principle, and one can think of such a system as a collection of atoms of the type just discussed. The success of Einstein's assumption indicates that each frequency can be treated independently of the other, however. It also is important to recall that an equilibrium is present which is conventionally thought of as arising from a series of collisions between the participating systems over a long period of time. This suggests that photons of energy hν only interact readily with each other or with photons possessing a multiple of this energy. The same conclusion can be inferred from the occurrence of the coherence phenomenon[43] in electromagnetic radiation.

It seems at least possible that the stability of the photon in its massless state is at the root of the observed quantum characteristics of the blackbody intensity distribution. The special conditions of velocity seen to be permitted in this state, namely $0 \leq v \leq c$, are at least suggestive in this regard. The presumed high density of zero-energy photons based on the results of the blackbody experiment clearly derives from the fact that this energy value is the minimal amount available to photons of *any given frequency*. The key point which distinguishes the blackbody radiation phenomenon from the other processes discussed previously in this chapter is clearly that a large collection of photons is required to describe the effect in a meaningful way. It is not surprising as a result that it is quite difficult to analyze this particular experiment in the kind of microscopic detail needed to deal with the general question of whether individual massless photons can exist or not.

D. COMPTON AND RAMAN EFFECTS AND BREMSSTRAHLUNG

The Compton effect[44] involves collisions between x-ray photons and weakly bound electrons and can also be interpreted in a very straightforward manner using conventional energy and momentum conservation arguments in conjunction with the Bohr frequency and de Broglie relations. In this experiment a photon with a given energy is scattered off an (essentially free) electron and another photon is observed after the collision with lower energy and momentum

than the first. It might be argued that the same (x-ray) photon is involved before and after the collision, but in view of experience with the absorption and emission of photons it is generally assumed that the first photon gives up all its energy initially and that afterwards this is distributed between the electron and a second photon. Again one conventionally speaks of annihilation of the first photon and creation of the second in the process, but one can just as well imagine that the first photon simply assumes a massless state upon collision, while another massless photon takes up the energy left over from the electron collision and appears as an x-ray photon, generally moving in a different direction than the first.

The Raman effect[45,46] is closely related to the Compton effect, and involves inelastic scattering of visible light off molecular systems. If the initial frequency is ν, it is found that photons emerge at right angles to the incident radiation with frequencies $\nu \pm \nu'$, where ν' is a characteristic infrared frequency which is small compared to ν. Again there might be a tendency to interpret incoming and outgoing photons as one and the same, only with changed energy, but the problematics are clearly the same in this regard as for the Compton effect. In both cases it is clear that a particle interpretation for the electromagnetic radiation allows for a quantitative description of these phenomena. There is no particular difficulty interpreting these effects in terms of massless photons located in the neighborhood of the pertinent collision processes.

Finally, it is pertinent in this connection to consider the Bremstrahlung phenomenon as well. In this case an x-ray photon is produced with an energy which is essentially equal to the decrease in an electron's kinetic energy caused by its collision with a heavy nucleus. The process can thus also be thought of as an interaction in which a massless photon picks up energy, similarly as in the emission processes discussed earlier. The fact that a third (heavy) body is required is again related to the energy-momentum conservation laws, especially the fact that a given energy value always corresponds to a greater momentum for a photon than for a neighboring electron by virtue of the disparity in their respective rest masses.

IV. PARTICLES RESULTING FROM HIGH-ENERGY INTERACTIONS

In the preceding chapters, a model for describing processes involving the interaction of electromagnetic radiation with matter has been examined whose main ingredient is the assumption of an e^+e^- "molecular" structure of the photon. Accordingly. the hypothesis of

disintegrating matter is replaced with the assumption of indestructible particles, including electrons and positrons, which in a specific state of binding can lose all their mass, thereby defying experimental observation but still retaining their own existence. The latter distinction seems subtle enough when formulated in this manner, but there is a potentially critical difference. Put simply, if particles can be created from pure energy, there is nothing fundamentally excluding the possibility that much larger objects can also be formed by this mechanism. It has been argued[47], for example, that whole universes could be created from the "energy equivalent to just a few pounds of matter." Later in the same article it is pointed out that the key element in the author's theory "is that quantum physics permits the spontaneous creation of something from nothing." If on the other hand, matter can never be created or destroyed by any mechanism, but instead only disappears from view under certain well-defined circumstances, the possibilities are much less fantastic. A much more sober view of the universe emerges, and consequently it behooves us to find out which of the above two hypotheses corresponds to the true facts.

It is thus important to see that the creation-annihilation concept does have definite relevance beyond the subject of the interaction of photons with other particles. In the preceding chapter it was noted. for example, that the formation of protons and antiprotons from photon collisions demonstrates that e^+e^- systems cannot be the only massless particle-antiparticle binaries which must be assumed if one hopes to do without the latter hypothesis . Once one has grasped the possibility that the creation-annihilation assumption may not actually be needed to explain relatively low-energy phenomena, there is thus a challenge to follow through on such argumentation in the high-energy regime as well. One is reminded that considerable impetus was given to the creation-annihilation concept at the time when it first became possible to carry out high-energy experiments in the laboratory. It is therefore to be expected that as the available energy from accelerators and related devices increases, the number of phenomena requiring similar explanations will tend to multiply, as has indeed been the case. New particles with non-zero rest mass have been identified over the past 70 years, and these have provided fertile breeding ground for new theories. In the present chapter the task of surveying such high-energy experiments will be taken up, with particular emphasis on the question of how the assumption of the creation and destruction of matter plays a role in the theories which are used to describe them.

A. THE NEUTRON

After the results of Rutherford's scattering experiments were understood[48], one had the model of atoms containing a relatively massive nucleus of positive charge being orbited by a number of electrons. The simplest atom is hydrogen with a single proton and electron. The heavier nuclei have rest masses which are nearly integral multiples of that of the proton, but charges which are always a smaller (exact) multiple of the electronic charge. On this basis it was first assumed that the nucleus itself generally contains both protons and electrons. The discovery of the neutron in 1932 by Chadwick[14] caused a shift in this position, however. The neutron was found to have a slightly larger mass than the proton and to have no electronic charge. It is meta-stable with a half-life of 1000 s, decaying into a proton and electron (or "beta-ray" in the earlier terminology). Other unstable nuclei were also known to undergo beta decay.

Several objections to the assumption of a proton-electron constitution for the nucleus quickly arose on the basis of these findings. The most important was based on the fact that the energy spectrum of the emitted electron is not mono-energetic, as would have to be expected from the laws of conservation of energy and momentum when a single particle decomposes into two fragments. One popular interpretation for this observation was supported by Bohr.[49] He suggested that the usual conservation laws might no longer be valid at high energies (0.8 MeV is released upon the neutron's decay). Pauli took a different view[37], however, which has since been generally accepted, namely that the continuous spectrum observed indicates that at least one more particle must be involved in beta decay. Accordingly, the energy lost in the process can be divided between several emitted particles in a continuous distribution without violating the above conservation laws. The name "neutrino" was subsequently coined by Fermi[50] for this new particle.

Other evidence was also found for the neutrino hypothesis, however. If a neutron were to consist of only a proton and an electron, it should possess integral spin and exhibit boson statistics, so that a nucleus such as ^{14}N (containing seven protons and seven neutrons) would behave as a fermion with half-integral nuclear spin. Since the opposite behavior is observed, it was suggested by Pauli that the neutrino (if indeed it is a single particle) is also a fermion with half-integral spin, and that its presence in the neutron thus explains this aspect of the nuclear

puzzle as well. This argument eventually carried the day and the third neutron decay product was later renamed the anti-neutrino $\overline{\nu}$.

What needs to be emphasized in the present context is that in the search for a quantitative theory of beta decay, confidence in the straightforward idea of the neutron being composed of a proton, electron and antineutrino was eventually lost. This development occurred primarily because of the realization that the relatively small radius of the neutron implies that an electron bound within it would have to possess an enormous amount of kinetic energy. A figure of 100-500 Me V could be computed for this quantity based on the de Broglie relation, which connects the magnitude of this radius (wavelength) with the electron's momentum. Since the known electromagnetic and gravitational forces are far too weak to explain how the electron could enjoy a net attraction inside the nucleus under these conditions, it was clear that an impasse had been reached. In addition, the fact that the magnetic moment of the neutron was measured[51] to be much smaller than that of the electron seemed to be totally inconsistent with such a composition.

The problem was ultimately circumvented by Fermi with his suggestion that the electron might be annihilated in the presence of a strong nuclear force. It is interesting to consider Fermi's original remarks on this point, as given in translated form by Wentzel[52]. "The simplest way for the construction of a theory which permits a quantitative discussion of the phenomena involving nuclear electrons, seems then to examine the hypothesis that the electrons do not exist as such in the nucleus before the ß emission occurs, but that they, so to say, acquire their existence at the very moment when they are emitted; in the same manner as a quantum of light, emitted by an atom in a quantum jump, can in no way be considered as pre-existing in the atom prior to the emission process. In this theory, then, the total number of the electrons and of the neutrinos (like the total number of light quanta in the theory of radiation) will not necessarily be constant, since there might be processes of creation or destruction of those light particles."

In at least one sense this was a significant departure from the original hypothesis used to interpret positronium decay, because up to that point charged particles were only thought to undergo creation or annihilation pair-wise with their respective antiparticles. If one continues to doubt as in the preceding chapters that electrons and positrons are really destroyed in positronium decay, however, then it is only consistent to question the very similar hypothesis made by Fermi in explaining the beta decay phenomenon. Yet if one insists on the continuous existence of the electron in an alternative theory, one must face up squarely to the need for

finding a potential which is strong enough to overcome the undeniably high kinetic energy the electron would possess in the small volume occupied by a nuc1eus. The problem is more severe than this, however, because in taking this approach one also must provide a suitable explanation for the role of the antineutrino itself in beta decay. Before considering these points further, a few additional details of the experimental observations should be carefully considered.

A whole series of nuclear reactions could be found which are closely related to one another. Written in the form of reactions these are:

β decay: $n \rightarrow p^+ + e^- + \bar{\nu}$

$\beta+$ decay $p^+ \rightarrow n + e^+ + \nu$

electron capture: $e^- + p^+ \rightarrow n + \nu$ IV. 1

$\bar{\nu}$ absorption: $\bar{\nu} + p^+ \rightarrow n + e^+$

ν absorption: $\nu + n \rightarrow p^+ + e^-$

The notation is schematic, with p^+ and n generally denoting constituents of heavier nuc1ei actually present. As always is the case, the c1assical chemist's balanced equation becomes a casualty of the assumption that matter such as electrons or neutrinos can be created or destroyed. None of the above five reactive equations is balanced in the traditional sense. In the first, an electron and antineutrino are created, while in the second, both (e^-, e^+) and $(\nu, \bar{\nu})$ pairs are created, with subsequent annihilation of e^- and $\bar{\nu}$ in the process of neutron formation. In the third, a $(\nu, \bar{\nu})$ pair is created and then the neutrino plus a captured electron from an associated atom are destroyed. Finally, an (e^-, e^+) pair is created in the fourth reaction, followed by the. annihilation of e^- and $\bar{\nu}$ in the neutron formation, while in the last case the neutron decomposition produces the same two particles, only to have the antineutrino destroyed along with the neutrino which induces the reaction.

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The hypothesis of massless particle-antiparticle binary systems is only partially successful in restoring particle balance to these equations, but strict adherence to the original definition of elements advocated by Boyle and others before him completes the process. In particular, when we surrender the idea that material particles can be created or destroyed in such interactions, we are forced to conclude that *the neutron is not an element at all.* Rather it is a compound of its known decay products, *i.e.* p^+, e^- and $\bar{\nu}$, akin to a triatomic molecule in the usual chemical notation. The fact that the mass of the neutron is greater than that of its separated constituents is perhaps surprising, but according to the Einstein mass-energy equivalence principle of STR this is what one must expect from the fact that the dissociation energy of the neutron is negative (-0.8 MeV). There is considerable precedent for such a situation, namely in the description of excimer complexes, which have had a great impact in the field of laser research. Excimers are bound (meta-stable) molecular states for collections of atoms with a ground state that is characterized by a repulsive potential curve and is therefore unbound. The fact that the neutron undergoes spontaneous decay is perfectly consistent with this analogy. Only the energies involved are far greater than those released in excimer lasers. In the language of scattering theory, the neutron is thus a resonance with a relatively long lifetime. The nature of the forces which hold the neutron together in this meta-stable state constitutes a major unanswered question in this model, but in this respect the situation is wholly similar to that for the e^+e^- tight-binding state which has been associated with the photon in Chapter II.

For the sake of clarity the conventional use of the term "elementary particle" for the neutron and related systems will continue to be made below.[53] Nevertheless, it is well to keep in mind that the definitions of atoms and elements that Democritus and Boyle espoused exclude the neutron from this classification because of its instability. With these preparatory remarks, we can rewrite the five beta decay reactions given in eq. (IV.1) in completely balanced form:
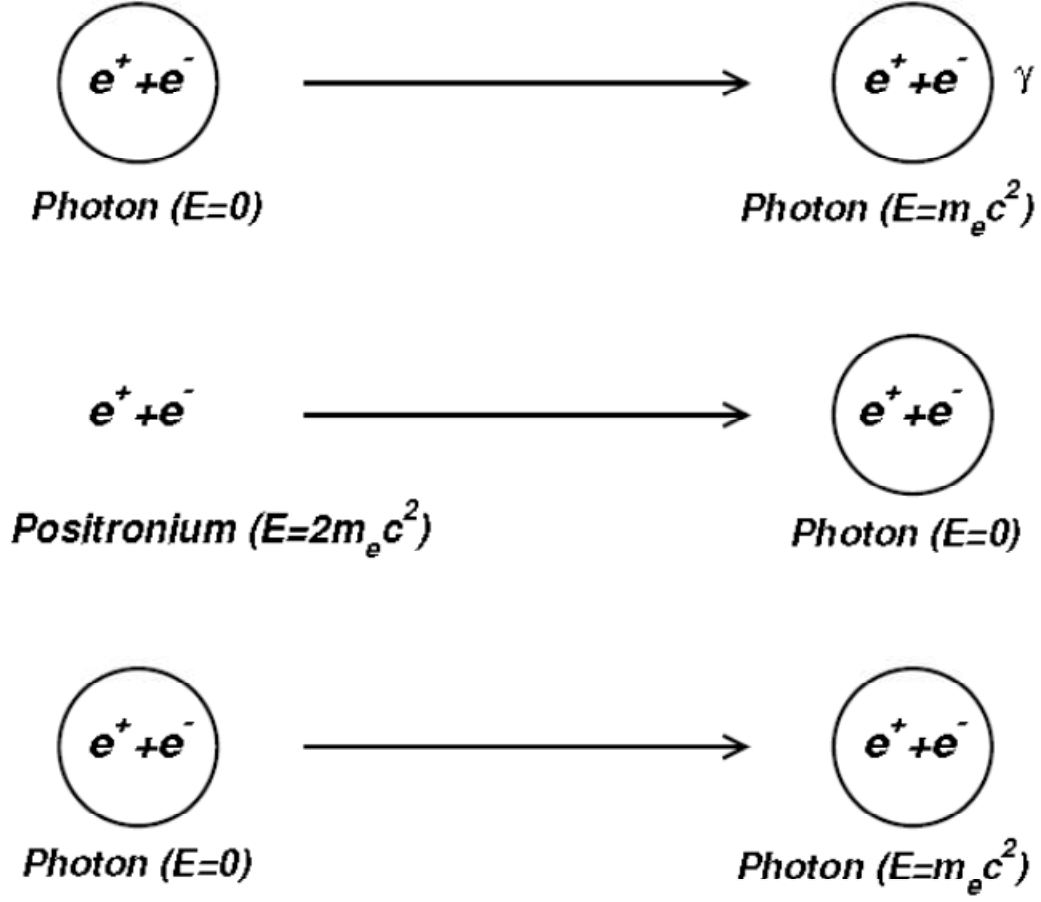
FIG. 4. Schematic diagram for the two-photon decay of (singlet) positronium. By assuming that the photon also has an e^+e^- composition, it is possible to describe this transition without assuming that particles are either created or annihilated in the process. In this model three e^+e^- binaries are involved, two of which are massless photons at the start of the process. They share the energy released by the positronium decay, and are observed as γ photons of equal energy at its conclusion. The final state of the original positronium system is another massless photon which, as its two counterparts at the start of the transition, escapes detection by virtue of its lack of energy.

i´) $p^+e^-\nu(n) \rightarrow p^+ + e^- + \bar{\nu}$

ii´) $e^+e^- + \nu\bar{\nu} + p^+ \rightarrow p+e^-\bar{\nu}(n) + e^+ + \nu$

iii´) $\nu\bar{\nu} + e^- + p^+ \rightarrow p^+e^-\bar{\nu}(n) + \nu$ IV.1´

iv´) $e^+e^- + \bar{\nu} + p^+ \rightarrow p^+e^-\bar{\nu}(n) + e^+$

v') $\nu + p^+e^-\bar{\nu}(n) \rightarrow p^+ + e^- + \nu\bar{\nu}$

In this way the number of distinct particle-antiparticle binaries has been kept to three in number, namely e^+e^-, $\nu\bar{\nu}$ and p^+p^-. The need for $n\bar{n}$ pairs is thereby avoided by virtue of the triatomic composition ascribed to a single neutron itself. Thus neutron-antineutron production involves the interaction of each of the three elemental binary systems, *i.e.*

$$p^+p^- + e^+e^- + \nu\bar{\nu} \rightarrow (p^+ + e^- + \bar{\nu}) + (p^- + e^+ + \nu) \rightarrow n + \bar{n}$$ IV.2

The economy of the assumptions in the above model is especially desirable when one attempts to construct a quantitative theory which is capable of generating wave-functions and properties for such massless particle combinations. The elemental balance in the above equations also has the advantage of making the additional assumption of charge conservation unnecessary. Since charge is an intrinsic property of each particle, never to be altered in the course of physical transformations, its conservation is always ensured thereby. In addition, we must continue to assume that energy and each of the components of the net linear and angular momentum are conserved in such processes, consistent with STR and the mass-energy equivalence principle. More generally, it can be said that there is an unwritten rule in the physical sciences according to which the number of theoretical assumptions should be held to an absolute minimum. In practical terms, an irrefutable contradiction based on experimental evidence should be established before adding any new postulate to the theoretical framework in whatever form. In this sense we can take a lesson from the conservation laws of classical physics, according to

which a truly remarkable variety of observations is understandable on the basis of only three fundamental principles[6].

B. THE NEUTRINO

The neutrino is associated with many puzzles that are still unresolved over 50 years after its discovery. It apparently bears no electric charge and thus is unaffected by electric fields. It also seems to be essentially massless, although this conclusion itself has been the subject of much controversy. During his lifetime Pauli compained[54] that the upper limit for its rest mass was set too high based on what was already know then. For a long time it was something of a bookkeeping device, merely accounting for energy otherwise missing in the beta decay spectrum. In 1956 the first successful experiment involving neutrinos as reactant species was performed by Reines and Cowans[55] and this achievement allowed considerably more confidence that the particle was more than a theoretical construction. In essence, Reines and Cowan carried out the first antineutrino absorption re action [eq. (iv)] in the laboratory. Extreme sensitivity was required. The actual mode of detection was the appearance of a photon signaling the reaction of the positron product with an electron, but the time between positive events was typically 30 hours. As a result it was clear that antineutrinos are extremely penetrating, with a chance of one part in 10^{12} of being captured while passing through the earth along a diametric ray. The almost complete absence of ionization products observed during the experiment also led to the conclusion that the particle has virtually no magnetic moment, but this raised difficult questions about what forces actually govern the neutrino's behavior. The lack of either electronic charge or a magnetic moment also left uncertain the exact relationship between ν and $\bar{\nu}$ themselves, since previously every other particle could be distinguished from its antiparticle on the basis of these properties.

An answer to this question was provided by Lee and Yang[56] when they concluded that parity might not be conserved in beta decay. A decisive experiment[57] suggested by their work was carried out shortly thereafter by Wu *et al.* on the longitudinal polarization of decay electrons emitted by the ^{60}Co nucleus in the presence of a strong magnetic field at low temperature, using a method proposed earlier by Rose and Gorter.[58] As ^{60}Co decays into ^{60}Ni the nuclear spin changes by one unit from I=5 to 4, and is referred to as a Gamov-Teller transition[59], as opposed

to the other possibility (Fermi transition) with ΔI=0. Because only the lowest-energy nuclear spin component is occupied in the presence of a strong magnetic field at low temperature, it was clear that the emitted electron must be ejected with its spin parallel to that of the ^{60}Ni nucleus. Under these conditions, a preferred direction for the departing electrons could be detected, namely opposite to that of the nuclear spin. One can summarize this result by saying that the electron behaves predominantly as a left-handed screw in beta decay. In addition, subsequent studies of nuclear recoil indicated that the antineutrinos turn exclusively as right-handed screws[60] in the same experiment.

Out of these investigations came a new theory of the neutrino proposed independently by Lee and Yang[61] ,Landau[62] and Salam[63] , according to which the neutrino and antineutrino could be distinguished on the basis of their respective left- and right-handedness, or alternatively their negative and postive helicities (defined as I·p/│p│). It was referred to as the two-component theory of the neutrino because it postulated that certain spin-momentum orientations are forbidden to this particle. The idea was not entirely new to theoretical physics as Weyl had proposed something very similar[64] nearly 30 years earlier from strictly mathematical considerations based on STR. Since the parity P was not conserved in this theory, Pauli initially criticized it[65]. The observed asymmetry in the ^{60}Co experiments of Wu *et al*. ultimately caused him to reverse his position[66], however, although he went to some length at the time to express his amazement at this development.

Charge conjugation is also violated according to the two-component neutrino theory, but Yang noted that the product CP should still be conserved[67], *i.e.* by simultaneously inverting the coordinate system and changing all particles into their antiparticles, so that a left-handed v becomes an (observable) right-handed v̄. A key assumption in this theory was that the neutrinos have zero rest masses, a possibility, which as already mentioned, has thus far not been refuted by experiment. The puzzle was later further complicated, however, by new experiments[68], also involving neutrinos, which provided strong evidence that even CP is violated in beta decay.
Since a theorem by Schwinger[69], Luders[70] and Pauli[71] proved that the product of CP and the time reversal operation T should be conserved under the most general of circumstances, this

observation seemed to indicate that neither T itself nor any two of these operators together are conserved in such processes.

The interpretation of the polarization phenomena discussed above has received support from a wide variety of other experimental observations, including studies on free neutron decay[72] and on muons[73.74]. In the first instance, it was found that the proton is predominantly left-handed, which ultimately led to the convention of referring to the particle emitted in neutron decay as the antineutrino $\bar{\nu}$ rather than the neutrino, as originally suggested by Fermi. In this way all antiparticles present in nuclei (and anti-nuclei) are right-handed, while their counterparts in charge conjugation are left-handed. Another important distinction between ν and $\bar{\nu}$ was demonstrated by the fact that antineutrinos do not undergo the capture reaction of eq.(IV.1.v) upon interaction with neutrons[75]. Attempts to observe a double beta decay in which no antineutrinos are emitted[76] have also never been successful. In the present context it should be noted that this result is also consistent with the belief in a definite composition for the neutron needed to achieve particle balance in the five beta decay reactions listed in the preceding section.

Another puzzle connected with neutrinos was discovered in 1964 by Bacahl and Davis[77] as a result of their investigation of nuclear fusion processes on the sun. The fusion reaction is closely related to eq. (iii), with neutrinos being set free in the process. Detailed knowledge of the solar reaction profile enables a relatively precise calculation of the magnitude of the associated neutrino flux reaching the earth, but the above experiments have invariably indicated a large discrepancy in the computed value. Less than half the expected neutrinos are ever observed. In addition there is evidence that the neutrinos involved in the beta decay of muons are not the same[78] as those associated with neutrons and heavy nuclei. One distinguishes then between electron (ν_e) and muon (ν_μ) neutrinos, and there is also theoretical evidence[79] that a third type (or flavor) also exists, namely the tau neutrino (ν_τ).

Yet the greatest puzzle of all in this connection seems to be why such charge less, extremely penetrating particles are involved in almost every high-energy reaction ever observed. Given this situation, the question of whether neutrinos can be created or annihilated seems a secondary issue, but one that nonetheless permeates any relevant theoretical discussion of the

nature of these particles, beginning with the Fermi theory of beta decay put forward over a half-century ago.

C. NUCLEAR STRUCTURE AND THE STRONG INTERACTION

The theory of nuclear structure relies firmly on the assumed elemental nature of the neutron[80]. Any nucleus can be described as containing a definite number of protons and neutrons, thereby providing a means of discussing nuclear reactions with balanced equations along the lines foreseen by Boyle in his general theory of chemical transformations. The fact that the neutron undergoes spontaneous emission complicates this simple picture, however, suggesting an alternative interpretation according to which the real elements are the electron and neutrino, in addition to the proton. The proton itself has been theorized[81] to decay as well, but careful experiments[82] have never found any positive evidence for this expectation. By any reckoning the proton is an extremely stable substance, with a lifetime far longer than that of the neutron.

The question of the nature of the forces which hold nuclei together is still largely open. One refers to them collectively as the strong interaction, a term which above all takes cognizance of the fact that the forces in question have properties which are far different from either the electromagnetic or gravitational type. In the time of Rutherford three nuclear decay modes were well known, involving α-, ß- and γ-rays respectively. In the meantime one can add at least two more decay products to this list, namely neutrons and the small nuclei which result from fission of their heavy counterparts. It is interesting to note that the creation-annihilation hypothesis is only invoked to explain two of these five possibilities, the beta decay discussed in the preceding two sections, as well as the gamma or photon emission process. In the other three cases the decay products have much larger rest masses, and thus are thought to be present in the original nuclei prior to decomposition as well as subsequent to it.

As noted previously, the justification for this distinction in the case of electron (beta) emission is the belief[50,52] that no potential could be sufficiently attractive to outweigh the large kinetic energy such a light particle would have in the confines of the nuclear volume. Yet the fact that the forces binding nuclei together are a matter of some uncertainty themselves lends a degree of tentativeness to this conclusion. On the other hand, if we assume that neither the

electron nor anything else can be annihilated under any circumstances, we are left with no choice but to look for a potential that is capable of binding an electron so strongly. In Chapter II a similar line of reasoning has led to the conclusion that the electron and positron form a much more tightly bound system than anything inferred from a solution of the electrostatic Schrödinger equation, one which corresponds to a binding energy of 1.02 MeV. For comparison purposes, it is interesting to note that nuclei are bound together on the average by roughly 8 MeV/nucleon, a number which is somewhat larger but of the same order of magnitude as that required in the present interpretation of a tightly bound electron-positron system. Moreover, it is over 200 times smaller than the p^+p^- binding energy which must be assumed on the same basis, namely 1.85 GeV.

The possibility may thus exist that in searching for a potential which binds particles with their antiparticles together with such force as to cause a total loss of mass relative to the reactant species, one is approaching the same goal as has long been sought in the context of nuclear interactions. Merely saying that the energies given off in the particle-antiparticle interactions can be computed on the basis of STR and the mass-energy equivalence relation does not after all conform to the standards physicists have adhered to in dealing with other types of elemental processes. In almost every other conceivable situation, a detailed description of the forces involved in causing systems to be either attracted or repelled by one another has become an important goal of subsequent research, and this line of approach has been rewarded with genuine scientific progress on numerous occasions in the past.

With this motivation, it is instructive to review what is known about the strong interaction from experimental studies of nuclear processes and what progress has been made in interpreting these observations theoretically. To begin with, one has a good idea from scattering experiments what the associated potential must look like. One distinguishes three types of fundamental processes, namely pp, nn and pn scattering. Especially when Coulomb repulsion effects are subtracted for proton pairs, it is found that the potential that causes nuclear binding is essentially the same in all three cases. On this basis the nuclear force is assumed to be charge-independent and, by inference, quite distinct from the electromagnetic force. Some evidence of at least a secondary role for electromagnetism is suggested by the existence of nuclear magnetic

and quadrupole moments, not to mention the positive electronic charge of all nuclei, but it is felt that such effects can be dealt with separately. One also might say that the discovery of the neutrino's involvement in nuclear reactions supports the charge-independence assumption in view of this particle's lack of either charge or magnetic moment.

Probably the most influential conclusion based on the above experimental findings was drawn by Heisenberg[83] in 1932. He postulated that the neutron and proton were simply two different states of the same system (nucleon) which would be perfectly degenerate in the (hypothetical) absence of the electromagnetic interaction. Out of this hypothesis ultimately evolved the theory of isospin[84,85], which has had a very great impact in both nuclear physics and the corresponding study of elementary particles.

Some of the first attempts to describe the nuclear force employed a non-relativistic Schrödinger equation[86] and an empirical potential of the type indicated by the scattering data. The most famous of these treatments employs the Yukawa exponential potential[87], $V(r) = -V_o(\alpha/r) e^{-r/\alpha}$, where r is the distance between nuleons (protons or neutrons), V_0 characterizes the depth of the potential and α, its range. The non-relativistic kinetic energy $p^2/2M$ is deemed appropriate for this purpose in view of the relatively large rest masses of the nucleons. By varying the two parameters V_0 and α, it is possible to fit a certain number of experimental results on the basis of the corresponding solutions of the Schrödinger equation, but the Yukawa procedure has never evolved into a truly quantitative theory of nuclear binding. Nonetheless the simplicity of the model on which it is based has led to much valuable insight into the subject. In particular, it emphasizes the extremely short range of the effect through the exponential form of the corresponding potential. For $r \gg \alpha$, the potential is effectively damped to zero, whereas for $r \leq \alpha$ it varies predominantly as r^{-1}. In the latter respect it is similar to the Coulomb potential, but the large magnitude of V_0 gives it a much stronger weight in the short internuclear distance region where the binding between nucleons is at its greatest. True to the creation-annihilation concept, no recognition is given to the possibility that either electrons or antineutrinos are involved directly in the nuclear binding process.

Another approach to the nuclear problem is modeled after quantum electrodynamics, focusing on the idea that the Coulomb force involves the exchange of photons between the interacting charged species. Using arguments based on the Heisenberg uncertainty principle, Yukawa postulated[88] that the particle regulating the: nuclear force must have a correspondingly short range, which in turn keeps it from being directly observable under normal circumstances. By assuming a velocity close to the speed of light, Yukawa was led to predict a rest mass for the new particle which was intermediate between that of an electron and a proton (200 m_{oe} was first proposed). The Coulomb force's range is thereby thought to be infinite, consistent with the null rest mass of the photon. The theory received support from experiment a decade later when the pi mesons (π^+ and π^-, with a rest mass of 273.13 m_{oe} each) were discovered[89] and shown to interact with nuclei. Attempts to construct a quantitative theory analogous to quantum electrodynamics were not generally successful,[90] however. The use of low-order perturbation theory to compute the mesonic interactions was concluded to be inadequate[91], despite the fact that a similar approach in quantum electrodynamics is found to be quite accurate. The anomalous spin magnetic moment of the electron amounts to a deviation of only one part per thousand[92] from the uncorrected value of Dirac theory, and can be predicted to an accuracy of six to seven significant figures with the perturbative inclusion of quantum electrodynamical effects. By contrast, the proton magnetic moment was measured by Stern and coworkers[51,93] to be 2.5 times the theoretical value deduced on the basis of the respective masses of the proton and electron. Nonetheless, the pi meson theory accounts for the known observations at least qualitatively, and the concept of a cloud of such (virtual) particles surrounding the proton has received broad acceptance, especially after it was found to be quite consistent with the results[94] of elastic scattering of high-energy electrons by nucleons.

Another theory which has had a great impact in this field is the nuclear shell model proposed by Goeppert Mayer[95] and Jensen.[96]. This work concentrates on certain regularities in the properties of nuclei as a function of the numbers of their constituent protons and neutrons, particularly the values of the nuclear spin and of the electric quadruple and magnetic dipole moments. It is possible to predict trends in these data by assuming that protons and neutrons are added in shells according to an *Aufbau* principle similar to that used in atomic structure theory to explain observed trends in the periodic table of the elements. The shell model traces its origins

to the independent-particle or Hartree model[97] and also to the Wigner coupling scheme,[98] which was successful in classifying nuclear states with the aid of a spin-independent zero-order Hamiltonian. Each shell can be identified with definite angular momentum quantum numbers l and j, with their familiar degeneracy factors determining degrees of maximum occupancy in accordance with the Pauli principle. Isomorphic pairs of shells are reserved for the individual protons and neutrons, whereby the stability of the proton shells always appears to be somewhat lower, reflecting the effects of their participation in the repulsive Coulomb interaction. On the basis of these considerations it was possible to explain the occurrence of certain "magic numbers" which had been noted empirically for the atomic and mass numbers of nuclei in surveying the aforementioned properties.[80,95,96]

For the present discussion the most interesting aspect of the shell model is the conclusion that forces are at work which are akin to the spin-orbit coupling interaction familiar in the classification of fine-structure effects observed in atomic spectra. Terms of this nature arise from a perturbative reduction of the Dirac equation,[1,99] as shown by Breiti,[100] Pauli[101] and others[102]. The origin of the spin-orbit interaction can be traced to the **v x E** term which appears as an adjunct to the magnetic field[103] upon application of a Lorentz transformation to the classical Maxwell equations.[104] The form of the spin-orbit interaction involved in nuclear binding is quite distinct[105] from the latter, however, because the magnitude of the effect is found to be much larger than one would expect on the basis of the small magnetic moments of nucleons.

The fact that the quantum numbers employed in such models are typical for a central potential has sometimes been remarked[106] to be inconsistent (or at least not obviously consistent) with the picture of the nucleus as a collection of protons and neutrons, *i.e.* without a fixed center comparable to that known for atoms. The success with which the experimental data can be ordered on this basis belies such criticism, but the *ad hoc* nature of its central potential assumption combined with its inability to obtain truly quantitative agreement with measured results have tended to prevent the nuclear shell model from being a complete triumph.[107] Nonetheless, the ability of such methods to account in an extremely detailed manner for the observed regularities in the properties of complex nuclei is a fact which must be reckoned with in trying to construct a more quantitative theory of nuclear structure in the future.

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It is impossible to do justice to the progress made in theoretical nuclear physics with such a brief survey, but at least one more series of developments needs to be considered in the present context. The liquid drop model proposed by Bohr[108] in 1936 points out similarities between nuclear matter and the liquid state. This model is based first and foremost on experimental measurements of radii and binding energies of a large series of nuclei, and as such is reminiscent of the theory of the chemical bond introduced into molecular physics and chemistry by Pauling.[109] Probably the most important result of an experimental nature is the finding that the magnitudes of nuclear radii are approximately proportional to the third root of the atomic mass number,[110] or alternatively, that the nuclear volume increases proportionately to the number of constituent nucleons. This is a very simple result which gives the impression that, even more than in molecular systems, for which a similar model is already very successful, the components of nuclei are not at all compressible. If one knows the volume of one nucleon (or better, the volume of two nucleons that are tightly bound together), it is possible to extrapolate this result to obtain the size of the entire nucleus. This close-packing condition is another important characteristic that must be derivable from the nuclear potential. To this can be added that the interactions between nucleons in the same shell are found to be much stronger than between those in different shells.[111]

What do the theories of nuclear structure tell us about the forces involved in positronium decay? If the pervasive view that nuclear forces occur primarily between elemental protons and neutrons is correct, the answer must be that there is little or no relation. On the other hand, if the neutron is looked upon as a compound of its decay elements, the electron, proton and antineutrino, a more significant relationship can be anticipated. For one thing, it then becomes essential to account for interactions between electrons and protons inside the nucleus, in which case it is not difficult to imagine that there might be similarities between these forces and those which might conceivably cause the electron and positron to be bound together much more tightly than in positronium. *Moreover, once this path is taken, it is only consistent to assume that the antineutrino also interacts strongly with either the electron or proton or both.*

The above possibilities will be explored in subsequent chapters, but for now it is worthwhile to consider how such a neutron hypothesis would affect the simple bookkeeping procedures used to describe nuclei in terms of the number of constituent protons (Z) and neutrons (N). In the first place, the number N becomes identical with the number of electrons in the nucleus, as well as the number of anti-neutrinos. The number of protons is no longer Z, but Z + N, while the total number of constituent particles is not Z + N, but Z + 3N. The latter difference is 2N and thus is always even. This fact means there is no alteration in nuclear statistics as a result of such an assumption, a point used by Pauli[37] in postulating the existence of the neutrino in the first place. Otherwise, particle balance in nuclear equations is ensured in this approach (even in cases for which beta-decay occurs in one form or another) if one inserts the appropriate particle-antiparticle binary systems as needed. When photons are involved, this means adding massless e^+e^- species as well. Conservation of energy can be handled in the usual way, in accordance with STR.

The resulting elemental balance in the description of such processes would be perfectly in line with the atomistic theories originated thousands of years before, as well as the more precise formulations proposed by Boyle and Dalton much later. What is clearly missing in the discussion to this point is a quantitative specification of the possible forces involved, one that would allow an accurate prediction of the differences in the masses of products and reactants in such processes. In its simplest form, this would mean the *ab initio* computation of the binding energy of the deuteron relative to its constituent proton and neutron separated to infinity, or even more fundamentally, with respect to two protons, an electron and an antineutrino at rest in their respective elemental states.

D. METASTABLE PARTICLES

The hypothesis that the neutron is a triatomic compound composed of each of its decay elements has far-reaching consequences beyond the realm of nuclear physics. There are many other metastable systems which are listed among the elementary particles, the smallest of which are the two muons and the three pions. They differ from the neutron mainly in that their lifetimes are much shorter ($10^{-10} - 10^{-16}$s) and that their decomposition energies are far greater (over 100 MeV in the case of the muons). In addition

their decay products do not include a proton, although electrons and neutrinos are emitted. Beyond this there are a large number of other metastable particles known, referred to as mesons and baryons on the basis of their rest masses.

In 1964 Gell-Mann[112] and Zweig[113] suggested that all of them could be interpreted as being composed of hypothetical fermions called quarks (or aces[113]) and their antiparticles. Probably the most novel aspect regarding this suggestion is the assumption that the new particles should possess non-integral electric charges, either $\pm\frac{1}{3}$ e or $\pm\frac{2}{3}$ e. In the meantime the number of quarks and antiquarks has grown to twelve[114] and the idea has achieved wide acceptance. Fermi and Yang[25] and Sakata[115] tried to identify a common denominator of elements in terms of the proton, neutron and lambda from which to construct all other known particles had only been partially successful, primarily because certain combinations which appear reasonable in this theory have never been observed. The quark concept is clearly consistent with the view that all matter consists of elements, but there is still some question as to whether particles of non-integral electric charge actually exist. One reads in the literature, to be sure, that individual quarks have been discovered experimentally, but the evidence is always indirect, referring to a successful prediction derived from the theoretical model.

The idea that all observed meta-stable particles might be composed of their decay products, either as elements or as other combinations thereof, is not necessarily in conflict with the quark model, although it is also not obviously consistent with it either. Just because a proton might have a quark composition, in other words, in no way precludes the possibility that a quantitative theory which employs protons as elements might not exist. The crucial test is whether the properties of the various meta-stable particles can be successfully computed with the help of any such theory. In the present context this would mean computing the rest masses of the muons and pions, for example, just as for the neutron, deuteron and other heavier nuclei, in a similar manner as one calculates binding energies and ionization potentials of atoms and molecules with the help of quantum electrodynamics.

There is a new aspect introduced by the study of the other metastable particles, however, namely that they exhibit multiple decay channels. Whereas the neutron is always observed to produce a proton, electron and antineutrino upon decomposition, several sets of decay products are known for most other meta-stable systems. *Does this not speak against the concept of a unique composition for them in terms of certain elements?* Not necessarily, because if massless particle-antiparticle binaries exist, other possibilities must be taken into account. One only has to look at a typical beta decay process discussed earlier in this chapter to see how the addition of e^+e^-, p^+p^- or $\nu\bar{\nu}$ species can effect a particle balance in reactions which are conventionally interpreted in terms of the creation-annihilation hypothesis.

The situation can be compared to that faced by the early chemists several centuries ago when a new substance was to be analyzed. A major distinction between the two cases exists, however, namely that when very large decay energies are involved, the masses of the participating systems cannot be used unambiguously to confirm a given structure. In other words, the concept of molecular weight is much less useful in elementary particle physics than it is in chemical investigations. All one knows with certainty is that if the rest mass of the combined system is less than that of its components it is a bound system, otherwise it must be meta-stable. The experimental fact that *all* the particles which decay into protons, electrons or neutrinos *have rest masses which are larger than the total for their respective fragments* again speaks clearly for the analogy with molecular excimers mentioned earlier. It remains to be seen whether a system of forces can be found which is capable of temporarily binding electrons and neutrinos together in muons and pions and the like, even though the total energy (mass) in the relevant states is much higher than that for the corresponding decay particles at rest.

V. VARIATIONAL THEORY FOR THE DECAY OF POSITRONIUM

In the preceding chapters it has been suggested that there is merit in considering the decay of positronium as an interaction in which an electron and positron are so strongly attracted to one another that the resulting binding energy is exactly equal to the sum of their rest masses, *i.e.* $2m_{oe}c^2$ or 1.02 MeV. A survey of the key experiments in modern physics has shown how such a development would fit into the theory of elementary particles and the energetics of their reactions. In the present chapter we will focus on the goal of finding a suitable potential which is capable of producing such a relatively large binding energy, while at the same time giving consideration to the possibility that the solution to this problem may have relevance for other types of interactions, particularly those involved in the study of nuclear physics.

A. A SHORT-RANGE POTENTIAL

The natural point at which to begin this investigation is with the nature of the potential which might be capable of bringing about such a strong attraction between an electron and a positron, although careful consideration must later be given to the manner in which the kinetic energy is treated as well. It is clear from the outset that this must be a distinctly relativistic problem, because the rest mass of the combined e^+e^- system is assumed to be much lower than the sum of those of the separated particles. Both the Schrödinger non-relativistic[86] and Dirac relativistic[99] treatments of positronium tell us that the lowest possible state for this system is analogous to the 1s state of the hydrogen atom. In this case the primary

interaction is Coulombic, exclusively so in the non-relativistic treatment and almost exclusively in the relativistic.

A two-component reduction of the Dirac equation leads to the characterization of a number of perturbative terms which are basically magnetic in nature. The most commonly employed such approximation is that of Breit-Pauli theory[100-102], including the Breit interaction. The perturbations are on the order of $\alpha^2/2 \cong 10^{-5}$ hartree ($\alpha = e^2/\hbar c = 137.036^{-1}$, the fine structure constant), and include the spin-orbit (same- and other-orbit), spin-spin, orbit-orbit and Darwin terms, as well as the mass-velocity correction to the non-relativistic kinetic energy[102]. These terms increase as Z^3 or Z^4 (spin-same-orbit) for atoms with nuclear charge Z. For positronium as well as the hydrogen atom they remain quite small, however, and their effects are observed only as fine structure in spectroscopic. studies. The potential terms all vary as r^{-3} and thus have relatively short ranges compared to the Coulomb interaction. This point bears further consideration, however, since binding energies of 1.0 MeV and higher are otherwise known only for nuclei, in which case there is clear evidence that short-range forces are involved to a high degree.

We can represent the presumed interaction schematically by plotting the total energy as a function of the average distance r between an electron and positron (Fig. 5). The 1s state of positronium can be thought of as corresponding to a minimum in total energy occurring at r = 2.0 bohr = 1.016 Å, *i.e.* roughly double the corresponding value for the H atom by virtue of the smaller reduced mass of the e^+e^- system. Toward larger separations the energy gradually increases to zero, *i.e.* the energy of the separated particles. The attractive Coulomb potential varies as r^{-1}, while the kinetic energy varies as $p^2 \approx r^{-2}$, from which it follows that the total energy itself at first *decreases* as the particles approach one another from a large distance. At the location of the energy minimum the shorter range of the kinetic energy term becomes the dominant factor, which explains why the total energy thereafter increases rapidly toward still shorter distances. It can be seen that these arguments are very close to those used by Bohr[5] in arriving at his theory of hydrogenic atoms in 1912.

The binding energy at the latter e^+e^- minimum is only 6.8 eV, which is quite small compared to the 1.02 MeV given off when positronium decays from the corresponding (1s) state. The possibility we wish to explore in this work is *whether* a *second energy minimum does not occur at* a *much smaller electron-positron separation.* It can be imagined, for example, that at some point the total energy stops increasing toward shorter distances because an attractive short-range potential term begins to overcome the effects of the increasing kinetic energy in this region. Such a potential term would have to vary at a higher inverse power of r than either of the other two terms in the non-relativistic electrostatic Hamiltonian, and would have to be relatively unimportant in the region of the first hydrogenic energy minimum. At the same time, it is evident that some effect with an even shorter range eventually must take over and cause the energy to increase (Fig. 5) once more toward even smaller distances after the assumed 1.02 MeV absolute minimum

value is attained. It is also clear that such a second energy minimum must be *totally absent in the corresponding hydrogen atom treatment.* Finally, it is only consistent to assume that an analogous double-minimum curve exists for the proton-antiproton system, but with a binding energy which is 1836 times larger, *i.e.* 1.85 GeV for its short-range minimum. At least one knows that this much energy is given off when the proton and antiproton interact, whereas no comparable loss of energy is observed for the combination of a proton and an electron.
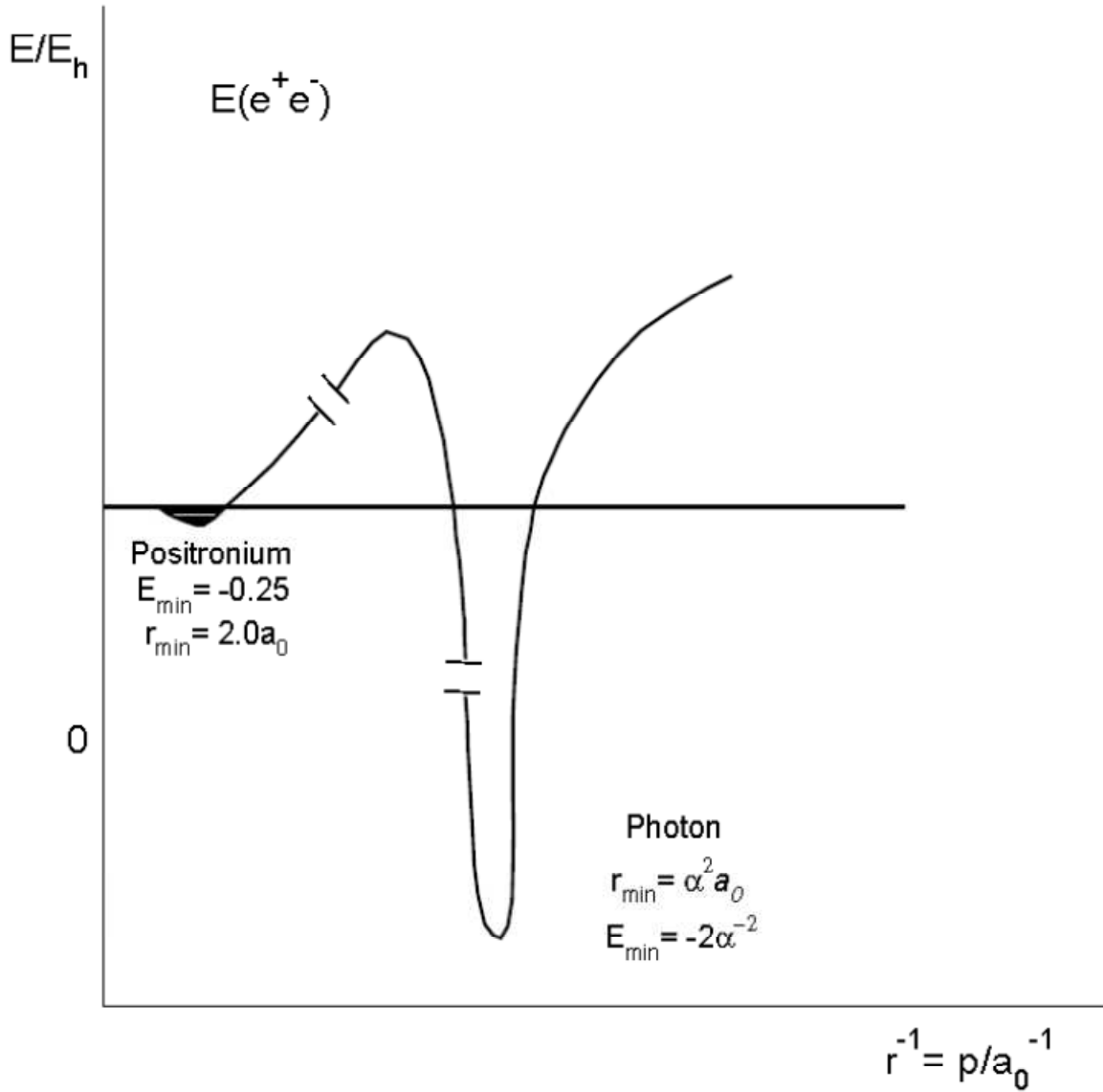
FIG. 5. Schematic diagram showing the variation of the internal energy of the e^+e^- system as a function of the reciprocal of the distance between the two constituent particles. At large separations the Coulomb attractive interaction dominates because of its long-range (r^{-1}) character. A minimum of energy of only -0.25 hartree is eventually reached, corresponding to the familiar

hydrogenic ls state of positronium, after which the total energy begins to rise because the shorter-range kinetic energy term begins to dominate. In the present model this trend is eventually reversed again at a much smaller e^+e^- separation, at which point attractive forces of even shorter range ($\approx r^{-3}$) begin to change more rapidly than the kinetic energy. Finally a second potential minimum, much deeper than the first, is reached which corresponds to a binding energy exactly equal to the sum of the rest energies of the electron and positron. At still smaller interparticle distances the total energy rises again, reflecting the effect of some momentum-dependent damping of the short-range potential in this region. A form for the potential is sought for which only the Coulomb energy minimum survives when the positron is replaced by a proton.

Concentrating on the comparison of e^+e^- with p^+e^-, the obvious question is how can the differences in the properties of the proton and positron lead to such an enormous distinction in their respective attractions to the electron. The traditional view embodied in the Schrödinger[86] and Dirac[99] equations for one-electron atoms holds that the large difference in mass of the two positively charged particles only plays a minor role in this connection, simply affecting the reduced mass of the electron. The magnetic moments (which have the charge-to-mass ratios as a factor) of e^+ and p^+ differ by a far greater amount because of the difference in rest masses, but this distinction is found to be of only minor importance in the Dirac equation treatment, in which effects such as the spin-orbit and spin-spin interactions depending on this quantity are accounted for explicitly.

Yet one knows from the outset that if there is indeed a much lower-lying state of the e^+e^- system than the familiar 1s species, it cannot be found among the solutions of the hydrogen atom Dirac or Schrödinger equations. To progress further in this regard it is necessary to do something differently. Especially since the effect that might cause such a novel tight-binding e^+e^- state seems almost certainly short-range in nature (Fig. 5), there is reason to give closer consideration to the above magnetic-type interactions. As noted previously, there are numerous Breit-Pauli terms which fall in this category, varying as the inverse cube of the distance between interacting particles. They are all of order $\alpha^2/2 \cong 10^{-5}$ hartree for typical atomic electron-nucleus separations, so this characteristic fulfills another requirement from Fig. 5, namely that such a short-range effect be relatively insignificant at these distances.

Most importantly, however, all these Breit-Pauli terms[102] depend on the product of the magnetic moments (or charge-to-mass ratios) of the interacting particles. For the spin-other-orbit, spin-spin and orbit-orbit terms each of these quantities appears once in the corresponding product. Thus these terms are weighted by a factor of 1836 (the ratio of the rest masses of proton and positron) larger for e^+e^- than for p^+e^-. For the spin-same-orbit and Darwin terms the distinction is less important because in these cases the square of the mass of one of the constituent particles is involved rather than the product of both. As a result, there is an extra factor of two for these interactions for e^+e^- than for the hydrogen atom, by virtue of the fact that the square of the charge-to-mass ratio of the proton is negligible compared to that of the positron. At atomic distances these distinctions are still relatively unimportant because in this range the

Coulomb interaction dominates, but it is not difficult to imagine the situation could be far different at shorter range.

Before considering this possibility further, however, it is well to note that the Breit-Pauli Hamiltonian terms as such also have some undesirable properties which make them unsuitable for a variational calculation, that is, one in which the charge distributions of the particles involved are allowed to assume optimal forms so as to minimize the total energy. Since these terms vary as r^{-3}, there is nothing to keep the total energy from decreasing beyond any limit. They do not therefore provide a possibility of a second energy minimum of the type indicated in Fig. 5, rather only the attractive branch to the long-distance side of it. This fact has long deterred giving serious consideration to the Breit-Pauli terms as having any truly dominant role to play in quantum mechanical calculations. There are other higher-order terms in a power-series expansion of the Dirac equation which need to be considered to properly understand their role in determining atomic fine structure. Specifically, the next terms in such expansions are of the order of $\alpha^4 r^{-4}$, and these higher-order effects prevent variational collapse in Dirac-equation solutions that would otherwise occur if only the $\alpha^2 r^{-3}$ terms were included.

A similar situation exists for the Lorentz force in classical electromagnetic theory. There one has a term of the form $|\mathbf{p} + e\,\mathbf{A}/c|^2$, where $\mathbf{A}$ is the vector potential. The cross term involving $\mathbf{p}\,\mathbf{A}$ is typically[116] also of the form $\alpha^2\,r^{-3}$ ($c = \alpha^{-1}$ in atomic units) for the interaction of a charged particle with an electromagnetic field, but it is damped at short range by the $|\mathbf{A}|^2$ term, which varies as $\alpha^4 r^{-4}$. An interesting possibility is nonetheless opened up by the fact that such repulsive terms *are of even shorter range and higher order in* α^2 than their Breit-Pauli counterparts. Including such terms might not keep the total energy in Fig. 5 from turning downward at short distances because of the attractive $\alpha^2 r^{-3}$ interactions, but they would insure that this trend not continue indefinitely toward still smaller interparticle separations, with the result that the proposed non-hydrogenic second minimum of energy could be formed. If one simply assumes a potential which is the difference of these two short-range terms, $-\alpha^2 r^{-3} + \alpha^4 r^{-4}$, it is easily shown that it possesses a minimum near $r \cong \alpha^2$. Such a distance corresponds to roughly 10^{-5} bohr, which is a typical separation for bound nucleons ($r = \alpha^2/2$ is normally given for the range of the nuclear force, for example). The possible connection between an e^+e^- tight-binding state and the nuclear force is thus reinforced by these considerations as well.

There are basically three arguments for pursuing this line of reasoning further. First, the theory of quantum electrodynamics has a very precisely defined range of applicability. Despite its ability to make extremely accurate predictions for interactions involving electrons and photons in an atomic environment, it is generally accepted that the theory in its established form is not capable[117] of describing high-energy interactions such as those involved in nuclear bonding. Any indication as to how *the framework of*

quantum electrodynamics could be adapted so as to become relevant to the description of short-range forces therefore merits serious consideration. Secondly, the analysis of the positronium decay process has provided a basis for associating the assumed e^+e^- tight-binding state with the photon itself. Given the prominent role of the photon in quantum electrodynamics, it seems likely that its internal structure would also have an important relationship to the electromagnetic force. Finally, examination of the multiplet structures and other properties exhibited by nuclei has already led to the conclusion[95,96] in the nuclear-shell model that spin-orbit or related terms[118] are almost certainly involved in this type of high-energy interaction . Taken together these observations suggest that a solution to the proposed problem may lie in an adaptation of the Dirac equation which does not detract from the reliability of the original theory's predictions for quantum-electrodynamics phenomena, but which at the same time enables an accurate treatment of interactions of much shorter range and higher binding energy. This eventuality would amount to an extension of the Bohr correspondence principle[119], which proved so effective in making the transition from classical to quantum mechanics at the beginning of this century.

B. KINETIC ENERGY CONSIDERATIONS

As remarked above, it is not satisfactory to use a non-relativistic form for the kinetic energy of the positron and electron if a large binding energy is assumed. In this respect the problem is significantly different than in the conventional treatment of nuclear binding, because there the kinetic energies of the nucleons are still relatively small compared to the energy equivalent of their rest masses. As Einstein showed[4] on the basis of the special relativity theory, the non-relativistic $p^2/2m_o$ term is actually just an element in the power series of the square-root quantity $(p^2c^2 + m_o^2c^4)^{1/2} - m_oc^2$. The Breit-Pauli approximation includes[102] a term of order p^4 to account for relativistic kinetic energy contributions, but just as for the corresponding potential terms, it is known that this correction leads to variational collapse when the electronic charge distribution is allowed to vary freely so as to minimize the energy. It is possible to circumvent this difficulty,[120] however, by employing the Einstein square-root expression directly in the Hamiltonian instead of relying on a truncated power-series expansion for it. The inconvenience of employing a square-root operator can be dealt with for general atomic and molecular systems by means of a standard matrix procedure. Each particle has its own kinetic energy and thus the square-root terms are treated as one-electron operators, exactly as their non-relativistic $p_i^2/2m_i$ counterparts in conventional quantum mechanical treatments.

This procedure brings with it another difficulty, however, namely how to separate the total kinetic energy into its internal and translational (center-of-mass) components. In the non-relativistic case it is

well known that the $p_1^2/2m_1 + p_2^2/2m_2$ term can be replaced exactly[121] by $p^2/2\mu + P^2/2M$ by a linear coordinate transformation (p is the internal and P, the center-of-mass, momentum respectively, μ is the reduced mass and $M = m_1 + m_2$ is the sum of the individual particle masses). The new coordinates are defined as $x = x_1 - x_2$ and $X = (m_1 x_1 + m_2 x_2)/M$, where x_1 and x_2 are Cartesian coordinates of the two particles, with analogous expressions for the y and z directions. Since the Coulomb potential only depends on the internal coordinates, it is thus possible to effect a separation of coordinates which leads to the familiar situation that the solutions of the corresponding Schrödinger equation can always be formed as simple products of the type Ψ (r) χ (R), *i.e.* as a product of a function of the internal coordinates with one involving only those of the center of mass.

This procedure can be generalized for any number of particles, but there is an important assumption to be noted. The derivation generally used[122] comes from classical mechanics and relies on the fact that cross-terms involving **p P** cancel out as a result of the coordinate transformation. The cancellation is perfect for the non-relativistic kinetic energy, but *use of the relativistic one-particle form,* in which p_1^2 and p_2^2 appear in separate square-root expressions, *does not lend itself to the same simplification.* If one of the particle's masses is much larger than the other, it is not difficult to find approximations from which the desired coordinate separation effectively results even in the relativistic case. Since the nuclear masses are so much larger than the electron's, it is therefore easy to justify this approach for atomic calculations, either for the Dirac equation itself or in the Breit-Pauli approximation. The difficulty is not so easily circumvented when both masses are equal, however, as is the case for the present e$^+$ e$^-$ interaction. It might be argued that it is intuitively obvious that the center-of-mass motion can always be separated out, but for quantitative work one would like to have a more solid basis for making this simplifying assumption. In general this question seems to have received relatively little attention in the literature, and it will be given more careful consideration later in this work in Chapter X.

The most straightforward method which presents itself under the circumstances is to simply work with the original Cartesian coordinates of each particle. The quantum-electrodynamics treatment of the positron uses a similar approach[16,123] in evaluating higher-order effects, including transition probabilities for the decay into photons out of various positronium electronic states, except that it imposes an additional condition of $\mathbf{P} = \Sigma \mathbf{p_i} = 0$. In such a two-particle application this means that $\mathbf{p_1} = -\mathbf{p_2} = \mathbf{p}$. No comparable transformation is employed for the spin coordinates of both particles in this approach, however. Radiative corrections to the Dirac-equation results can also be computed by employing the same condition for the center-of-mass momentum.[16,123] When more than two particles are involved complications arise in attempting to generalize this procedure, however, particularly in the relativistic treatment of the kinetic energy. In the transformed coordinate system, one of the particles is effectively singled out as a reference for the internal coordinates, so that $\mathbf{x_i'} = \mathbf{x_i} - \mathbf{x_l}$ is now employed instead of $\mathbf{x_i}$,

for example (with $\mathbf{x_l}$ itself being retained in the new basis). As a result the expression for the conjugate momentum of x_l is $\mathbf{p_1} = - \sum_{i \neq 1}' p_i'$, *i.e.* a sum of internal momenta rather than a single such quantity as in the two-particle case.

In order to develop a computational scheme which can be conveniently applied to systems containing more than two particles it therefore seemed advisable to avoid making any type of transformation to internal coordinates, and hence to simply employ the original Cartesian coordinates of each constituent particle directly in constructing the corresponding quantum mechanical Hamiltonian operator governing the interactions of a given system. Use of such a coordinate system at all stages of the theoretical treatment carries with it the complicating feature that internal and translational characteristics are mixed together in the resulting wavefunctions, but in view of the precautionary remarks given above and the exploratory nature of the proposed calculations, this disadvantage seemed to be of acceptable proportions. In this connection it is worth recalling that $\Sigma\mathbf{p_i}$ commutes with each $\mathbf{p_i}$ and $\mathbf{r_{ij}}$ (particle separation) quantity, so that the translational energy operator itself must have a common (complete) set of eigenfunctions[124] with any Hamiltonian containing exclusively these kinds of variables. Hence any exact solution of a corresponding Schrödinger equation must always be characterized by a definite value of the translational energy. This observation underscores another assumption in the usual separation-of-variables argument for internal and center-of-mass coordinates, however. The Breit-Pauli terms mentioned in the last section contain momentum factors as well as internal distances. Consequently, even when the non-relativistic kinetic energy is employed, the desired separation is not complete for a Hamiltonian containing these types of interactions. Again this presents no real problem for calculations of one-electron atoms, in which the masses of the constituent particles differ greatly, but for a system consisting of only e^+ and e^-, there is need for more careful consideration.

Finally, it should be mentioned that there is a covariant two-electron equation[125] which does employ a separation of internal and translational coordinates just as for the Dirac equation. It is characterized by two separate time coordinates, whereas in what has been described above it is always assumed that there is only one such variable, and that it can be separated from its spatial counterparts in the usual way by virtue of the time-independent nature of the corresponding Hamiltonian operator. Since the stated *modus operandi* in the present study is to depart from the purely quantum-electrodynamics treatment of the e^+e^- system, and especially since no other heavy system is present, it is preferable to work with a relatively simple Schrödinger-equation formulation of this problem which does not make any assumptions regarding the way in which the internal motion is separated from that of the center of mass.

C. SUGGESTED DAMPED FORM FOR THE BREIT-PAULI TERMS

A prerequisite for constructing a Schrödinger equation to investigate high-energy processes is the use of a potential which is suitably bounded. The assumed tight-binding e^+e^- state most likely is the result of a short-range interaction which is strongly attractive over a given region of inter-particle separation but even more strongly repulsive at still smaller distances (see Sect. V. A). The Breit-Pauli approximation employs attractive terms fitting this description but lacks corresponding shorter-range effects which would produce the desired second minimum in the e^+e^- total energy curve sketched in Fig. 5. We have seen how the Breit-Pauli kinetic energy term (including the mass-velocity correction) can be replaced by the Einstein free-particle square-root expression to avoid variational collapse without giving up the advantages of having a reliable approximation to the Dirac equation results for hydrogenic atoms. It remains to find a similar means of dealing with the Breit-Pauli potential terms.

If we look upon the spin-orbit and related interactions as terms in a power series it seems reasonable to look for a closed expression which approaches this result in the low-energy relativistic regime encountered in calculations of atoms with moderately heavy nuclei, something analogous to the Einstein relativistic kinetic energy, in other words. At least one knows from the form of the Lorentz electromagnetic force that the next higher-order terms after those of $\alpha^2 r^{-3}$ spin-orbit type vary as $\alpha^4 r^{-4}$. Simply adding such terms to the Hamiltonian has several disadvantages, however. It falls short of the goal of replacing the Breit-Pauli interactions with closed expressions which themselves correspond to infinite-order power series. In addition, it is difficult to carry out computations with an operator varying as r^{-4} because not all integrals which would be required in a variational treatment are finite, in particular not those involving only s-type basis functions.

The form of the desired potential (Fig. 5) is reminiscent of that observed in nuclear scattering, as mentioned earlier, and this suggests the following alternative. The r^{-3} and r^{-4} terms already discussed can be grouped together as $\alpha^2 r^{-3} (1 - \alpha^2 r^{-1} + ...)$. The terms in parentheses are the beginning of a power series expansion of the exponential function $\exp(-\alpha^2 r^{-1})$, which in turn is quite similar to that appearing in the Yukawa potential[87], thus making explicit the connection with the nuclear force description. By multiplying the Breit-Pauli α^2 terms with such an exponential function, we have a potential which is capable of producing the second minimum for e^+e^- in Fig. 5 while at the same time retaining the correct behavior needed at relatively large inter-particle separations to properly describe the conventional positronium (hydogenic atom) states. Since the damping effects produced by the exponential factor are relativistic in nature, it seems somewhat more likely that the corresponding argument is a function of the momentum of the particles rather than the distance between them, *i.e.* of the form $\exp(1 - \alpha^2 p)$, with $r^{-1} \sim p = |p|$. This choice has computational advantages as well, because it means working with individual

quantum mechanical operators which depend on the coordinates of a single particle rather than two. From the Lorentz classical Hamiltonian we can also anticipate that a given particle's momentum p is multiplied by its charge-to-mass ratio q/m. Finally, to obtain the desired binding energy for a given particle-anti-particle system it is convenient to introduce a free parameter A as an additional factor in the exponential argument.

The negative of the resulting potential is plotted in Fig. 6 (atomic units are employed throughout) as a function of the reciprocal inter-particle distance. For this purpose we use the approximate representation $V(r) = -\alpha^2 r^{-3} \exp(-\alpha^2 r^{-1})$. For high particle velocities it can be assumed that the kinetic energy varies as pc in the range of interest, which can therefore be represented in an analogous manner as $r^{-1} \alpha^{-1}$ in atomic units, *i.e.* as a straight line. This diagram is useful in analyzing how binding can be achieved with such an exponentially damped potential over a very narrow range, consistent with the total energy curve shown in Fig. 5. For small momenta typical of electrons in atoms, the kinetic energy far outweighs the short-range potential because of the factor of α^2 in the latter expression. Coulomb effects are omitted from consideration for the time being.
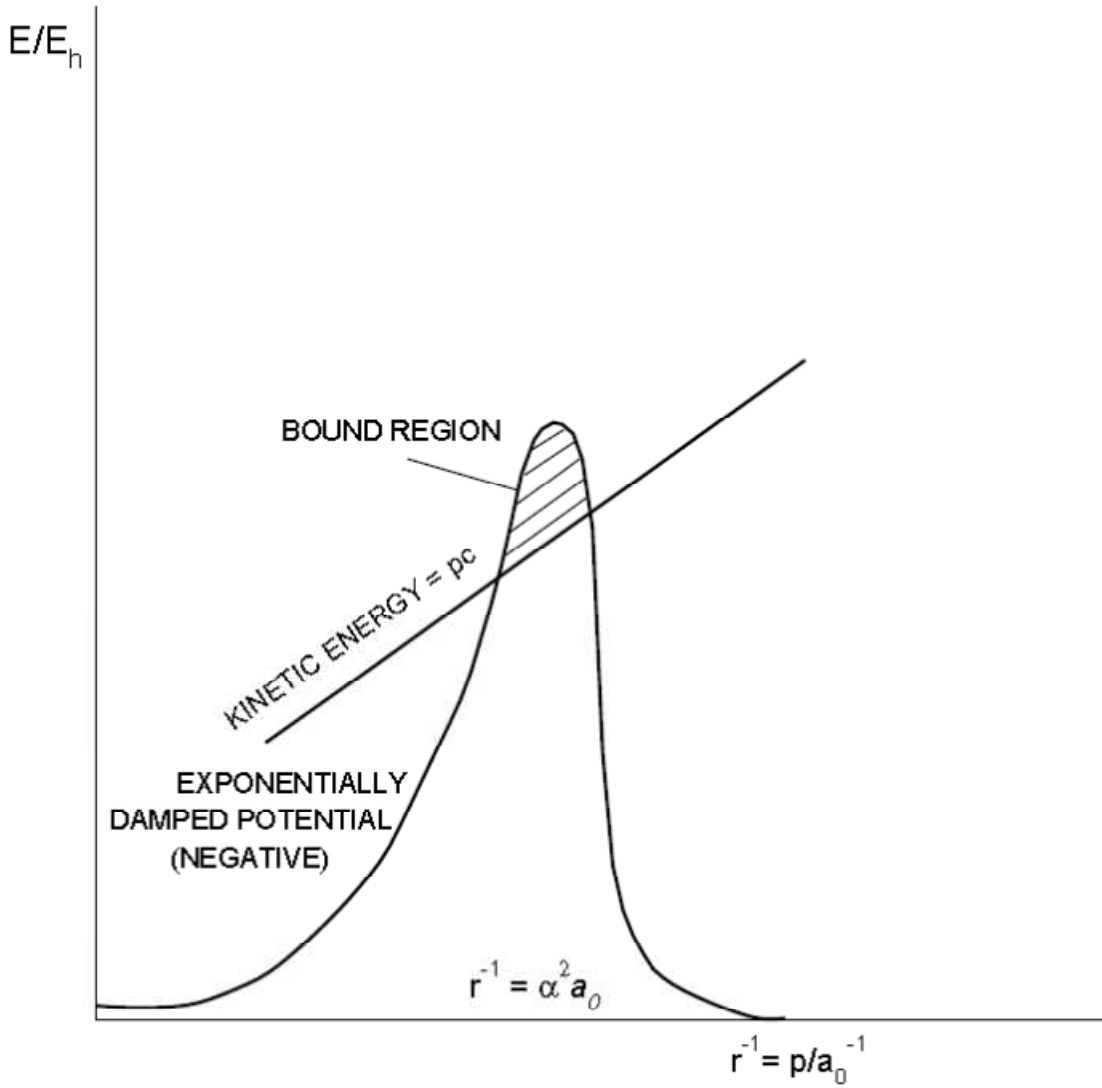
FIG. 6. Schematic diagram exploring the nature of a short-range potential required to produce the type of e^+e^- total energy curve depicted in Fig. 5. At relatively small inter-particle distances ($r \approx \alpha^2$) the relativistic kinetic energy varies nearly linearly with momentum $p \approx r^{-1}$. In order to obtain strong binding within a very narrow range of the e^+ - e^- distance, the negative of the attractive potential term must reach a maximum shortly after it crosses the kinetic energy line from the long-distance side of the diagram, and then drop off again very sharply. Such an extreme cancellation effect requires a potential which fulfills at least three conditions: a) a small coupling constant (order α^2), b) a shorter range ($\sim r^{-3}$) than the kinetic energy and c) a momentum-dependent damping which is exponential in nature.

The absolute value of the potential increases as the cube of the momentum (or inverse distance) while the kinetic energy changes in a nearly linear manner, so it can be imagined that the two quantities eventually

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become equal at some point and binding becomes possible. The exponential damping becomes noticeable in the same region, however, and thus the above term does not increase as quickly as before and finally reaches a maximum. At the same time, the kinetic energy continues to increase linearly with p and eventually a second crossing with the negative of the potential occurs in Fig. 6. The area in which the negative of the potential exceeds the kinetic energy corresponds to a very small range of r, but the amount of binding with which it is associated can still be quite large. For example, at $r = \alpha^2$ the undamped Breit-Pauli potential is of the order α^{-4} hartree, compared to the kinetic energy's order of α^{-3} hartree. Since the assumed binding energy for the e^+e^- system is 1.02 MeV or $2\alpha^{-2}$ hartree, it is clear that an enormous cancellation must occur because of the damping of the potential to obtain physically acceptable results.

This state of affairs is probably the strongest argument for employing an exponential damping to produce such a large degree of binding over a narrow range of interparticle separation. The fact that the Coulomb energy is also of order α^{-2} hartree for $r = \alpha^2$ suggests that it is not possible to ignore this effect either, however, despite its relatively long-range character. Nonetheless, the predominant feature in the tight-binding scenario given above is clearly the delicate cancellation at small interparticle separations between the exponentially damped Breit-Pauli terms and the relativistic kinetic energy.

D. SCALING PROPERTIES OF THE BREIT-PAULI HAMILTONIAN

One of the key requirements for the Schrödinger equation under discussion is that it leads to maximum binding energies for particle-antiparticle systems of $2Mc^2$, consistent with the Einstein mass-energy equivalence relation. One postulates that a Hamiltonian exists which has the required energy as its *minimal* eigenvalue instead of assuming that annihilation occurs and the total mass of the particles simply appears as the equivalent amount of energy. It is well known that the Schrödinger and Dirac equations for purely electrostatic potentials both have the property that their binding energies are proportional to the reduced mass of the electron in hydrogenic atoms, and this result is easily generalized for systems containing other charged particles such as muons or antiprotons. More interesting in the present context is the fact that the proportionality between energy and mass also holds when the various Breit-Pauli relativistic corrections are added to the Hamiltonian.

To show this let us assume that a solution to the Schrödinger equation is known for a particle-antiparticle pair with charge $\pm q$ and rest mass m_0. Furthermore, its lowest energy eigenvalue is taken to be $-2m_0c^2 \equiv -2m_0\alpha^{-2}$ (in atomic units), corresponding to the eigenfunction

$\Psi(r)$. The Hamiltonian itself consists of a series of kinetic and potential energy operators of the type discussed earlier, including exponential damping factors $F(p, q, m_o)$:

$$H(p, r, q, m_o) = (p^2 \alpha^{-2} + m_o^2 \alpha^{-4})^{1/2} - m_o \alpha^{-2} \qquad \text{V.1}$$
$$-q^2 r^{-1} - q^2 m_o^{-2} \alpha^2 r^{-3} F(p, q, m_o)$$

If the coordinates are scaled so that

$$p' = M_o m_o^{-1} p \text{ and } r' = M_o^{-1} m_o r \qquad \text{V.2}$$

the original Hamiltonian becomes:

$$H(p, r, q, m_o) = M_o^{-1} m_o \{(p'^2 \alpha^{-2} + M_o^2 \alpha^{-4})^{1/2} - M_o \alpha^{-2}$$
$$-q^2 r'^{-1} - q^2 M_o^{-2} \alpha^2 r'^{-3} F(p', q, M_o)\} \qquad \text{V.3}$$
$$= M_o^{-1} m_o H(p', r', q, M_o),$$

provided $F(p, q, m_o) = F(p', q, M_o)$. The corresponding Schrödinger equation in the scaled coordinate system thus becomes:

$$H(p', r', q, M_o) \Psi(r) = -2 M_o \alpha^{-2} \Psi(r) \qquad \text{V.4}$$

i.e. by multiplying both sides of the Schrödinger equation for the original Hamiltonian by $M_o m_o^{-1}$. As a result it is seen that $\Psi(r)$, or the function $\Psi'(r')$ obtained by making the corresponding coordinate substitution for it, is an eigenfunction of the analogous Hamiltonian for a particle-antiparticle system of the same charge q as before, but with rest mass M_o instead of m_o. Its energy eigenvalue is $-2M_o \alpha^{-2}$, exactly as required by the mass-energy equivalence relation.

Moreover, this result is quite general, since it is easily seen that the above scaling procedure has the effect of producing an entire spectrum of Schrödinger equation eigenvalues which differ by a factor of $M_o m_o^{-1}$ from those obtained for the original particle-antiparticle system. Furthermore, by choosing the argument of $F(p, q, m_o)$ to contain the ratio p/m_o, as suggested by the form of the Lorentz electromagnetic force Hamiltonian discussed in Sect. V.A., the requirement that this damping factor be unaffected by such a coordinate scaling is clearly fulfilled. The Breit-Pauli interactions also contain angular orbital momentum terms not included explicitly in the above Hamiltonian, but these are clearly unaffected by the above scaling procedure because they either involve only products of r and p, or in the case of the spin interactions, are completely independent of spatial coordinates. It is thus shown that the desired proportionality between binding energy and rest mass of the constituents of a particle-antiparticle binary system holds for the Breit-Pauli interaction as long as the charge q of the individual particles remains the same. This is clearly the case in comparing the e^+e^- system to p^+p^-, so the

original objective sought at the beginning of this section is guaranteed by the use of such a Hamiltonian.

This result also tells us that the proportionality constant A in the exponential damping factor $F(p,q,m_o) = \exp[-A\alpha^2(q/m_o)p]$ must be the same for proton-antiproton interactions as between electron and positron. Alternatively, one might have assumed a different constant for the electron than for the proton, in which case one would have had to adjust the inverse mass dependence of the present exponential argument in order to obtain the desired scaling property. From the point of view of economy of assumptions and relation to established theoretical models, the former arrangement is clearly superior. The units for the constant A are left somewhat open by the choice of working in the atomic unit system. The exponential argument as a whole must be dimensionless, but the fine structure constant α can be defined as either dimensionless $e^2/\hbar c$ or simply as

c^{-1} among other possibilities). Since p/m_o has units of velocity, one can look upon one of the α factors as c^{-1} and the other as being dimensionless, in which case A must have units of inverse electric charge, so that Aq is also dimensionless.

E. MAGNETIC PHENOMENA: NEUTRINO INTERACTIONS

The charge-to-mass ratios appearing in the Breit-Pauli relativistic corrections are closely related to the magnetic moments of the corresponding particles. The Bohr magneton ß of a system is defined as $\hbar/2$ times the charge-to-mass ratio and the magnetic moment is usually measured in terms of this quantity. Strictly speaking, the magnetic moment is a vector proportional to the angular momentum of the particle, and in classical theory the Bohr magneton would simply be the corresponding proportionality constant. Magnetic measurements on electrons in atoms have shown that such a straightforward relationship is oversimplified, and have led to the introduction of a supplemental factor, the gyromagnetic ratio g. The discovery of spin angular momentum[126] brought with it the equally surprising result of g = 2 for the electron's spin magnetic moment[l03,127]. Later on, it was found that the correct value is actually slightly greater than this result[92,128,129], however. Moreover, the corresponding value for the proton[51,93] was measured to be several times larger than that of the electron.

The question that arises in the present context is whether the observed g values should be included explicitly in the Hamiltonian used to describe the particle-antiparticle states which have been suggested above. Since the scaling results discussed in the last section no longer hold if one multiplies the q/m values in the Breit-Pauli terms with a different factor for each type of particle, however, it seems clear that the measured g values should not appear in such a Hamiltonian. Moreover, this choice is also consistent with the accepted explanation[129] for the observed non-integral g values for spin magnetic moments, namely that they arise because the corresponding particle is affected by external factors (virtual photons or virtual mesons) which prevent the measurement of its properties in a pure (bare) state.

Having made this decision, it can be noted that the only experimental constants appearing in the damped Breit-Pauli Hamiltonian are the electric charges and rest masses of the interacting particles. It is therefore clear how to define a specific Hamiltonian for a system involving given numbers of electrons and protons and their respective antiparticles, particularly the e^+e^- and p^+p^- combinations of primary interest. There remains an important question, however, namely *how to deal with neutrino interactions.* The obvious approach is to simply substitute the corresponding values for the charge and rest mass of such particles and to solve the corresponding Schrödinger equation, but the observed electrical neutrality of the neutrino seems to preclude any chance of success for such a procedure. There is one possibility worth considering, however, *provided the rest mass of the neutrino is also exactly zero,* as seems possible based on experimental observations. In that case, the charge-to-mass ratio cannot simply be computed by dividing the above two quantities because the quotient of zero with itself is undefined. From a purely theoretical standpoint this eventuality would seem to invalidate the conclusion that the Breit-Pauli terms necessarily vanish for the chargeless neutrino, *i.e.* because q_{ov}/m_{ov} itself might still be different from zero. Such a possibility seems to clash with the experimental observations bearing on the magnetic moment of the neutrino, however, which indicate that it is also of vanishingly small magnitude,[55] thereby implying that the corresponding charge-to-mass ratio is still effectively zero.

The idea that the neutrino might possess a non-zero magnetic moment which could help to explain its observed activity in nuclear reactions is not new, and was in fact proposed by Pauli[130] as a general property of massless neutral particles based on the Dirac theory.[131] He later stated[37] that such a hypothesis "doesn't seem to me at all well founded." The history of the relationship

between the magnetic moment and the charge-to-mass ratio of a given particle has not been without its surprises, however. As already noted, the gyromagnetic ratio, for example, is an experimental quantity *invented* for the expressed purpose of accounting for discrepancies between observations and overly simplistic theoretical models. It is therefore wise to exercise some care in making conclusions about a particle's charge-to-mass ratio based on magnetic measurements alone.

In this connection it is important to recall that the special theory of relativity emphasizes that electric and magnetic fields are intertwined, with their respective magnitudes differing from one inertial system to another moving relative to it. Viewed from the standpoint of the affected particle, *there is never a magnetic force acting upon it,* because it is by definition stationary in its own intertial system (v = 0). Considerations of this nature led Lorentz[132] to point out that a Galilean transformation does not leave the laws[28] of electricity and magnetism invariant, and they eventually enabled Einstein[4] to formulate the special relativity theory in the first place. By this reasoning, it would seem to follow that since a chargeless particle *can never experience an electrical force* according to Coulomb's law, it should consequently experience *no electromagnetic force at all in its own inertial system.* Further, because of the principle of relativity this conclusion leads to another, namely that a chargeless particle must be unaffected by any electromagnetic field, regardless of the relative state of motion of the observer to it. If an electrically neutral system is composed of several charged particles, the analogous conclusion is negated by the possibility that its individual components move at different velocities, but for a true (chargeless) element this situation is excluded.

In order for q/m_o to be non-zero for the neutrino, however, its rest mass must be exactly zero and (for $E \neq 0$; see Sect. II.C) it must move with the velocity of light (v=c). In this case a Lorentz transformation to a different inertial system is no longer defined since $\gamma = (1 - v^2/c^2)^{-1/2}$ is then infinite. Consequently, the above argument based on the theory of special relativity does not really prove that a *chargeless, massless* particle is unaffected by electromagnetic fields in *all* inertial systems, although it perhaps suggests that this is most likely the case. It seems only prudent to note, however, that the preceding discussion has been based exclusively on classical electrodynamics. It might be anticipated that a truly definitive answer to the question of whether a massless neutrino can possess a non-zero charge-to-mass ratio q_v/m_{ov} without exhibiting a

magnetic moment in the usual sense can only be obtained on the basis of a quantum mechanical formulation of this problem.

The possibility of a non-zero charge-to-mass ratio of the neutrino is interesting in several other contexts as well, however. It provides a ready explanation for the existence of both a neutrino and an antineutrino, for example. If q_v/m_{ov} is non-zero, then different signs are possible for it, just as for charged species and their antiparticles. This characteristic might then be closely related to the helicity property of neutrinos measured in longitudinal polarization experiments[57,72-74]. It might also explain why there are apparently different types of neutrinos observed in neutron and pion-muon beta decays[78]. Neutrinos with distinct $|q/m_o|$ ratios would be expected to exhibit different behavior, despite their mutual lack of charge and rest mass.

In the present discussion, the point of immediate interest is whether the assumption of a non-zero q/m_o value for the neutrino leads to a solution of the corresponding Schrödinger equation (with the exponentially damped Breit-Pauli interactions mentioned above) for a $\nu\bar{\nu}$ complex which has zero binding energy relative to its separated particles. *i.e.* - 2 m_o c^2 in analogy to that of the e^+e^- and p^+p^- systems. Zero binding energy does not necessarily mean that a potential well does not exist, as Fig. 5 shows. There could be a large barrier separating the two minima in this case as well as for e^+ e^-, and thus the $\nu\bar{\nu}$ system might also be characterized by a relatively small optimal inter-particle separation[133]. Calculations investigating this point will be discussed in the following chapter.

F. TRANSLATIONAL ENERGY AND SHORT-RANGE INTERACTIONS

In Sect. V.B it was noted that the usual separation of internal and center-of-mass motion is not possible for a Hamiltonian containing the Einstein relativistic kinetic energy operator and the various Breit-Pauli terms. This observation deserves closer examination before moving on to the task of carrying out explicit calculations based on the corresponding Schrödinger equation. The condition of zero translational energy requires that the expectation value of $\mathbf{\Sigma p_i}$ vanish, which for a binary system such as positronium or the hydrogen atom implies that the momenta of each particle are equal and opposite to one another at all times and spatial positions.

Since $\mathbf{p}$ = m$\mathbf{v}$, this means that particles of equal mass must always move with equal speeds in opposite directions relative to their midpoint to fulfill the condition of zero translation. The Breit-Pauli interactions are not only short-range but also momentum-dependent, and so the only way to obtain a large attraction on the basis of such terms is for the expectation values of p_i and r_{ij}^{-3} to both be large for a given probability distribution. From the above argument, however, it is clear that a positron and an electron, with their equal rest masses, can stay in close proximity to one another while still moving at high velocity, without having net translation for the system as a whole. Hence a high degree of binding can result from this type of short-range interaction under these circumstances.

By contrast, if the masses of the two particles are quite different, as is the case for the proton and electron in the hydrogen atom, they must move with widely different speeds to avoid net translation. Particles whose speeds differ by a factor of 1836 can only stay close to one another while fulfilling this condition *if neither is moving very fast,* from which it can be seen that the possibilities for obtaining a tightly bound state under these circumstances are much less pronounced. On this basis, it is easy to imagine how the e^+e^- system might benefit much more strongly from inclusion of the Breit-Pauli interactions in the Hamiltonian than does the hydrogen atom. In particular, e^+e^- might possess a state of much lower energy than the familiar 1s species which results primarily because of the long-range Coulomb effect, whereas no comparably tight-binding state could be expected to be available to the hydrogen atom.

At the same time the scaling property discussed in Sect. V.D tells us that the range of the p^+p^- interaction must be shorter than for e^+e^- by a factor equal to the ratio of the respective rest masses of the proton and electron. The v_1 = -v_2 condition for motion without net translation also applies to the p^+p^- system and so one can explain the even larger binding energy of p^+p^- on the same basis. Their larger mass allows the proton and antiproton to approach each other far more closely on the average than the electron and positron while avoiding net translation of either system. The exponential damping factor F (p, q, m_o) would also play a crucial role in this distinction, particularly the inverse mass dependence of its argument, since it begins to effectively negate the influence of the Breit-Pauli terms at much larger separations for the electron and positron than for their heavier counterparts. These qualitative observations also suggest that it might be important *not* to assume that the internal motion can be totally separated

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from the translational when it comes to short-range interactions at relativistic speeds. This topic will be treated in more detail in Chapter X, but for the present we will simply represent the proposed interactions directly in terms of the Cartesian coordinates for the constituent particles of a given system, *i.e.* not employ the usual transformation to center-of-mass and internal coordinates.

G. EXPLICIT REPRESENTATION OF THE EXPONENTIALLY DAMPED HAMILTONIAN

The explicit form of the Hamiltonian discussed above is given in Table I. For simplicity we will refer to it subsequently as the XBPS Hamiltonian, the abbreviation standing for "Exponentially-damped Breit-Pauli Schrödinger equation." As usual, there is a one-particle interaction for each constituent in the system, and a set of two-particle operators for each pair of such species. The Hamiltonian of Table I is given explicitly for only two (representative) particles with respective charges and rest masses q_i (q_j) and m_{oi} (m_{oj}), but in view of the above discussion it is easily generalized for the description of any number and combination of different particle types. The only one-particle term in the Hamiltonian is the relativistic kinetic energy. Other common one-electron interactions such as the Coulomb nuclear attraction for an electron are to be found among the two-particle interactions. The rest energy $m_{oi}c^2 = m_{oi}\alpha^{-2}$ (atomic units are used throughout) is subtracted from the Einstein square-root operator in the usual way to represent the kinetic energy. This procedure effectively defines the zero of energy as that of the (infinitely) separated particles at rest.

The first two-particle interaction listed in Table I is the Coulomb term and it is the only one which is unchanged relative to the standard Breit-Pauli treatment.[102] The remaining terms are all exponentially damped, but fall into distinct categories, depending on whether they are multiplied with $q_1 q_2 / m_{o1} m_{o2}$ or q_1^2 / m_{o1}^2 (or q_2^2 / m_{o2}^2). In the first group are the spin-other-orbit, orbit-orbit and spin-spin terms, the latter including a δ-function component. The spin-same-orbit and Darwin interactions comprise the second category. Since all particles are to be treated equivalently, it is essential that each two-particle interaction be symmetric with respect to interchange of the corresponding indices, as provided for in the original Breit-Pauli Hamiltonian. The Coulomb term obviously satisfies this condition, since r_{12} is a scalar quantity denoting the magnitude of the distance separating the two particles.

The most significant change in the operators of Table I compared to those in the conventional Breit-Pauli Hamiltonian is their multiplication with exponential damping factors. For the terms of orbit-orbit type, it is assumed that two such factors are needed: $\exp(-A\alpha^2|(q_i/m_{oi})p_i|)$, for i= 1 and 2. To insure that the Hamiltonian be hermitian, it is necessary that the same pair of factors appear on both sides of the original Breit-Pauli terms, since $\mathbf{p_i}$ and $\mathbf{r_{ij}}$ do not commute with one another. The arguments of all the exponential terms are defined to be negative-definite, hence the absolute value sign in these expressions. The damping factors for the spin-same-orbit and Darwin terms have a somewhat more complicated form, however. Since the q/m_o factors appear squared for these terms in the original Breit-Pauli Hamiltonian,[102] it seems consistent to also use squares of the corresponding damping factors for the same particle in this instance instead of products of two different kinds as before.

The sign of the interaction in the latter class of operators requires additional comment as well. Since the pre-multiplying factor is either q_1^2/m_{o1}^2 or q_2^2/m_{o2}^2, it is not possible on this basis alone to specify whether the interaction is attractive or repulsive. This information is contained in the product of the charges of the two particles, which appears explicitly in the orbit-orbit-type terms but not for the Darwin or spin-same-orbit operators. It is thus necessary to define a sign convention based on the product $(q_1/m_{o1})(q_2/m_{o2}) \equiv G(1,2)$ according to which a positive result corresponds to a negative sign for both terms, while the opposite choice is made if the result is negative. Similarly as in the other case, the $\exp(-2A\alpha^2|(q_i/m_{oi})p_i|)$ factor must appear on both sides of the corresponding Breit-Pauli terms in order to preserve the required hermitian character. The absolute value sign guarantees that the argument of the exponential is negative, *i.e.* the factor always reduces the absolute magnitude of the original Breit-Pauli interaction for the same charge distributions.

Table I. Definition of quantum mechanical operators present in the exponentially damped Breit-Pauli Hamiltonian employed throughout the present study (α is the fine-structure constant; atomic units employed throughout). The indices i and j are used generically to represent two interacting particles; the quantities q_i (q_j) and m_{0i} (m_{0j}) are the electric charges and rest masses of the ith (j^{th}) particle, A is the exponential damping constant (units of e^{-1}; see text), and $\mathbf{p_i}$ ($\mathbf{p_j}$), $\mathbf{s_i}$ ($\mathbf{s_j}$) and $\mathbf{r_{ij}}$ are the standard vectorial symbols for the linear and spin angular momenta of a single particle and the distance between the ith and j^{th} particles, respectively.

Designation		Operator
Relativistic Kinetic Energy (one-particle only)	KE	$\left(p_i^2\alpha^{-2} + m_{0i}^2\alpha^{-4}\right)^{\frac{1}{2}} - m_{0i}\alpha^{-2}$
Coulomb	C	$q_i q_j r_{ij}^{-1}$
Spin-same-orbit (exponentially damped)	SsO	$-\dfrac{\alpha^2}{2}G(i,j)$

$$\times\left\{\left(\frac{q_i}{m_{0i}}\right)^2 \exp\left(-2A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)\right.$$

$$\times\left(\mathbf{r_{ij}}\times\mathbf{p_i}\cdot\mathbf{s_i}\right)r_{ij}^{-3}\exp\left(-2A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)$$

$$+\left(\frac{q_j}{m_{0j}}\right)^2 \exp\left(-2A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)$$

$$\left.\times\left(\mathbf{r_{ji}}\times\mathbf{p_j}\cdot\mathbf{s_j}\right)r_{ij}^{-3}\exp\left(-2A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)\right\},$$

$$\text{where } G(i,j) = \begin{cases} 1, & \text{if } \dfrac{q_i q_j}{m_{0i}m_{0j}} > 1 \\ -1, & \text{if } \dfrac{q_i q_j}{m_{0i}m_{0j}} < 1 \end{cases}$$

| Spin-other-orbit (exponentially damped) | SoO | $-\alpha^2\left(\dfrac{q_i}{m_{0i}}\right)\left(\dfrac{q_j}{m_{0j}}\right)$ |

$$\times\exp\left(-A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)\exp\left(-A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)$$

$$\times\left(\mathbf{r_{ji}}\times\mathbf{p_j}\cdot\mathbf{s_i} + \mathbf{r_{ij}}\times\mathbf{p_i}\cdot\mathbf{s_i}\right)r_{ij}^{-3}$$

$$\times \exp\left(-A\alpha^2 \left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)\exp\left(-A\alpha^2 \left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)$$

Table I continued

Designation		Operator
Darwin Term (exponentially damped)	D	$-\pi\dfrac{\alpha^2}{2}G(i,j)$

$$\times\left\{\left(\frac{q_i}{m_{0i}}\right)^2\exp\left(-2A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)\right.$$

$$\times\delta(\mathbf{r_{ij}})\exp\left(-2A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)$$

$$+\left(\frac{q_j}{m_{0j}}\right)^2\exp\left(-2A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)$$

$$\left.\times\delta(\mathbf{r_{ij}})\exp\left(-2A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)\right\},$$

$$\text{where } G(i,j)=\begin{cases}1, & \text{if } \dfrac{q_iq_j}{m_{0i}m_{0j}}>1\\[2ex]-1, & \text{if } \dfrac{q_iq_j}{m_{0i}m_{0j}}<1\end{cases}$$

| Orbit-orbit (exponentially damped) | OO | $-\dfrac{\alpha^2}{2}\left(\dfrac{q_i}{m_{0i}}\right)\left(\dfrac{q_j}{m_{0j}}\right)$ |

$$\times\exp\left(-A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)\exp\left(-A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)$$

$$\times\left[(\mathbf{p_i}\cdot\mathbf{p_j})r_{ij}^{-1}+\left(\mathbf{r_{ij}}\cdot(\mathbf{r_{ij}}\cdot\mathbf{p_i})\mathbf{p_j}r_{ij}^{-3}\right)\right]$$

$$\times\exp\left(-A\alpha^2\left|\frac{q_i}{m_{0i}}p_i\right|\right)\exp\left(-A\alpha^2\left|\frac{q_j}{m_{0j}}p_j\right|\right)$$

| Spin-spin (exponentially damped) | SS | $-\alpha^2\left(\dfrac{q_i}{m_{0i}}\right)\left(\dfrac{q_j}{m_{0j}}\right)$ |

$$\times\exp\left(-A\alpha^2\left|\frac{q_i}{m_{0i}}\mathbf{p_i}\right|\right)\exp\left(-A\alpha^2\left|\frac{q_j}{m_{0j}}\mathbf{p_j}\right|\right)$$

$$\times \left[\left(\mathbf{s_i} \cdot \mathbf{s_j} \right) r_{ij}^{-3} + 3 \left(\mathbf{r_{ij}} \cdot \mathbf{s_i} \right) \left(\mathbf{r_{ij}} \cdot \mathbf{s_j} \right) r_{ij}^{-5} \right]$$

$$\times \exp \left(-A\alpha^2 \left| \frac{q_i}{m_{0i}} \mathbf{p_i} \right| \right) \exp \left(-A\alpha^2 \left| \frac{q_j}{m_{0j}} \mathbf{p_j} \right| \right)$$

Table I continued

Designation		Operator
Spin-spin δ (exponentially damped)	SSδ	$-\dfrac{8\pi\alpha^2}{3}\left(\dfrac{q_i}{m_{0i}}\right)\left(\dfrac{q_j}{m_{0j}}\right)$
		$\times\exp\left(-A\alpha^2\left\vert\dfrac{q_i}{m_{0i}}\mathbf{p_i}\right\vert\right)\exp\left(-A\alpha^2\left\vert\dfrac{q_j}{m_{0j}}\mathbf{p_j}\right\vert\right)$
		$\times\mathbf{s_i}\cdot\mathbf{s_j}\delta\left(\mathbf{r_{ij}}\right)$
		$\times\exp\left(-A\alpha^2\left\vert\dfrac{q_i}{m_{0i}}\mathbf{p_i}\right\vert\right)\exp\left(-A\alpha^2\left\vert\dfrac{q_j}{m_{0j}}\mathbf{p_j}\right\vert\right)$

It might be argued that the above sign convention for the Darwin and spin-same-orbit terms can be satisfied more simply by replacing q_1^2/m_{o1}^2 by $q_1 q_2/m_{o1}^2$. The result would clearly be the same as before only if $|q_1| = |q_2|$, however.[127] This is the case for interactions between protons and electrons and their antiparticles, but in a more general formulation capable of dealing with other types of particles as well, it seems preferable to employ the former definitions. In this way the charge of a given particle only appears divided by its own rest mass in the damped Breit-Pauli interactions. This prescription at least formally allows for the treatment of chargeless, massless neutrinos with such a Hamiltonian, without introducing the types of singularities which otherwise would arise when coupling constants involving ratios of the charge of one particle and the rest mass of another are employed.

Returning to the immediate focus of attention, the e^+ e^- system, it can be seen that the XBPS Hamiltonian of Table I exhibits a special symmetry in this case, namely it commutes with the charge conjugation operation C. This means that the corresponding eigenfunctions must be either symmetric or anti-symmetric with respect to interchanging the electron and positron coordinates. The same result clearly holds for any particle-antiparticle binary system. This is a different situation than one normally encounters in conjunction with the Pauli principle[134], in which permutation symmetry results because of the *indistinguishability* of the component particles. As a result it is not necessary to assume that the only physically meaningful solutions are of one symmetry-type, for example, anti-symmetric in the case of fermions. The symmetry in question arises not because the particles are indistinguishable, but rather because their (different)

properties can be exchanged without affecting the form of their mutual interaction. This characteristic applies to all states of positronium, including those of hydrogenic type. In view of its special nature, however, there seems little point in incorporating this symmetry into the basis functions employed in explicit e^+e^- calculations with the XBPS Hamiltonian. In general, it will be assumed that the many-particle basis consists of products of Slater determinants.[135] for each particle type, *i.e.* anti-symmetry is assumed only for indistinguishable fermions. Bosons arise naturally as even products of fermions[25] and therefore require no additional symmetrization procedure.

VI. COMPUTATIONS FOR MASSLESSS PARTICLE-ANTIPARTICLE BINARIES

The central hypothesis explored in the previous chapters is that elemental matter can neither be created or destroyed. Rather it is argued there exist strong attractive forces between elementary particles and their antiparticles which lead to massless binary systems whose non-observability is understandable in terms of the Bohr frequency relation ($E = m = 0$ implies $\nu = 0$ and $\lambda = \infty$). The XBPS Hamiltonian given in Table I has been deduced in accordance with such expectations and calculations employing it will be considered below, starting with the electron-positron system.

A. DETAILS OF THE COMPUTATIONAL METHOD

The computational methods employed are modeled after those of electronic structure calculations for atoms and molecules. In essence a matrix representation of the XBPS Hamiltonian is formed with the aid of products of one-particle functions. The first step is to compute integrals for each of the original Breit-Pauli operators[102] (except for the mass velocity terms) over a set of basis functions. Because of the decision to treat all particles in an equivalent manner, with no assumptions about fixed probability distributions for any of them, it can be noted that the Hamiltonian is symmetric with respect to all operations of the full rotation group

plus inversion. Accordingly, it is reasonable to localize all basis functions at a single center, simply the origin of the coordinate system. As mentioned in the last chapter, the only one-particle operator under the circumstances is that of the (relativistic) kinetic energy. All the other Breit-Pauli terms in the Hamiltonian, including the Coulomb interaction. are treated exclusively as two-particle operators[102].

To construct the most general possible digital computer program it was decided to employ the same set of basis functions to describe each type of constituent particle. This approach seems reasonable because in a system with no net translation, the momenta of the particles should be similar, which implies basis functions of roughly the same extension for each of them. In the last analysis it is always possible to employ individually optimized basis functions for each particle type by simply combining all such sets and making them available to describe the probability distributions of all the particles involved.

In this approach it is only necessary to compute the various one- and two-particle interaction integrals for electrons in the initial stage of the calculation. These results can then be conveniently adapted for treatment of particles other than electrons at a later stage. It was decided to use real cartesian gaussian functions to construct the one-particle basis, although it would also be convenient to use Slater-type exponential functions as more commonly employed in atomic calculations. Since no exact solutions for the Schrödinger equations of primary interest are known, the primary consideration was to choose a basis capable of describing general continuous functions which vanish at infinite distance from the origin.

An option for use of one-center potentials is available in the program which enables conventional atomic calculations to be carried out as well, in which case only two-particle interactions for electrons are assumed, although this restriction could also easily be removed. In such applications the charge of the nuclear center is required as input for the initial integral computations. Otherwise no input is needed other than the exponents and coefficients of the contracted cartesian gaussian basis functions. Formulae for the calculation of cartesian gaussian integrals for all the Breit-Pauli operators may be found elsewhere[136]. The computer program for evaluation of these integrals for the present study has been written by Chandra and implemented by Phillips, Liebermann and the author[137].

Because of the presence of spin variables in the one-particle functions it is desirable to carry out the overall treatment by employing a basis of eigenfunctions of the total angular

momentum operators j^2 and j_z. The corresponding spin-orbitals are formed with the help of the ladder operator technique[138] and conform to the standard Condon-Shortley conventions[139]. The transformation of the Breit-Pauli integrals from a basis of spatial functions to the desired spin-orbitals is effected in two steps: the complex spatial eigenfunctions of l^2 and l_z are employed in the first transformation, followed by a second change of basis to the j^2, j_z eigenfunctions. In the process, the number of basis functions is doubled in the usual way because of the duality of the spin representation for fermions. The final two-particle integrals are classified according to quartets of the four quantum numbers, n, l, j and m_j, whereby n simply numbers the different spatial basis functions from unity upwards. Because of the spherical symmetry of the Breit-Pauli terms it is only necessary to specify three m_j values explicitly, since the only non-zero two-particle integrals are characterized by $\sum_i m_j^i = 0$.

Two other indices are required, referred to as Γ and μ. The index μ simply refers to the different operators employed, while r is an ordering index with only two values. A standard order of indices is defined, whereby the integral $<\varphi_a(1)\ \varphi_b(2)\ \varphi_\mu(1,2)\ \varphi_c(1)\ \varphi_d(2)>$ is characterized by $\Gamma = 1(2)$ if the standard order can be reached by an even (odd) number of permutations of the orbitals of the same particle. As is common practice[140], the spin-same-orbit term is treated as two separate operators, one for $\mathbf{l_1}\ \mathbf{s_1}$ and one for $\mathbf{l_2}\ \mathbf{s_2}$. All other Breit-Pauli terms are symmetric with respect to particle exchange prior to multiplication by the various q/m_o coupling constants and can therefore be treated as symmetric sums at this stage of the computations, i.e. before the charges and rest masses of the actual constituent particles are introduced. More details concerning these computer programs will be given elsewhere[141].

B. TRANSFORMATION TO THE MANY-PARTICLE-TYPE BASIS

In Chapter V it was argued that the original Breit-Pauli operators need to be adapted so that a Schrödinger equation employing them can lead to a massless state of a particle-antiparticle binary system, i.e. one whose binding energy is exactly equal to the sum of the rest masses of its constituents. The suggested changes have always involved functions of the momentum operator, specifically a square-root and several exponentials. These operators lead to integrals needed for their matrix representation which are relatively complicated to evaluate in a direct manner. This

is particularly true for the two-particle exponentially-damped Breit-Pauli terms (Table I). Closed expressions for the relativistic kinetic energy do exist[142], but in order to deal with all the required operators in a consistent manner, an approximate integral evaluation technique must be employed. It was thus decided to apply the matrix procedure referred to above[120] for the treatment of both the square-root and exponential function operators. This involves use of the resolution of the identity formalism[143] and thus results employing it approach their exact values only as the basis set employed to obtain the matrix representation of the Hamiltonian nears completeness. Numerical tests[144,145] comparing the exact and matrix representation values for relativistic kinetic energy integrals indicate that this level of approximation is suitable for the purpose at hand. The situation is more complicated for the damped Breit-Pauli terms of a two-particle nature, because it becomes impractical to saturate the basis to a similar extent as in the square-root operator tests. Nonetheless, it is always possible to judge the numerical stability of the final results of the calculations by comparing with analogous findings obtained with basis sets of different size. Since all the operators which are primarily responsible for these difficulties are functions of the momentum operator, it is possible to proceed in a very similar fashion for all of them.

Briefly, one first needs to form the non-relativistic kinetic energy matrix for electrons in the assumed gaussian basis and then to diagonalize it. If the basis functions were momentum eigenfunctions, the desired relativistic square-root integrals could be obtained exact1y by simply replacing the $p^2/2$ eigenvalues by the corresponding results for the operator in question. A related diagonal matrix can be formed even if momentum eigenfuctions are not used, but then it is necessary to subsequently carry out a reverse transformation to the original basis. The resulting non-diagonal matrix is then used for all subsequent computations. In the case of the damped Breit-Pauli terms it is necessary to carry out an additional matrix multiplication[144] involving four one-electron exponential matrices and the original two-particle Breit-Pauli counterpart discussed in Sect. VI.A. The use of this matrix technique has the disadvantage of rendering the overall treatment non-variational, but again use of a suitably flexible basis set minimizes this effect.

It is at this stage of the computations that the charges and rest masses of the various constituent particle types are first needed. Each of the one-particle matrices for the relativistic kinetic energy and the exponential damping factors requires such input values explicitly. The corresponding matrices are generated for each particle type from the $p^2/2$ non-relativistic

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counterpart. It should be clearly distinguished between "particle types" and "particles" in this connection. At this stage it is only necessary to know what kinds of particles are contained in the system at hand, and not how many of each. Once these one-particle matrices are generated it is necessary to carry out a series of four-index transformations for each Breit-Pauli operator μ and pair of particle types p. Because of the nature of the total XBPS Hamiltonian it is necessary to distinguish between $\rho = (1, 2)$ and $\rho = (2,1)$, *i.e.* the order of particle-types is significant. This becomes obvious at the next step in the procedure, in which each transformed matrix is multiplied with an appropriate set of coupling constants formed from the charges and rest masses of the constituent particles (see Table I).

These results are then added together to form the final one- and two-particle Hamiltonian matrices, which are stored for further use in the many-particle phase of the theoretical treatment. The four-index (two-particle) integrals are ordered by means of quartets of indices for each basis function, as distinguished by their respective values for the quantum numbers n, 1, j and m_j discussed above. In addition there are separate values for each of the two Γ permutations, as well as for each pair of particle-types ρ. The operator index μ has thus effectively been replaced by the particle-pair index ρ at this stage of the treatment as a result of the additions and scalar multiplications of the individual operator matrices to form the Hamiltonian two-particle matrix. The kinetic energy matrix elements are only ordered with respect to n and l because they do not vary with j and m_j. In addition, only diagonal ρ values are needed because of the one-particle nature of this operator.

C. COMPUTATION OF ENERGIES AND WAVEFUNCTIONS: VARIATIONAL CI APPROACH

The method employed to obtain approximation solutions for the Schrödinger equation discussed above is primarily the configuration interaction approach. A self-consistent field calculation is first carried out to generate an orthonormal basis of one-particle functions which allows an optimal description in terms of a single configuration. A coupling scheme is employed which amounts to using J - ½K (J, Coulomb and K, exchange operators, in the .conventional notation[146,147]) for interactions of the same particle-type and only J for all others. For this purpose appropriate averages over multiplet interactions are assumed[141]. The exact form of the SCF

procedure is of little consequence in the present applications, however, because a full CI calculation is ultimately carried out, in which case the final eigenvectors and eigenvalues of the many-particle XBPS Hamiltonian are independent of the choice of one-electron basis. In essence the SCF procedure is used primarily to obtain an orthornormal basis whose use simplifies the evaluation of CI matrix elements (Slater-Condon rules)[148].

The next step in the theoretical treatment is then the four-index transformation of the XBPS Hamiltonian matrix to the SCF one-particle basis. Advantage is taken of the high degree of symmetry blocking produced by the use of one-particle eigenfunctions of j^2 and j_z. The resulting matrix elements are reordered into groups with the same l, j, Γ and ρ values for the orbital quartets, *i.e.* all n and m_j components are stored contiguously. Because of the transformations to SCF orbitals, it is now necessary to distinguish in general between kinetic energy matrix elements involving the same l but different j.

Subsequently all configurations of orbital products which are possible in a given basis are formed based on the occupation numbers of the various particles. The latter information must be provided by the user at this point. The desired total J angular momentum quantum number and the corresponding parity P are also given as input, and only configurations which lead to multiplets of the desired symmetry are retained. Linear combinations of the many-particle product functions (anti-symmetrized within each particle-type, as discussed at the end of the last chapter) which are eigenfunctions of J^2 with $M_J = J$ are constructed and Hamiltonian matrix elements are computed with respect to this basis. For this purpose, the Bunge-Davidson projected determinant technique[149] ensures that only the minimal number of determinantal matrix elements be computed. In general the Slater-Condon rules are employed with a procedure closely related to the Table CI technique[150] used in computations for molecular systems. It should be noted that imposing the condition of zero net translation for the combined system, as discussed in Sect. V.B, makes the Slater determinant form for the associated wavefunctions considerably less advantageous than in the present treatment. This is because the coordinate transformation required for the above purpose inevitably singles out a particular particle in order to define a set of relative spatial coordinates[122]. In a relativistic treatment for which the reference particle's spin-functions need to be considered explicitly, this circumstance represents a considerable obstacle to the design of a generally applicable many-particle computational procedure.

The resulting XBPS Hamiltonian matrix is then diagonalized by conventional methods[151], provided the dimension of the full CI is not very large. Tests have been carried out in such cases to demonstrate that the full CI eigenvalues are the same for different choices of one-electron (SCF) orbitals deriving from the same gaussian basis. For applications involving larger systems, for which the full CI method becomes impractical, a subset of the corresponding space is employed which is generated by taking all single and double substitutions relative to a series of key reference configurations (MRD-CI approximation[152]). In this case the Davidson diagonalization procedure[153] is employed to obtain the desired roots, whereby accurate starting vectors are highly desirable for this purpose. In the present chapter dealing with particle-antiparticle binary systems, only full CI computations are considered so that the only approximation employed in obtaining the XBPS solutions is the use of a finite basis in the corresponding Hamiltonian matrix representation.

As mentioned in Sect. V.B, the square-root form of the relativistic kinetic energy as well as the presence of the various momentum factors in the Breit-Pauli potential terms eliminates the possibility of introducing the usual separation of internal and center-of-mass motion in the present theoretical treatment. As a result, the total energy resulting from the above CI calculations generally contains a contribution corresponding to net translation of the system. It is therefore desirable to also compute the expectation value of the translational energy $T = [(\sum_i p_i)^2 \alpha^{-2} + M_o^2 \alpha^{-4}]^{1/2} - M_o\alpha^{-2}$, for which purpose the rest mass M_o of the combined system must be provided as input. The procedure employed again involves the matrix representation technique described in the last section.

It is first necessary to calculate the $(\sum_i p_i)^2$ matrix in the same basis used throughout, which is accomplished conveniently by dividing the operator into its one- and two-particle contributions: $\sum_i p_i^2$ and $2\sum_{i>j} p_i p_j$. For this purpose one-particle matrix elements for the p_i operator are required. There is an important selection rule in this case. namely $L' = L \pm 1$, i.e. only pairs of functions satisfying this relation give non-vanishing results. A basis set consisting of only s functions, for example, can only lead to vanishing $(\sum_i p_i)^2$ expectation values for a given many-particle wavefunction if the diagonal $\sum_i p_i^2$ term itself vanishes, which as a sum of

positive-definite one-particle quantities is impossible. The two-particle terms can lead to a substantial reduction in $<(\sum_i p_i)^2>$ relative to what is obtained with only one-particle terms, however, if more than one l value is present in the basis set employed, but even in this case, it is very difficult to obtain an even approximately vanishing result for $<(\sum_i p_i)^2>$ for the XBPS wavefunctions in the present theoretical treatment.

In principle, knowledge of the total energy $<E>$ and $<(\sum_i p_i)^2>$ values allows the direct computation of the rest mass of the combined system, but this possibility is of little practical use when less than exact results are available. Hence the reliance on a separate input value for this quantity is preferred, as noted above. There are several procedures for obtaining the internal energy E_o at this level of approximation. The first involves the variational calculation of the total energy's expectation value $<E>$. The corresponding wavefunction is then used with the help of the M_o input value and the expectation value of $(\sum_i p_i)^2$ to evaluate $<T>$, which is then subtracted from $<E>$. A second possibility is to treat the operator H - T in a variational calculation directly. This requires a matrix representation of T in the many-particle basis, which is only practical for relatively small one-particle basis sets. The resulting wavefunctions are infinitely degenerate in principle, since any degree of translation leads to the same E_0 value for a given internal state. In practice it is generally preferable to seek solutions with minimal translational energy, and this goal is best accomplished by dealing with H alone at the variational stage, as discussed first. The expectation value for the translational energy can also be computed in the perturbational approach by employing the above matrix representation of T, as opposed to first computing $<(\sum_i p_i)^2>$.

D. CALCULATION OF NON-HYDROGENIC STATES OF THE e^+e^- SYSTEM

The basic strategy for obtaining an e^+e^- state with a binding energy of $2m_ec^2$ is to vary the constant A in the XBPS Hamiltonian (Table I) until the lowest energy possible for a given number and type of basis function corresponds to the desired value. Without the exponential damping (A = 0), this goal can never be reached because the Hamiltonian is not bounded from below in this case. The first basis chosen contains two s- and two p-type primitive Gaussians with initial exponents of 1.0×10^7 and 1.0×10^8 a_o^{-2} for both types. The exponents were multiplied

by a scale factor η and a full CI calculation was carried out for symmetries spanned by this basis. The computations show that the lowest energy results for a state of 0^- symmetry (J=0 and negative parity). By varying both η and the damping constant A, it is found that a minimum in energy of the desired value ($-2\alpha^{-2}$ =-37557.7 hartree) occurs for the lowest 0^- state for $\eta = 0.11$ and A= 1.054 e^{-1}. As expected, the value of the minimal energy increases with the magnitude of A for all η and advantage is taken of this relationship in subsequent optimizations. These calculations thus demonstrate that the XBPS Hamiltonian can be subjected to standard optimization techniques and the energies of the corresponding full CI secular equations can be suitably adjusted with a single free parameter.

The next step was to define a larger basis set consisting of five s- and five p-type primitive Gaussians. The exponents were initially assumed to form a geometrical progression: $\beta_N = \beta_1\gamma^{N-1}$. The value of γ was taken to be 2.0 after some initial experimentation. As before, the energy optimizations are carried out with respect to a single scaling parameter η which multiplies all the β_i values. The exponents are taken to be the same for the s and p sets ($\beta_1 = 0.25 \times 10^8$ a_o^{-2}). This is already a reasonably large s,p basis, consisting of 80 spin-orbitals or 40 for each particle, considering that a full CI optimization is to be carried out. It is nonetheless considerably smaller than those commonly used to study hydrogenic systems on a definitive basis[144,145], but it will serve the present purpose adequately, namely to examine the description of the proposed tight-binding states in the XBPS model.

Again it is found that the most stable state of the e^+e^- system has 0^- symmetry. The corresponding wavefunction consists of products of $s_{1/2}$ and $p_{l/2}$ ($\pm$ denotes positron or electron function respectively) one-particle functions. Four products of spin orbitals are required for any pair of exponents:

$$0^- = p_{1/2}^+ (m_j = \tfrac{1}{2})\ s_{1/2}^- (m_j = -\tfrac{1}{2}) - p_{1/2}^+ (m_j = -\tfrac{1}{2})\ s_{1/2}^- (m_j = \tfrac{1}{2})$$

$$+ p_{1/2}^- (m_j = \tfrac{1}{2})\ s_{1/2}^+ (m_j = -\tfrac{1}{2}) - p_{1/2}^- (m_j = -\tfrac{1}{2})\ s_{1/2}^+ (m_j = \tfrac{1}{2}). \qquad\qquad \text{VI.1}$$

This function is seen to not only possess singlet spin but also to be symmetric with respect to the charge conjugation operation (see Sect. V.G for a discussion of the latter symmetry property for particle-antiparticle pairs). The optimal value for the scale factor η is 0.095 and the correct

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binding energy of $2\alpha^{-2}$ is obtained for A= 1.078 e^{-1}. One should expect that the value of the damping constant increases with improvement in the one-particle basis and this behavior is observed. The change in A relative to its 2s,2p value is 2.3%, which indicates that we are already relatively close to the limit attainable with s and p basis functions. It is thus of interest to look at the results of the 5s,5p calculations in more detail below.

TABLE II. Total full CI energy (in hartree) of the lowest states of various symmetries of the e^+e^- system obtained employing the 5s,5p basis with scale factor $\eta = 0.095$ and exponential damping constant $A = 1.0775\ e^{-1}$ for the XBPS Hamiltonian of Table I.

Symmetry	First Root	Second Root
0^+	372025.142	561471.371
0^-	-37656.717	483278.726
1^+	596788.279	646488.246
1^-	592821.494	639727.142
2^+	785793.989	806032.930
2^-	691742.083	763807.974
3^+	853632.944	164188.023

TABLE III. Energy contributions (in hartree) of various operators (see Table I for definitions) for the 0_s^- ground state of the e^+e^- system obtained employing the 5s,5p basis with scale factor $\eta = 0.095$ and exponential damping constant $A = 1.0775\ e^{-1}$ for the XBPS Hamiltonian.

Operator	Expectation Value
Kinetic energy	1992262.978
Coulomb	-3233.894
Spin-same-orbit	-378340.098
Spin-other-orbit	-830993.650
Darwin Term	8889.119
Orbit-orbit	-407592.397
Spin-spin	-418648.776
Spin-spin δ	0.000
Total energy	-37656.717

The energies of the most stable e^+e^- states of each symmetry obtained in this treatment are given in Table II, from which it is seen that the lowest-lying 0 species is favored by a large margin over the other states, being the only one which is bound with respect to the separated

particles at this level of treatment. The 0^- total energy is broken down into contributions from each of the terms in the XBPS Hamiltonian in Table III. The total kinetic energy is 1.99 x 10^6 hartree or 54.2 MeV, so it is clear that the two particles are tightly bound. The main attractive contributions come as expected from the damped Breit-Pauli interactions. The spin-same-orbit, orbit-orbit and spin-spin terms are each in the order of -4 x 10^5 hartree, while the spin-other-orbit interaction is double this amount. The relative order of these contributions is easily understandable in terms of the constants multiplying each operator in the Breit-Pauli interactions themselves[102] (see Table I). The cancellation between these attractive potential terms and the two repulsive quantities is very delicate, as seen by the fact that the binding energy is only 1.88 % of the total kinetic energy. By way of comparison, it should be recalled that the binding energy is exactly equal to the kinetic energy in non-relativistic treatments of atomic systems (by virtue of the virial theorem).

It is important to note that a state composed of only s orbitals must give zero contribution by symmetry for each of the above Breit-Pauli potential terms, so it is not difficult to understand that substantial $p_{1/2}$ character is present in the optimal 0^- wavefunction. It is well known that the $p_{1/2}$ orbital is stabilized by spin-orbit coupling in atomic calculations, so this observation is also not surprising from that point of view. The binding is considerably enhanced for the e^+e^- system relative to the hydrogen atom because of the much larger magnetic moment of the positron compared to that of the proton, as demonstrated in Table III. In this sense the large binding of the 0^- state can be thought of as resulting from a tremendously large increase in the $p_{1/2}$ - $p_{3/2}$ multiplet splitting in the spectrum of hydrogenic systems. It might be thought that a $p_{1/2}$ $p_{1/2}$ configuration would be even more stable on this basis, but the added kinetic energy of $p_{1/2}$ *vis-a-vis* $s_{1/2}$ is decisive in avoiding this result. If the calculations are carried out under the condition of vanishing translational energy ($\mathbf{P} = 0$; see Sect. VI.B), it follows that the corresponding function of the electron-positron separation would be a p orbital (more specifically a $p_{1/2}$ species coupled with the spin of the reference particle to produce a 0^- solution).

The 0^- state's composition is noteworthy in another way, namely that it possesses an exactly vanishing expectation value for the Darwin δ-function term appearing in the original Breit-Pauli Hamiltonian, as can be verified using the above sample wavefunction. This means that the electron and positron never occupy the same spatial position when their spins are identical. The expectation value of the Darwin term in the XBPS Hamiltonian itself (Table I) is

non-zero in value (Table III), but this result stems from the form of the exponential damping factors employed in this case (same as for the spin-same-orbit term). The δ-function term in the spin-spin operator also has a vanishing expectation value for the 0^- wavefunction, with and without the XBPS adaptations (Table III). This result shows that the avoidance property for electron and positron is not restricted to species of the same spin component since the $s_1 s_2$ scalar product allows α and ß spins to have a non-zero interaction in general. In view of the opposite charges of the electron and positron it might be thought that these particles would prefer to be in the same region of space much more than these results would indicate, but one should not forget that the Darwin term itself is repulsive in this case, and thus its influence should tend to be minimized in a variational treatment. This consideration does not completely explain the observed behavior, however, as it must be assumed that such a relationship between the two charge distributions also maximizes the effects of the attractive terms in the XBPS Hamiltonian, such as the spin-spin, spin-orbit and orbit-orbit interactions. Since the vanishing magnitude for the various δ-function terms is so clearly tied up with the 0^- composition for the e^+e^- wavefunction, there is good reason to expect that similar results will be found regardless of the size of the one-particle basis set employed. In the $\mathbf{P} = 0$ limit for this state discussed above, even the expectation value of the exponentially damped Darwin term must vanish exactly since it is forbidden by symmetry from having an admixture of s spatial character. Such a function must also be symmetric with respect to the charge conjugation operation, in agreement with what has been found in the calculations above for the 0^- wavefunction with non-vanishing translational energy.

The exceptional nature of the 0^- e^+e^- state also gives added support to the hypothesis formulated in Chapter II to the effect that such a massless state of positronium is identical with that of the photon at rest. Although this result might be thought to be inconsistent with such an assumption because the photon is normally assigned 1^- symmetry, it should be recalled that the photon state given the latter designation does not correspond to a massless system. The experiments which demonstrate that the photon has one unit of angular momentum[30,31], for example, are based on a radiative emission process. Since the dominant mechanism in such transitions is electric-dipole in character, it follows that a change in angular momentum of one unit must have occurred in the process, for both the atomic system *and the photon itself* (see Fig.

3). In the creation-annihilation hypothesis, attention is centered on the change in the atom's angular momentum, but the premise in the XBPS model is that a massless photon is present prior to the transition and thus that its angular momentum is also altered upon emission. On this basis one can conclude from the rule for the vector addition of angular momenta that the original massless photon must have possessed one of three possible J values: 0, 1 or 2. Since it .is difficult to imagine how a system with zero momentum, as must be assumed for a photon with E = 0, can have other than zero angular momentum, the finding that J = 0 is greatly preferred in the present calculations for the proposed e^+e^- tight-binding state is actually easily reconcilable with the results of the photon angular momentum measurements.

Since the electric dipole moment has negative parity, one is also led to conclude that *the photon state after emission has a different parity than the initial massless state.* This deduction would appear to contradict the presently calculated finding that the symmetry of the latter is 0^-, but closer consideration shows that the assignment of negative parity to the state of the emitted photon is perfectly arbitrary. Parity designations of particles are always based on assigning one of two possible values to some standard system, and as such it is not possible to speak of absolute parity determinations[154] on the basis of experimental evidence alone. If one believes that the photon is created from nothing, it is perhaps natural to assign even parity to the initial photon state, and consequently odd parity to a photon generated as a result of an electric-dipole transition. The present calculations suggest a definite structure for the massless photon state, with 0^- symmetry, so that in this view one is led to conclude that the state of a photon observed after an electric-dipole transition is actually 1^+, i.e. the opposite parity as conventionally assumed.

At the same time, one can point to more general discussions[155] of the dynamics of relatively light particles such as electrons or neutrinos when confined to small (nuclear-like) volumes which clearly suggest that it is highly unlikely that they exist in other than the lowest possible angular momentum state under these circumstances, which again in the case of a photon would be J = 0. Higher values of J are easily conceivable in conjunction with the translation of a tightly-bound system, however, which corresponds to the natural condition of a photon with non-zero energy. With regard to the positronium decay process, it is interesting to note that the present assignment for the photon's massless state implies that the most common process involving a singlet initial state corresponds to a $0^+ \rightarrow 0^-$ transition (see Fig. 4). Such a process is

well known to be forbidden by any radiative mechanism involving only a single photon, consistent with what is observed (Sect. II.A). A possible two-photon transition would proceed with the aid of a $1^\pm$ virtual photon state, in which case the two decay photons would possess respectively 1^+ and 1^- symmetry, i.e. of opposite parity to one another but of the same total J value. Since there is no net change of total angular momentum in the overall process, it follows that the two photons must have complementary polarizations, again as observed. On the other hand, there appears to be no definitive means of establishing their relative parity experimentally. All in all, it can be concluded that the calculations appear to be perfectly consistent with both experimental observations and fundamental theoretical considerations with regard to the symmetry properties expected for particles of light.

The variation of the energy of the 0^- state with the scaling parameter η and the exponential damping constant A is shown in Fig. 7. The minimal energy always becomes higher as A is increased, as already noted. The E vs. η curve is closely related to the schematic total energy diagram given in Fig. 5, whereby η plays much the same role as the reciprocal of the square root of the inter-particle distance r by virtue of the scaling properties of Gaussian functions. For example, the expectation value <r> approaches zero to the right of Fig. 7 as η increases. The damping of the Breit-Pauli terms (Table I) produces a sharp minimum consistent with this interpretation, and the depth of the minimum is seen to be very sensitive to the value of the constant A. Qualitatively, it is easy to imagine from this diagram that a system trapped in such a deep potential well would be extremely stable. At distances to the left of the minimum's location the energy increases sharply, passing well beyond the zero value for the separated particles. Eventually as the Gaussian exponents are decreased to their hydrogenic values ($\eta \cong 10^{-8} - 10^{-9}$), the total energy peaks and then the positronium ls minimum is reached on the basis of the same Hamiltonian. A change of state occurs along the way, however, so that the $s^{1/2} s^{1/2}$ configuration becomes most stable. At this point the mean values of the exponential damping factors are very nearly unity, but they have the advantage of allowing a variational treatment of the hydrogenic states while still retaining a form of the Breit-Pauli interactions in the Hamiltonian. At distances smaller than that of the location of the deep potential minimum in Fig. 5, the energy is seen to increase very sharply as a result of the steep decrease in the magnitudes of' the exponential damping factors in this region combined with the corresponding increase in the magnitude of the kinetic energy (see Fig. 6). Altogether, a consistent picture emerges of a tightly bound e^+e^- state

with a binding energy of exactly $2m_{oe}c^2$, resulting primarily from an exponentially damped attractive potential of relatively short range.
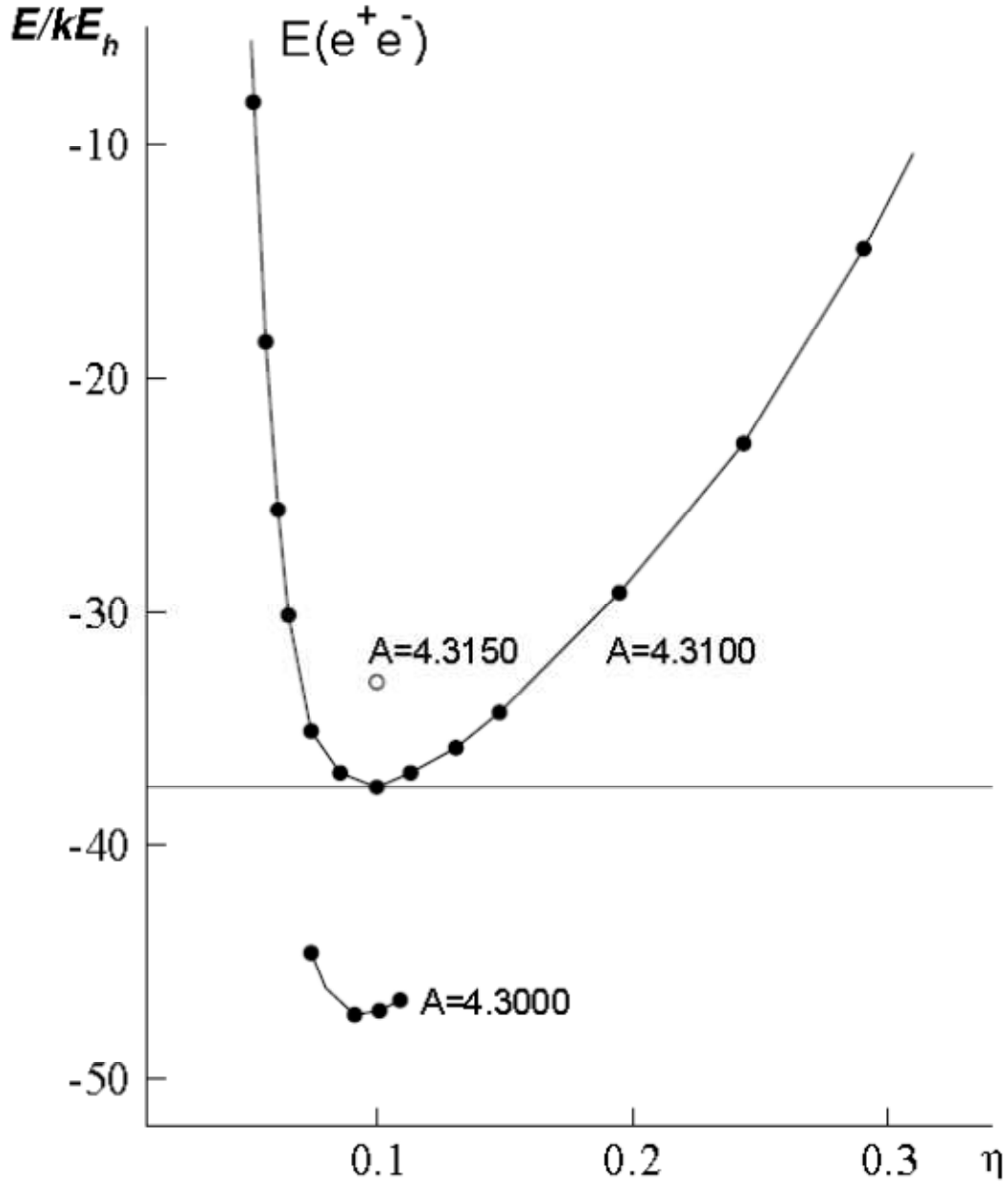
FIG. 7. Variation of the computed total energy (in khartree) of the e^+e^- system as a function of the 5s,5p basis set scaling factor η in the XBPS treatment for various values of the damping constant A. The horizontal line at the center of the diagram corresponds to the negative of the rest energy $(2m_{oe}c^2)$ of the system. A value of A is sought which leads to this energy result for the optimum choice of η. Results for several other A values are also shown for comparison.

E. CONSIDERATION OF TRANSLATIONAL EFFECTS IN THE XBPS CALCULATIONS

The calculations discussed thus far have yet to consider the magnitude of the translational component of the total energy. As discussed in Sect. VI .C, this can be done in several ways, the simplest of which in the present context involving the computation of the expectation value of $(\Sigma \mathbf{p_i})^2$ for the wavefunction already obtained. On this basis it is found that the translational energy is 1.3483×10^6 hartree, a very considerable amount. This result needs to be kept in perspective, however, when comparing with conventional calculations in which the center-of-mass motion is factored out. We have therefore carried out the analogous treatment for the hydrogen atom (employing a slightly larger 10s,5p basis), i.e. by also treating the translational and internal motion together explicitly. A total energy of

-0.4660 hartree results, which is 0.0340 hartree higher than the non-relativistic Schrödinger equation value. The expectation value of the translational energy <T> obtained with this wavefunction is 0.015 hartree, which corresponds to a mean center-of-mass momentum of 7.42 a.u. Assuming that momentum increases proportionally with r^{-1} and comparing the value of the 0^- e^+e^- expectation value Table III (Coulomb term, -3233.89 hartree) to the unit value known for the H atom ground state leads to an estimate of the translational momentum of 2.400×10^4 a.u., which upon multiplication with $c = \alpha^{-1}$ corresponds to a translational energy for a massless system of 3.29×10^6 hartree, roughly two-and-one-half times larger than the above computed value. The $p_{1/2}s_{1/2}$ form for this state helps to minimize the expectation value of T because it leads to a $<\mathbf{p_1}\,\mathbf{p_2}>$ cross term which is necessarily negative (it is proportional to $-|< p_{1/2}\,\mathbf{p}\,s_{1/2}>|^2$). In other words, it is also possible to look upon the relatively high stability of the 0^- state as resulting from its ability to minimize the translational energy with this type of wavefunction.

Since the goal of the present exercise is to obtain a state with $E_o = -2m_ec^2$, i.e. with no translational energy, it is clear that the previous optimization procedure based on <H> alone has an element of inconsistency connected with it. An alternative procedure is to minimize $<E_o> = <H> - <T>$ in the scaling procedure, choosing the damping constant A so that the lowest *internal* energy is $-2m_{oe}c^2$. This can be done in two ways, as discussed in Sect. V.B. In the first method, the variational calculations are done relative to the XBPS Hamiltonian as before, but the decision as to which η and A values to employ is based on the criterion of producing a minimum value for $<E_o>$ of the desired magnitude rather than for <H>. This procedure leads to an increase

in the optimum scaling factor to $\eta = 0.27$ (three times larger than before). Moreover, the A damping constant must be increased to 1.622 e^{-1} (roughly 50% higher than in the $\langle H \rangle$ optimization) in order to have the $\langle E_0 \rangle$ minimum vs. η to have the desired value. The value of $\langle T \rangle$ is decreased in the process to 8.0857×10^5 hartree. Numerically this result can be seen to arise from a competition between the effect of increasing A (which cuts down the influence of the short-range potential terms) and that of increasing η at the same time. The tendency for η to increase is consistent with the effect discussed qualitatively in Sect. V.F, namely that in a state which makes heavy use of short-range interactions there is a definite advantage to having the center-of-mass motion kept as small as possible. This allows the velocities of the two equally massive particles to maintain roughly the same speed and thus draw more closely together on the average.

There is a difficulty of another kind with the latter method of optimization, however, namely that its charge distributions are determined from CI secular equations involving the total Hamiltonian, whereas the criterion used for determining η and A values involves something other than the corresponding energy eigenvalue. The alternative procedure of using $\langle H - T \rangle$ directly in the variational procedure is even less satisfactory, however, because as mentioned earlier there are an infinite number of translational states with the same eigenvalue E_0, and so there is no guarantee that the lowest-energy state obtained in this type of optimization will have a low value of $\langle T \rangle$.

It needs to be emphasized that the primary goal of the present calculations is to obtain a wavefunction for a massless $e^+ e^-$ state, which means $\langle T \rangle = 0$ and $\langle H \rangle = \langle E_0 \rangle = -2m_{oe}c^2$. If the one-particle basis set is complete, both of the first two optimization procedures discussed must lead to identical results for the lowest-energy eigenvalue and corresponding wavefunction, whereas for the variational procedure based on $\langle H-T \rangle$, this result is only one of an infinite number of possibilities and thus seemingly unlikely. Nonetheless, the $\langle H-T \rangle$ optimization does lead to one important result, namely it yields a better lower limit for the value of the exponential damping constant A needed to obtain the desired lowest eigenvalue in an exact solution of the damped Breit-Pauli Schrödinger equation. The energies obtained for $\langle H-T \rangle = \langle E_o \rangle$ must be lower than those deriving from the perturbative approach using eigenfunctions of H alone to evaluate them because of the variational nature of the former procedure. This point is verified in actual calculations, and leads to an A value of 1.772 e^{-1} needed to produce the desired minimal $\langle E_0 \rangle$

value of $-2m_{oe}c^2$, some 10% larger than that obtained with the perturbation theory approach first discussed.

It should be noted that further improvements in the one-particle basis while maintaining a fixed A value will cause not only a decrease in the minimum total energy $<H>$ obtained for the e^+e^- system but also in the corresponding translational energy $<T>$. The qualitative arguments mentioned above show how this occurs in practice, namely as $<T>$ decreases, the absolute magnitudes of the attractive short-range potential terms increase as a consequence, leading to a lowering of the total energy as well. For exact wavefunctions the decrease in $<H>$ and $<T>$ must be exactly equal in going from higher to lower levels of translation, and this result can be obtained even when the internal and center-of-mass motion are not completely separated from one another.

A more thorough discussion of this topic will be given in Chapter X, but for the moment it is important to recall the arguments of the preceding chapter which indicate that the presence of a square-root form for the relativistic kinetic energy as well as the momentum dependence of the various short-range potential terms in the XBPS Hamiltonian precludes the possibility of completely uncoupling the translational motion from the internal. In particular, a transformation to the usual center-of-mass coordinate system leads to cross terms in the Hamiltonian involving both relative and translational momenta. Such terms are totally absent in the non-relativistic treatment, and as a result a perfect separation of the two types of coordinates is possible in this case. For relativistic momenta, the situation is more complicated, however. The cross terms couple translational functions of different (angular) l quantum number but the same (total momentum) |k|, so that the value of the translational energy is conserved in the process, as it must be because of the commutation of H and T (Sect. V.B>. At the same time, however, internal functions of different angular momentum quantum number also become mixed, which means that the simple product wavefunction form for the two kinds of factors is lost for large k values in such a relativistic approach. One of the main consequences of this coupling of the two types of motion is that it can cause particles of the same rest mass to remain farther away from one another on the average as a result of the increased translational energy of the combined system. This characteristic is expected to be most noticeable for states in which short-range r^{-3}-type interactions dominate.

The coupling of different l values for states of high translational energy indicated in the above discussion suggests that one-electron functions of high angular momentum may be important in describing the desired e^+e^- tight-binding state when employing basis sets whose functions depend on the coordinates of only a single particle. To test this possibility an additional series of calculations has been carried out employing a basis set consisting of 2s, 2p and 2d primitive Cartesian Gaussian functions. As before with the 5s,5p test calculations in Sect. VI.D, the same two exponents are used for each pair of functions of the same l value. A constant ratio of 2.0 is taken between these two exponent values, and it is found that optimum results are obtained when the larger of these two quantities has a value of $0.14 \times 10^8 \, a_0^{-2}$. The influence of the d functions is quite substantial, as indicated by the fact that the damping constant A must be increased to a value of $1.2534 \, e^{-1}$ relative to the 2s,2p result of $1.0540 \, e^{-1}$ mentioned earlier in order to obtain the desired binding energy of $2m_{oe}c^2$. The 0^- state continues to have the lowest energy in the full CI treatment, as can be seen from Table IV. The d basis functions occur in $d_{3/2}p_{3/2}$ 0^- products and such configurations make up 29.074% of the total CI wavefunction based on the sum of the coefficient squares computed (Table V).

The expectation value of the translational energy in this basis is 0.8896×10^6 hartree, a 37% reduction compared to the corresponding 2s,2p result. When the optimization is carried out with respect to variational $\langle H\text{-}T \rangle$ results for the 0^- e^+e^- state, one obtains a higher value for the damping constant A of $1.7300 \, e^{-1}$, as compared to $1.6995 \, e^{-1}$ in the corresponding 2s,2p treatment. Thus the gap between these results for the two types of optimization methods is reduced to $0.4766 \, e^{-1}$ with the addition of the 2d orbitals relative to the corresponding 2s,2p result of $0.6455 \, e^{-1}$. The $\langle E_o \rangle = \langle H\text{-}T \rangle$ eigenvalues for the lowest-lying states of each symmetry in the 2s,2p,2d basis are given in Table VI, and they show that although 0^- remains the most stable species, there is now a second state with negative energy, namely 1^+. Since one expects the order of pure translational states (when employing relatively small basis sets) to be $0^+ < 1^- < 2^+$..., this result is consistent with the formation of a translational continuum based on a 0^- internal state, i.e. $0^- = 0^- \times 0^+$ as lowest eigenvalue, $0^- \times 1^- = 1^+$ as second lowest.

TABLE IV. Total full CI energy (in hartree) of the lowest states of various symmetries of the e^+e^- system obtained employing the 2s,2p,2d basis with scale factor $\eta = 0.07$ and exponential damping constant $A = 1.2534\ e^{-1}$ for the XBPS Hamiltonian of Table I.

Symmetry	Lowest Root
0^+	475145.036
0^-	-37475.722
1^+	594854.649
1^-	811619.239
2^+	1181673.209
2^-	1242603.690
3^+	1409590.063
3^-	1464134.359
4^+	1793907.516
4^-	1619616.897
5^+	1965090.791

A breakdown of the various energy contributions in the 0^- wavefunction in the 2s,2p,2d basis is given in Table VII for the case in which $\langle H \rangle$ is minimized to give the desired e^+e^- binding energy. Comparison with Table III for the 5s,5p basis shows that most expectation values have decreased in absolute magnitude as a result of the addition of the d functions. The lone exception is the orbit-orbit term, which is 28% larger in absolute magnitude than before with the 5s,5p basis. The translational energy decreases far more than the total kinetic energy as a result of the addition of the d functions, so it is clear that something more delicate than a general increase in the diffuseness of the overall 0^- wavefunction has occurred.

TABLE V. Self-consistent field (same for e^+ and e^-) (a) and CI (b) coefficients for the XBPS calculation of the O_s^- ground state of the e^+ e^- system obtained employing a 2s,2p,2d basis (exponents α_1 and α_2) with scale factor $\eta = 0.07$ and exponential damping constant A = 1.2534 e^{-1}.

a)	Orbital	SCF Coefficients (e^+, e^-)	
		$\alpha_1 = 0.14 \times 10^8\ a_0^{-2}$	$\alpha_2 = 0.70 \times 10^7\ a_0^{-2}$
$1s_{1/2}$		-0.96575	1.80548
$2s_{1/2}$		2.28957	-1.70735
$1p_{1/2}$		-0.52009	1.41377
$2p_{1/2}$		1.91052	-1.38629
$1p_{3/2}$		-0.65233	1.50719
$2p_{3/2}$		1.86950	-1.28411
$1d_{3/2}$		-0.42116	1.31229
$2d_{3/2}$		1.66813	-1.11262
$1d_{5/2}$		-0.49857	1.36280
$2d_{5/2}$		1.64665	-1.05015

b)	Configuration (orbital occupations)		CI Coefficient
	e^+	e^-	
1)	$1s_{1/2}$	$1p_{1/2}$	0.427541
2)	$1p_{1/2}$	$1s_{1/2}$	0.427541
3)	$2s_{1/2}$	$1p_{1/2}$	-0.272544
4)	$1p_{1/2}$	$2s_{1/2}$	-0.272544
5)	$1s_{1/2}$	$2p_{1/2}$	0.136486
6)	$2p_{1/2}$	$1s_{1/2}$	0.136486
7)	$2s_{1/2}$	$2p_{1/2}$	-0.280952
8)	$2p_{1/2}$	$2s_{1/2}$	-0.280952
9)	$1p_{3/2}$	$1d_{3/2}$	0.263695
10)	$1d_{3/2}$	$1p_{3/2}$	0.263695
11)	$1p_{3/2}$	$2d_{3/2}$	0.093143

12)	$2d_{3/2}\ 1p_{3/2}$	0.093143
13)	$2p_{3/2}\ 1d_{3/2}$	0.167764
14)	$1d_{3/2}\ 2p_{3/2}$	0.167764
15)	$2p_{3/2}\ 2d_{3/2}$	0.197512
16)	$2d_{3/2}\ 2p_{3/2}$	0.197512

TABLE VI. Total internal full CI energy (in hartree) of the lowest states of various symmetries of the e^+e^- system obtained employing the 2s,2p,2d basis with scale factor $\eta = 0.04$ and exponential damping, constant $A = 1.730 \ e^{-1}$ for the XBPS Hamiltonian of Table I.

Symmetry	Lowest Root $< H - T >$
0^+	133281.417
0^-	-37748.278
1^+	-5469.698
1^-	137451.214
2^+	144787.077
2^-	127712.122
3^+	227313.394
3^-	272696.240
4^+	373004.635
4^-	322805.296
5^+	440007.121

TABLE VII. Energy contributions (in hartree) of various operators (see Table I for definitions) for the 0_s^- ground state of the e^+e^- system obtained employing the 2s,2p,2d basis with scale factor $\eta = 0.07$ and exponential damping constant $A = 1.2354 \ e^{-1}$ for the XBPS Hamiltonian.

Operator	Expectation Value
Kinetic energy	1832136.978
Coulomb	-3049.568
Spin-same-orbit	-322271.655
Spin-other-orbit	-683215.569
Darwin Term	4439.576
Orbit-orbit	-523387.686
Spin-spin	-342127.801
Spin-spin δ	0.000
Total Energy	-37475.725

On the basis of these results it seems safe to conclude that further extensions in the one-particle basis sets will simply lead to a gradual increase in the value of the damping constant A needed to have the total energy expectation value for the e^+e^- system equal $-2m_{oe}c^2$. It seems plausible that in the limit of basis set optimizations, the expectation value of the translational energy of the lowest-energy wavefunction will approach vanishing magnitude. Similarly there is a good indication that the symmetry of the lowest e^+e^- state will be 0^- when this limit is reached. Rather than going further with basis set optimizations for this system, however, attention will be turned to how a similar type of approach performs when applied to other two-particle systems. This study will include the other particle-antiparticle binaries mentioned above, as well as the hydrogen atom, whose theoretical treatment is normally found to be wholly similar to that of the electron-positron system treated first.

F. THE PROTON-ANTIPROTON BINARY

As pointed out in Sect. V.D, there is a scaling property for the XBPS Hamiltonian (Table I) which requires that for every e^+e^- wavefunction there exists a corresponding p^+p^- solution with an energy eigenvalue which is exactly m_{op} times greater in magnitude. The desired p^+p^- eigenvector can always be obtained from the relation:

$\Psi_{e+e-}(r) = \Psi_{p+p-}(m_{oe}r/m_{op})$. In other words for the p^+p^- system everything is played out in a coordinate system which is contracted by a factor of 1836 relative to that of e^+e^-. The same property exists for the translational energy operator T, so all results obtained above for the electron-positron system can be converted over to p^+p^-. In particular, this result allows one to employ the same value for the exponential damping constant as before, thereby giving this quantity more of a general character than might otherwise be assumed.

It might come as a surprise to see that the required binding energy of $2m_{op}c^2 = 1.876$ GeV comes from short-range effects which are generally associated with magnetic interactions, i.e. Breit-Pauli terms, because the magnetic moment of the proton is so small compared to that of the electron. Closer examination of the effects involved, however, underscores the fact that the relatively large mass of the proton is actually quite beneficial in forming a tight-binding state which takes extensive advantage of such short-range interactions. To begin with, there is the

obvious fact that the kinetic energy associated with a given momentum value is substantially smaller for a proton than for an electron. The occurrence of the mass in the denominator of the exponential arguments of the XBPS Hamiltonian (Table I) is an even more significant factor, however, since it leads to a drastic reduction in the exponent of the damping function for a given momentum *vis-a-vis* that for a lighter particle (Sect. V.D). This means the short-range potential terms are still going down in energy for inter-particle distances much shorter than the $r \cong \alpha^2$ value favored by the e^+e^- system (Fig. 5). As usual, the exponential damping ultimately prevents the situation from getting out of hand, but the corresponding p^+p^- energy minimum occurs for $r \cong \alpha^2 / m_p$ or 2.9 x 10^{-8} a_0 (1.5 x 10^{-18} m), making this probably one of the shortest-range interactions occurring in nature.

The small magnetic moment of the proton only means that magnetic-type interactions occurring at typical atomic separations (bohr radius a_0) are nearly negligible compared to those of electrons. By contrast, at the much smaller inter-particle distances preferred by the p^+p^- system, even the Coulomb attraction is far from negligible. For $r = \alpha^2 / m_p$ the expectation value for this relatively long-range interaction would be -938 MeV, for example, which would be one-half of the total binding energy of the assumed p^+p^- system. From this point of view, the corresponding Breit-Pauli energy contributions, though considerably larger at such distances, do little more than counter the system's enormous kinetic energy, which at $r = \alpha^2 / m_{op}$ can be estimated at 254 GeV (using $p \approx r^{-1}$ for each particle). The XBPS calculations for e^+e^- indicate that the total p^+p^- kinetic energy is somewhat smaller than this value, with an estimate of 99 GeV resulting from multiplication of the pertinent value in Table III by the m_{op}/m_{oe} ratio required by the scaling procedure.

By either of the above measures the cancellation of the kinetic energy due to the Breit-Pauli terms is seen to be almost total, with a net binding energy contribution of less than 2% compared to the total kinetic energy in either system. As is evident from Fig. 6, it is difficult to imagine that anything other than an exponentially damped potential could achieve such a delicate balance on a general basis. On the other hand, one of the arguments against assuming that electrons are actually present in nuclei (Sect. IV.A) has been that no such potential can supposedly be found. The present experience strongly suggests that this position should be reevaluated. The Hamiltonian employed in the above investigation allows for an equivalent variational treatment of systems with binding energies varying between 1.0 MeV and nearly 2.0

GeV, respectively, as well as a reliable description of conventional atomic systems by virtue of its close association with the Dirac and Breit-Pauli formulations of quantum electrodynamics interactions.

G. THE INTERACTION OF A PROTON AND AN ELECTRON

It remains to be considered whether a strongly bound state also results from an analogous treatment of the electron-proton system. The answer is clearly that *no such second minimum is found corresponding to an inter-partic1e distance smaller than 1.0 bohr* (Table VIII). Moreover, the reason for this distinction between the e^+e^- and p^+e^- systems is easily understandable from the earlier calculations with the XBPS Hamiltonian. The results of Table III for the e^+e^- system are changed dramatically by the substitution of a proton for the positron because of the disparity in the masses of these two particles. The magnitudes of the various Breit-Pauli terms are substantially reduced upon making this substitution because of the importance of the q/m_o factors in the corresponding operators (Table I). Only the spin-same-orbit and Darwin terms with the electron's $(e/m_{oe})^2$ prefactor survive for all practical purposes. By contrast the kinetic energy is only halved because of the change in particle mass, thereby destroying the delicate balance mentioned in the last section between the attractive and repulsive components of the total energy for both the e^+e^- and p^+p^- binary systems.

TABLE VIII. Total full CI energy E (in hartree) for the lowest state of the p^+e^- hydrogen atom system obtained employing the 3s,2p,2d basis for various scale factors η and a fixed value of the exponential damping constant $A = 1.2648$ e^{-1} for the XBPS Hamiltonian of Table I (* indicates minimum).

η	E
0.30	710408.14
0.20	560400.24
0.10	374890.12
0.08	329689.40
0.07	305334.70
0.06	279471.535
0.02	148434.705
0.005	64715.8077
5.0×10^{-4}	12839.8600
5.0×10^{-5}	1623.358959
5.0×10^{-6}	159.118214
5.0×10^{-8}	0.173309
3.0×10^{-8}	-0.195469
2.0×10^{-8}	-0.339412
1.0×10^{-8}	-0.425769
9.0×10^{-9}	-0.428911
8.0×10^{-9} (*)	-0.430521
7.0×10^{-9}	-0.430349
6.0×10^{-9}	-0.428057
5.0×10^{-9}	-0.423152
4.0×10^{-9}	-0.414828
3.0×10^{-9}	-0.401501
1.0×10^{-9}	-0.333257
5.0×10^{-10}	-0.281758
1.0×10^{-10}	-0.168834

The results of Table III correspond to an e^+e^- inter-particle distance on the order of $r \cong \alpha^2$, but as we have seen, the maximum binding for the p^+p^- system occurs when the corresponding

separation is m_{op}/m_{oe} times smaller. At this distance all energy values in Table III can be multiplied by a factor of 1836 to obtain the corresponding p^+p^- results because of the scaling property discussed in Sect. V.D. If the mass of the electron is substituted for that of the antiproton at such a small distance, the kinetic energy is only slightly reduced according to the classical Einstein free-particle expression. On the other hand, the corresponding damping factors for the electron are very close to zero for such high momenta, and so only the two BreitPauli terms with the $(e/m_{op})^2$ pre-factor are left (relatively) unaffected. As a result the required cancellation of the attractive and repulsive terms no longer takes place and an extremely large positive total energy results for the p^+e^- system in this range of inter-particle separation. No choice of basis set succeeds in binding the electron and proton more strongly together than is the case for the hydrogenic 1s state. For example, the use of electronic functions which are optimal for the strongly bound e^+e^- system along with proton functions which are 1836 times more compact (corresponding to their optimal p^+p^- counterparts) also produces only negative binding energies.

These results can again be understood on a qualitative basis by considering the p^+e^- system in its own center-of-mass coordinate system. The condition of equal and opposite momenta (see Sect. V.F) now requires that the electron move at a speed which is 1836 times greater than that of the proton. This requirement forces the particles to avoid each other by wide margins if high speeds are to be maintained, making the type of short-range momentum-dependent interaction represented by the Breit-Pauli interactions very ineffective in producing binding under these circumstances. Such distinctions in the relative particle masses for binary systems are of only minor importance when the Coulomb interaction dominates, however, *which explains why we normally regard* positronium as just another *hydrogenic system.* By simply decreasing the values of the exponents employed for the Gaussian basis functions, one eventually finds that the variational treatment of the XBPS Hamiltonian leads to binding energies in the 0.5 hartree range (Table VIII) expected for the ground state of the hydrogen atom. No amount of exponent optimization produces a second minimum at shorter inter-particle separations for this system, in marked contrast to what is observed when employing the analogous Hamiltonian for the e^+e^- and p^+p^- systems.

H. EXTENDING THE XBPS MODEL TO NEUTRINO INTERACTIONS

In Sect. V.E it was remarked that if the rest mass of the neutrino is exactly zero, its charge-to-rest-mass ratio might be non-vanishing despite its lack of electric charge. By providing the XBPS Hamiltonian with such q/m_o factors in the various exponentially damped Breit-Pauli terms it is then conceivable at least in theory for the neutrino to undergo attractive interactions with other particles. In this section we will test this possibility by means of explicit calculations. The first question that arises is what value to use for q_v/m_{ov}. Consideration of the scaling arguments discussed in Sect. V.D shows, however, that this choice is not critical for the $\nu\bar{\nu}$ system itself. If we simply assume that the charge-to-mass ratios for the neutrino and antineutrino are equal but of opposite sign, we obtain a Hamiltonian (see Table I with the following representative terms:

$$H (p, r, q_v, m_{ov}) = p\alpha^{-1} - (q_v/m_{ov})^2\alpha^2 r^{-3} F(p, q_v, m_{ov}). \qquad \text{VI.2}$$

This operator is identical in form to that employed in Sect. V.D, except that all the terms with factors of q_i or m_{oi} which do not appear as ratios of one another are missing because of the assumed vanishing magnitudes of the neutrino's charge and rest mass.

With this special form of the XBPS Hamiltonian it is possible to scale the coordinates in a slightly different manner than before, namely with

$$p´ = (q_v/m_{ov})(Q/M_o)^{-1} p$$
$$\text{and } r´= (q_v/m_{ov})^{-1} (Q/M_o) r, \qquad \text{VI.3}$$

so as to obtain the following result:

$$H(p, r, q_v, m_{ov}) = (q_v/m_{ov})^{-1}(Q/M_o)$$
$$x\{p´/\alpha^{-1} – (Q^2/M_o^2)\alpha^2 r´^{-3} F(p', Q, M_o)\} \qquad \text{VI.4}$$
$$= (q_v/m_{ov})^{-1} (Q/M_o) H (p´, r´, Q, M_o),$$

where $H (p´, r´, Q, M_o)$ is the corresponding Hamiltonian in the primed coordinate system for particles with a different value of $|q/m_o|$. The operations are exactly the same as in the mass

scaling procedure of Sect. V.D, but the relative charge-to-mass ratios are involved instead of relative masses. It must be recalled, however, that the simple relation between the two different Hamiltonians in this case only holds for $q_v = m_{ov} = 0$. Under these conditions the lowest possible energy eigenvalue for the $v\bar{v}$ system, *i.e.* $2m_{ov}c^2$. is exactly zero as well. As a result, a wavefunction $\psi(r)$ satisfying the corresponding Schrödinger equation with this eigenvalue can be converted by means of the above coordinate transformation [eq. VI.3] into an eigenfunction $\psi(r')$ of the primed Hamiltonian, which by virtue of the above scaling property also possesses the desired vanishing energy. In short, if we can find a Hamiltonian of this type which gives $E = 0$ as the lowest eigenvalue for the $v\bar{v}$ system with a given $|q_v/m_{ov}|$ value, we can easily repeat the process for any other charge-to-mass ratio. It can be noted that for this situation to hold, it is essential that the argument of the damping exponential operator F also contain a q/m_o factor.

The next point that merits discussion is whether it makes sense to talk of a bound system with the same energy as its separated products, in this case E=0 . The answer is clearly yes as long as the combined and dissociated systems are separated by a reasonably large energy barrier. With reference to Fig. 5 for the e^+e^- system, it is easy to construct an analogous total energy curve for the $v\bar{v}$ system which satisfies this requirement (Fig. 8). Because of the absence of the long-range Coulomb interaction in this case, we must expect that the energy first goes up as the neutrino and antineutrino approach each other from a long distance. The kinetic energy itself rises more slowly with decreasing separation than for the e^+e^- system because it varies linearly with momentum for systems with no rest mass. As a result, one doesn't expect the atomic-like potential minimum of Fig. 5 in this case, but rather that the energy should increase steadily until very short inter-particle separations of roughly $r \cong \alpha^{3/2}$ (for $q_v/m_{ov} = 1.0$ a.u.). A maximum seems likely at that point, similarly as for e^+e^- , because the Breit-Pauli short-range interactions start to change more rapidly than the kinetic energy. After this point it is simply necessary that the exponential damping halt the attractive tendency when the total energy has again reached the value of zero. The kinetic energy begins to dominate once more at still smaller separations, so that a potential well can be formed in which the $v\bar{v}$ system can exist indefinitely in the absence of external forces.

E/E_h

$E(\nu\tilde{\nu})$

Photrino

$r_{min}= \alpha^2(q_\nu/m_{0\nu})^2$

$E_{min}=0$

0

$r^{-1}= p/a_0$
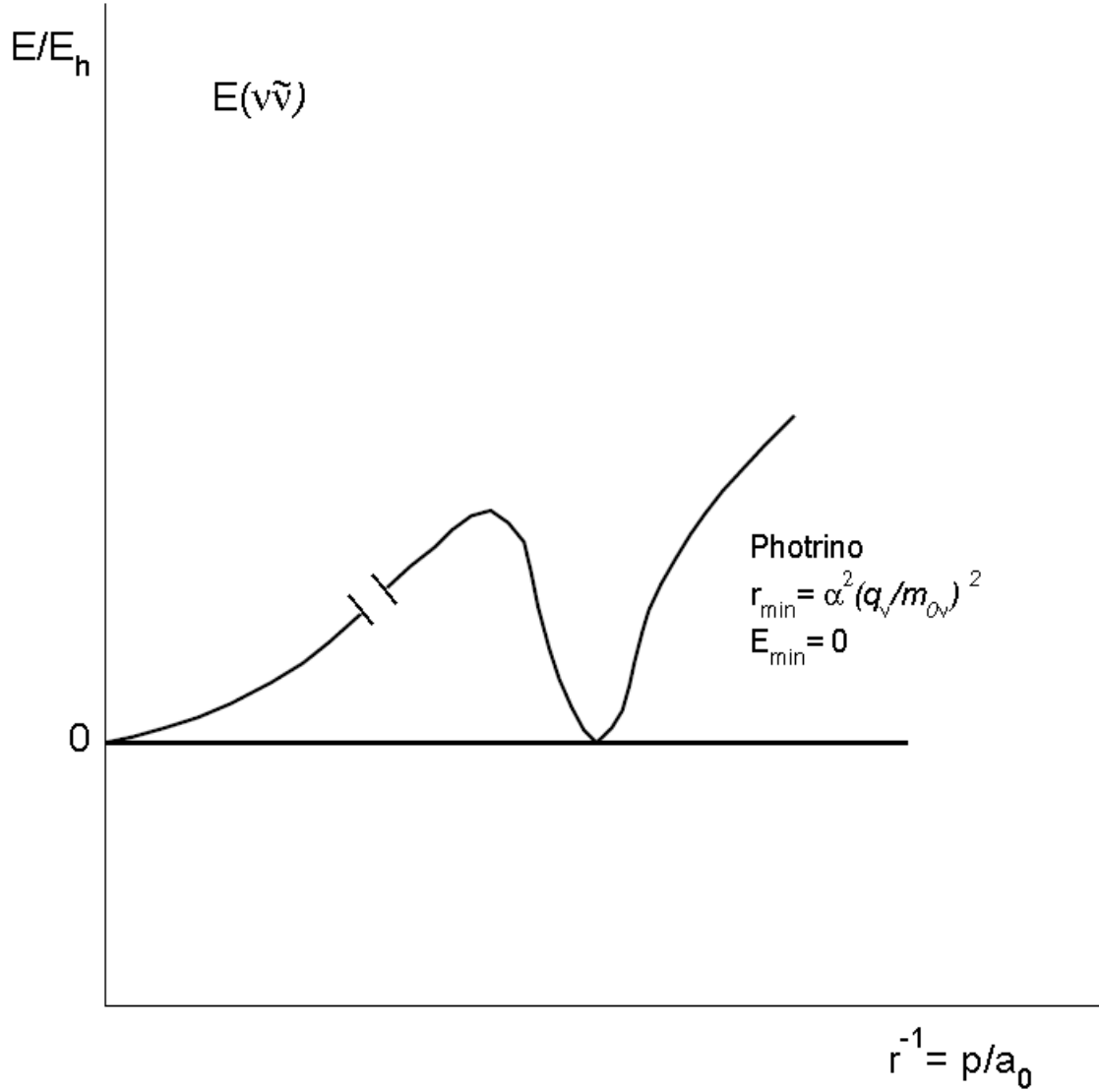
FIG. 8. Schematic diagram showing the proposed variation of the $\nu\,\overline{\nu}$ system's internal energy as a function of the reciprocal of the distance between the two constituent particles. Only one minimum is expected, in contrast to the e^+e^- case shown in Fig. 5, at which point the total energy vanishes exactly, i.e. corresponding to a binding energy of $2m_{0\nu}/c^2$ for the tightly bound $\nu\,\overline{\nu}$ (photrino) system.

The present scaling argument tells us that it really does not matter what value is chosen for $|q_\nu/m_{0\nu}|$ to demonstrate this effect, but to obtain the most straightforward possible comparison with the e^+e^- system, it is reasonable to take it to be unity. If the former 5s,5p basis is employed (see Sect. VI.D) in the corresponding XBPS calculations, it is found that the zero-energy minimum occurs for a damping constant A which is only slightly smaller than the corresponding e^+e^- and p^+p^- value (Fig. 9). This result is easily understood if one uses the optimum scale factor and A value derived from the e^+e^- calculations, for which the lowest-energy eigenvalue is

$-2m_{oe}c^2$. This means that the damped Breit-Pauli terms (Table I) will have the same magnitudes for the $\nu\bar{\nu}$ system (with $|q_\nu/m_{o\nu}|=1.0$ a.u.) as for the e^+e^- system in Table III. The Coulomb component is missing, however, as are the m_oc^2 terms in the kinetic energy. The two electronic $m_{oe}c^2$ terms appearing with minus signs in the kinetic energy expression for e^+e^- make up the entire difference in the assumed binding energies of the two systems, so we can effectively ignore them in the following discussion. This means that there are only two effects remaining which can cause a non-zero binding energy for $\nu\bar{\nu}$ when employing the same damping constant as before for e^+e^-. One of these favors the latter system (Coulomb attraction), while the other favors $\nu\bar{\nu}$ [i.e. pc vs. $(p^2c^2 + m^2c^4)^{1/2}$]. Of these, the Coulomb effect is larger in magnitude for the inter-particle distance range in question, but the energy difference (2680 hartree) is relatively small compared to the e^+e^- binding energy. Thus to obtain an exactly zero binding energy value for the $\nu\bar{\nu}$ system in the same basis set, it is only necessary to lower the value of the damping constant A by 0.0007 e^{-1}.

This result ignores translational effects, however, and so it is interesting to carry out the optimizations based on the E_o expectation value, $<H> - <T>$, rather than on that of the total energy alone (see Sect. VI.E). As before with e^+e^-, when this is done the required value of A must increase in order to obtain the same binding energy as before. If the same value of A (1.7725 e^{-1}) is employed which gives the correct $2m_{oe}c^2$ binding energy value of -37557.7 hartree for e^+e^- using the $<E_o>$ criterion, a total internal energy of 897.2 hartree results for $\nu\bar{\nu}$, which is about three times smaller than in the analogous treatment employing the optimal A value (1.0775 e) for e^+e^- based on the $<H>$ computations.
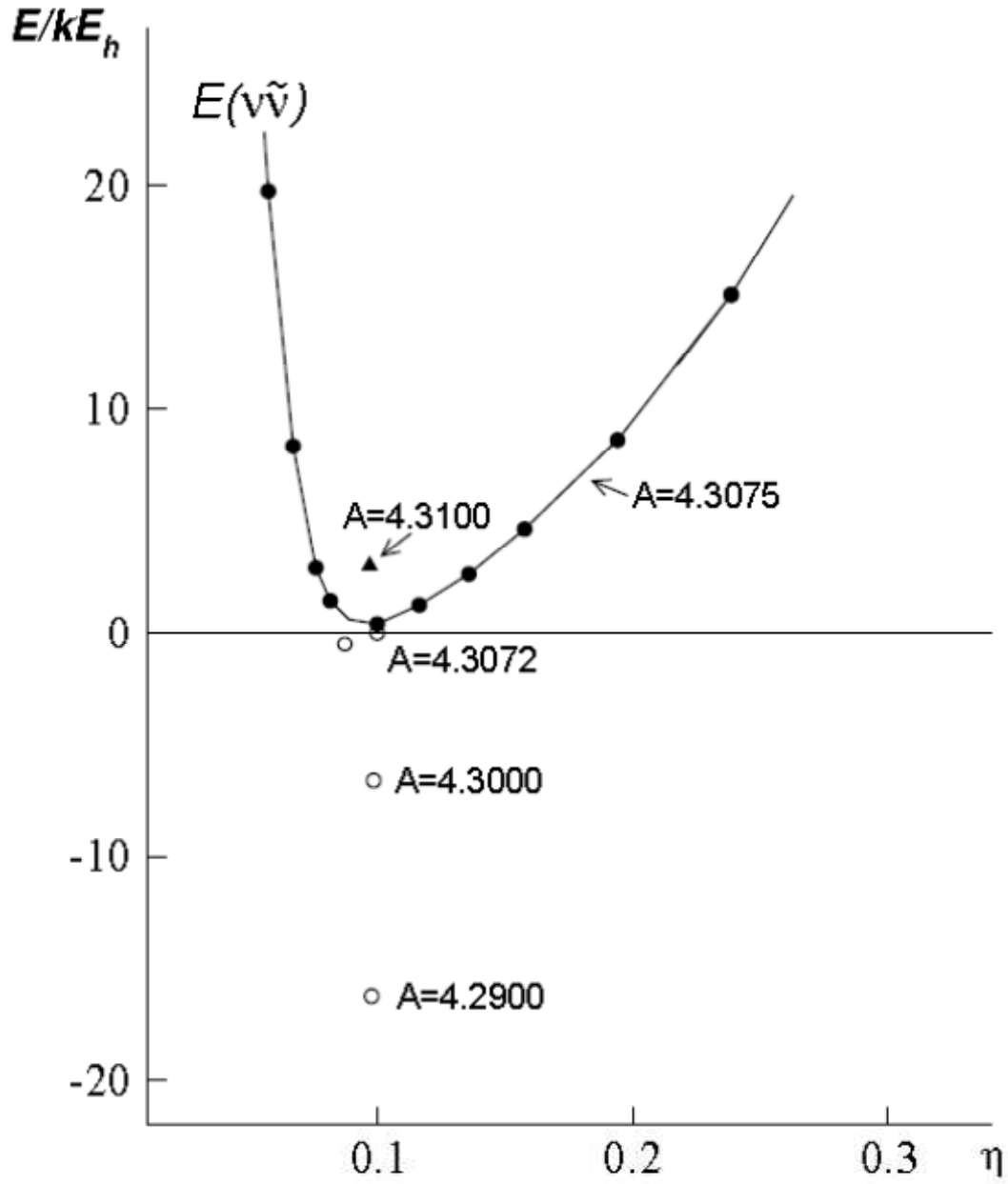
FIG. 9. Variation of the computed total energy (in khartree) of the $\nu\bar{\nu}$ system as a function of the 5s,5p basis set scaling factor η in the XBPS treatment for various values of the damping constant A. The horizontal line at the center of the diagram corresponds to the null rest energy $(2m_{ov}c^2)$ of the system. A value of A is sought which leads to this energy result for the optimum choice of η. Results for several other A values are also shown for comparison.

In view of the fact that the present full CI calculations fall short of providing exact solutions of the Schrödinger equations under consideration, it seems justified to conclude that for all practical purposes the same damping constant in the XBPS Hamiltonian leads to minimal binding energies for the three particle-antiparticle binary systems studied which are each equal to $-2m_oc^2$, in accordance with the expectations of the theory of special relativity. This result is exact for e^+e^- and p^+p^-, but only accurate to a good degree of approximation for the $\nu\bar\nu$ system based on the calculations carried out thus far. Moreover, the second scaling theorem discussed above [eq. VI.4] shows that this situation holds for any choice of the charge-to-mass ratio assumed for the neutrino. The latter finding thus offers a means of describing more than one kind of neutrino in the present model, consistent with what has been inferred from the results of various experiments[78] and other theoretical considerations[79] (see Sect. IV.B).

I. NON-IONIZING PROPERTY OF NEUTRINOS

The total energy curve of Fig. 8 for a $\nu\bar\nu$ interaction derives its attractive characteristics from the short-range Breit- Pauli terms in the XBPS Hamiltonian by virtue of an assumed non-zero charge-to-mass ratio for the constituent particles. This assumption needs to be reconciled with the observation that neutrinos have essentially no magnetic moment. In at least one sense the total energy variation shown in Fig. 8 is consistent with the experimental findings related to this issue, however, namely with the lack of ionization exhibited by neutrinos and especially their ability to penetrate essentially unhindered through dense matter[55].

For a particle to cause ionization it is necessary that it be attracted to an electron (or other charged system) to a close proximity. Because of the lack of a Coulomb interaction the total energy for the hypothetical $\nu\bar\nu$ system should rise as the two constituents come closer together. Before the attractive Breit-Pauli terms can reverse this trend, it is necessary that the neutrinos approach each other very closely, to at least $r = \alpha$. The possibility of undergoing a long-range attractive force thus clearly distinguishes electrons (Fig. 5) from neutrinos (Fig. 8). If we look upon ionization as requiring some kind of orbital motion for the colliding (point) particles, this distinction can be crucial since it means that the centrifugal force must exceed any opposing attractive force in the $e^-\bar\nu$ or $\nu\bar\nu$ interactions at all but extremely small inter-particle separations. In this case, a hyperbolic trajectory would be expected almost universally. There is always a

large centrifugal barrier which needs to be overcome, and the particles must come very close to one another before this is feasible, even if a fairly large q/m_o value is assumed for the neutrino. An automobile with such a total energy profile could race down the highway with reckless abandon, certain to be repelled whenever it came very close to any other object. The only danger would be that it might approach something so closely at extremely high speed that the short-range attractive force would finally be able to overcome the repulsive force. Such a picture is very reminiscent of the behavior of neutrinos in the Reines-Cowan experiment[55], in which an extremely small cross section for neutron formation of ~10^{-43} cm^2 is observed.

The conclusion that a non-zero q/m_o value necessarily implies a non-zero magnetic moment would also seem to overlook the possibility that the mass of a neutrino is only zero at rest. The ratio of the charge to the *relativistic* mass is a more relevant quantity in determining the results of magnetic interactions, and this is exactly zero whenever the neutrino is in motion because no corresponding change in its electronic charge occurs as a result. The dominant formula used in describing cyclotron dynamics, $p = mv = qBr$, leads to the conclusion that a chargeless particle cannot achieve orbital motion with a finite radius, for example. The key assumption in formulating the XBPS Hamiltonian is that the coupling constants are ratios of charge to rest mass, and without this provision the short-range minimum in either Fig. 8 or 9 would not be possible. In general, it should not be forgotten that virtually everything one knows about electromagnetism stems from experiments with individually charged particles (see also Sect. V.E), even if the macroscopic system being observed possesses zero *net* electric charge. A suitably relativistic treatment of electromagnetism emphasizes that electric and magnetic fields are always inextricably intertwined[156], so the inability of either producing or being affected by an electric field is at least suggestive that a chargeless particle (as opposed to a collection of charged particles with no net charge) is not affected by magnetic fields either.

Support for this view also comes from the classical experimental result that the magnetic force is no more short-ranged than its electric counterpart. The force per unit length on an electric wire, for example, varies as the product of its current and that of a neighboring wire and is inversely proportional to the first power of the distance between them[157]. There is little hint of a short-range effect in this behavior. The theoretical explanation for this result lies in the (nearly) constant velocities of the electrons involved in such experiments, which implies that the quantity $mvr/r \equiv l/r$ is constant as well. In quantum mechanical interactions the more typical situation is

that l (or s) is constant, so that an energy term of order $l^2 r^{-3}$ varies as the inverse cube of the inter-particle distance rather than as r^{-1}. Since the Lorentz transformation is linear[36,158], it is not possible to change the distance dependence of an interaction simply by moving with a constant velocity away from the event. This circumstance makes it possible for electric and magnetic fields to be transformed among one another despite their apparently different mathematical forms. The fact that particles don't always move at constant velocities opens up the possibility that related short-range interactions also exist, however, as suggested by the appearance of the Breit-Pauli operators[102]. This eventuality raises interesting questions about the manner in which the damped Breit-Pauli terms in Table I vary upon application of a Lorentz transformation, and these merit careful consideration. For the present, however, we can conclude that a non-zero charge-to-rest-mass ratio for the neutrino does offer a way of rationalizing its extreme penetrability through dense matter, while at the same time leaving open the possibility of its undergoing a strong interaction under certain well-defined circumstances.

J. RESULTS OF CALCULATIONS FOR OTHER BINARY SYSTEMS

There are 21 distinct binary systems that can be formed from the proton, electron and neutrino and their respective antiparticles. Of these three are of the particle-antiparticle type already discussed, while the other 18 divide into pairs related to one another by charge conjugation. One such example is $p^+ e^-$, which has the same Hamiltonian as $p^- e^+$ by virtue of the fact that the charges of the respective constituents appear as products of one another in all interactions (or as an absolute value in the exponential damping functions). Three of the remaining pairs are diagonal cases involving two identical particles. No binding is expected as a result, in accord with the results of explicit calculations. The various Breit-Pauli terms are attractive for certain angular momentum states, so this is not such an obvious result. The strength of the attraction is never enough to outweigh the kinetic energy contributions, however. Another three pairs are easily dispensed with as well, namely $p^+ \nu$, $p^+ \overline{\nu}$ and $p^+ e^+$ and their charge-conjugated partners. If we assume a negative q/m_o value for ν, the $p^+ \nu$ system is seen to be quite similar to $p^+ e^-$, at least as long as the absolute value of the charge-to-mass ratio is close to unity. In all cases it is assumed that the damping constant A in a given basis is the same as has been found in the $e^+ e^-$ calculations discussed above.

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This leaves two such pairs of binary systems unconsidered, $e^-\nu$ and $e^-\bar{\nu}$ and their charge-conjugated positron systems. Of these, only the ones with q/m_o values of opposite sign for their constituents are interesting in view of the experience with the other binary systems. Because of the fact that electrons and antineutrinos commonly appear together as decay products of neutrons, it is tempting to associate a positive q/m_o value with $\bar{\nu}$. The similarity to e^+e^- is expected to be particularly great if this quantity is assumed to be unity, in which case the results of Table III again become pertinent. Especially if the translational energy is excluded from consideration, it has been found above that e^+e^- and $\nu\bar{\nu}$ are characterized by very nearly the same wavefunction in their respective lowest-energy states. The maximum binding energy of the $e^-\bar{\nu}$ system can thus be anticipated to fall midway between the corresponding values for e^+e^- and $\nu\bar{\nu}$. This expectation is very nearly fulfilled in explicit calculations with the XBPS Hamiltonian in which it is assumed that $q/m_o = 1.0$ a.u. for the $\bar{\nu}$ species. A binding energy of -15872.194 hartree is computed when the same 5s,5p basis is employed which was found to be optimal for both e^+e^- and $\nu\bar{\nu}$ (see Sects. VI.D and VI.H).

Since it seems certain that no such bound $e^-\bar{\nu}$ system actually exists (it would correspond to the mutual annihilation of the two particles in the conventional description for particle-antiparticle pairs), it can be concluded that such a high value for the $\bar{\nu}$ charge-to-mass ratio is unacceptable. We can eliminate any computed binding with the electron by decreasing the above q/m_o value, however, and repeating the scaling optimization procedure for the corresponding $e^-\bar{\nu}$ system (while again maintaining the damping constant A at its previous value). The resulting minimal energies .are plotted in Fig. 10 as a function of the assumed q/m_o value. There is a simple relationship between this quantity and the optimum scaling parameter η, namely they are found to be inversely proportional to one another. This result is consistent with the second scaling theorem discussed in Sect. VI.H, which shows that the optimal value of η is inversely proportional to the square of the $|q/m_o|$ value assumed for the neutrino in a treatment of the hypothetical $\nu\bar{\nu}$ system.
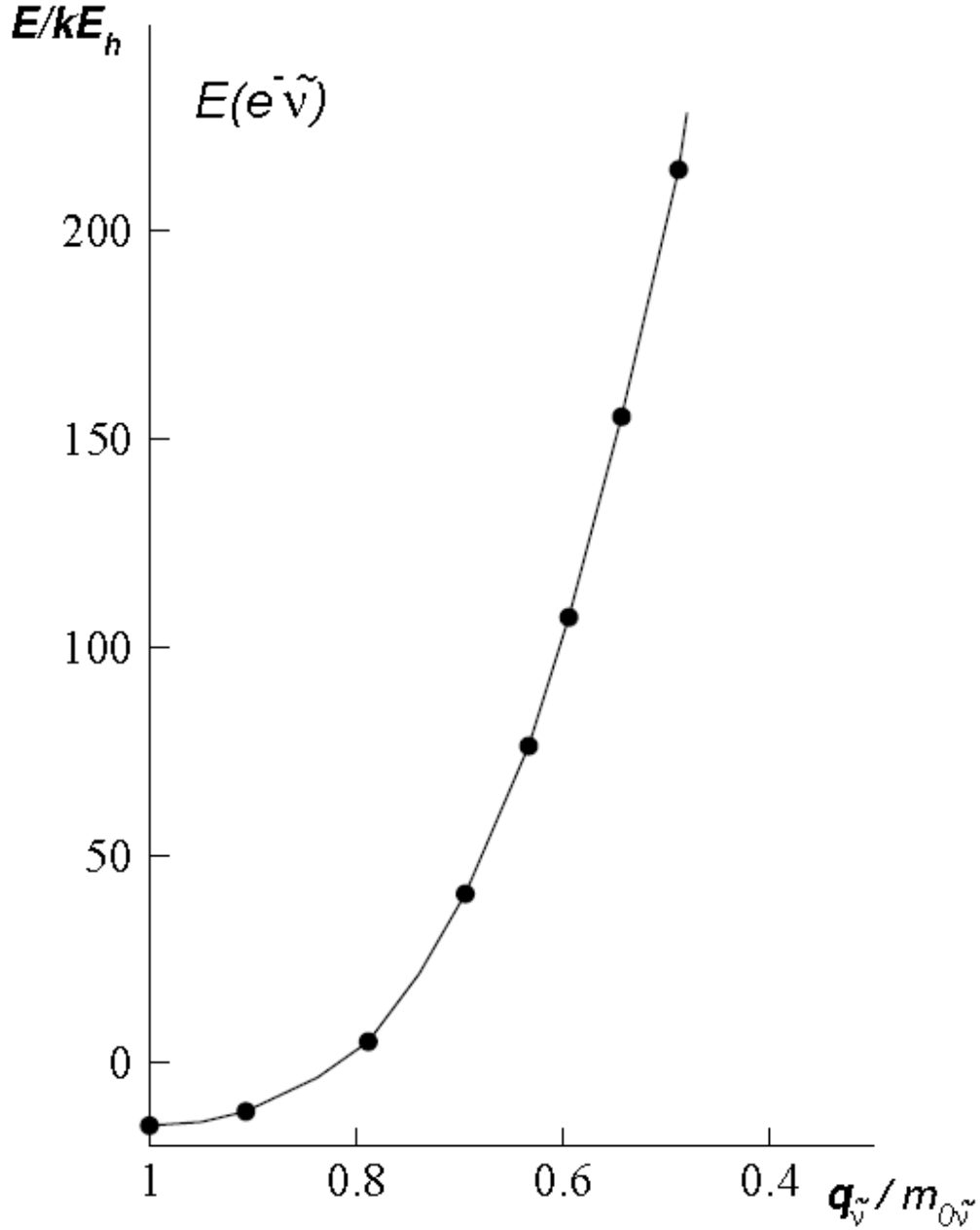
FIG. 10. Variation of the minimum value of the computed total energy (in khartree) of the $e^-\tilde{\nu}$ system (obtained by optimizing the scale factor η) in the XBPS treatment employing a 5s,5p basis set, given as a function of the antineutrino charge-to-rest-mass ratio q/m_0. The value of the exponential damping constant A assumed throughout (1.0775 e^{-1}) is taken from the results of the analogous e^+e^- calculations in the same basis.

On the basis of the above considerations it appears likely that the $|q/m_o|$ value assumed for the neutrino and antineutrino' cannot be greater than 0.7 a.u. and still avoid the prediction of a bound $e^-\bar{\nu}$ system in nature. A value exceeding zero by a good margin is still tenable on this basis, however, since it would allow for a $\nu\bar{\nu}$ system with a zero binding energy which is

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nonetheless separated by a large barrier from its dissociation products. The above scaling arguments show that the inter-particle distances involved must decrease as the value of $|q_v/m_{ov}|$ is lowered. It also should be noted that the symmetry of the lowest-energy $e^-\bar{v}$ state is also found to be 0^-, so that the analogy with the e^+e^- tight-binding state appears to retain its validity over a wide range of $|q_v/m_{ov}|$ values. There is a tendency to emphasize the $p_{1/2}$ character of the electronic function over $s_{1/2}$ as the antineutrino charge-to-rest-mass ratio decreases, as seems reasonable from the nature of the Breit-Pauli terms and the requirement of minimizing the total energy for the resultant $e^-\bar{v}$ system (the charge-conjugation symmetry is clearly missing for this binary system).

VII. NUCLEAR BINDING IN THE XBPS MODEL

The questioning of the creation-annihilation hypothesis considered in the previous chapters has led to an alternative interpretation of particle-antiparticle interactions in terms of a Schrödinger equation whose Hamiltonian contains momentum-dependent short-range operators. According to this model the electron and positron can be bound so strongly to one another that there is a total loss of mass relative to their respective free-particle states. Since nuclear binding processes are well known to be accompanied by distinctions in the total masses of products and reactants, in accordance with the predictions of the theory of special relativity, it is natural to speculate that the same types of interactions might be involved in the e^-e^+ interaction. The exponential form of the attractive potentials employed in the XBPS Hamiltonian follows at least partially from this line of reasoning.

There is another similarity connecting these two types of phenomena as well, however, which again is tied up with the supposition that matter can be created and destroyed by the gain or loss of energy. As discussed in Sect. IV.A, the accepted view of the role of the electron and antineutrino in nuclear interactions is that they are created whenever a neutron decays but that they are not present in the bound nuclei themselves[52]. The alternative interpretation which will be pursued in the present chapter is that the neutron is akin to a tri-atomic molecule, i.e. composed of a proton, an electron and an antineutrino. It has negative binding energy and is thus analogous to an excimer system commonly encountered in molecular physics studies. A key element in the ensuing theoretical model is that neutrinos are capable of being strongly attracted

by other particles, but only at very short range. The $e^-\bar{\nu}$ model system treated in the last section is the starting point of this investigation.

A. INITIAL CALCULATIONS OF THE $p^+e^-\bar{\nu}$ SYSTEM

The XBPS Hamiltonian (Table I) has a single free parameter, the exponential damping constant A, and it has been fixed by the requirement that the binding energy of each of the three particle-antiparticle binary systems be $2m_oc^2$ in a full CI treatment employing a given one-particle basis set. Otherwise, all that is needed to completely define a system's Schrödinger equation are the electric charges and rest masses of the component particles. In the case of the antineutrino $\bar{\nu}$ these are assigned to be zero in each instance, but the possibility that their ratio has a non-zero value is left open as a means of explaining this particle's role in the nuclear binding process. The actual value for the antineutrino's charge-to-rest-mass ratio has not been specified as yet, primarily because it has been found that the requirement of a vanishing binding energy for the $\nu\bar{\nu}$ binary system can *be satisfied for any choice of this quantity's magnitude.* A potential solution to this problem is provided by the "tri-atomic hypothesis" for the neutron's structure, however. Accordingly, we will simply demand that the magnitude of the antineutrino's q/m_o value be such as to lead to the experimental total energy of the neutron when solving the XBPS for the $p^+e^-\bar{\nu}$ system to which it is assumed to correspond in the present model. Specifically, the binding energy of the latter system should be equal to -28758 hartree (0.7825 MeV), which corresponds to the difference in the rest masses of the proton-electron combination and that of the meta-stable neutron.

The first series of calculations carried out to investigative this hypothesis employs the simple 2s,2p Gaussian basis mentioned in Sect. VI. D (exponents of 2.0 x 10^8 and 1.0 x 10^8 a_o^{-2} in both cases). A scale factor η is then defined as before to be optimized so as to obtain the minimum energy possible for a given choice of $q_\nu/m_{o\nu}$. Strictly speaking, only a local minimum is of interest in this case, since the desired energy is greater than that of the separated products, but the range of η involved is anticipated to be in the same neighborhood as for the e^+e^- calculations considered earlier. The minimization is carried out in terms of the total energy, although the magnitude of the corresponding translational energy will also be determined. Accordingly, the value chosen for A is 1.0540 e^{-1}, since it gives the correct binding energy in the

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analogous treatment for the e^+e^- system (Sect. VI. D). For all q/m_o values for $\bar{v}$ it is found that the lowest energy eigenvalue in the interesting range of η occurs for a state of $1/2^-$ symmetry. A minimum with close to the desired total energy (+28900 hartree) occurs for a scale factor η of 0.18 and a q/m_o value of + 0.5733 a.u. The corresponding variation of energy with η is shown in Fig. 11, while the dependence of the minimal total energy as a function of the assumed q/m_o value for $\bar{v}$, i.e. as obtained by optimizing η in each case, is given in Fig. 12.
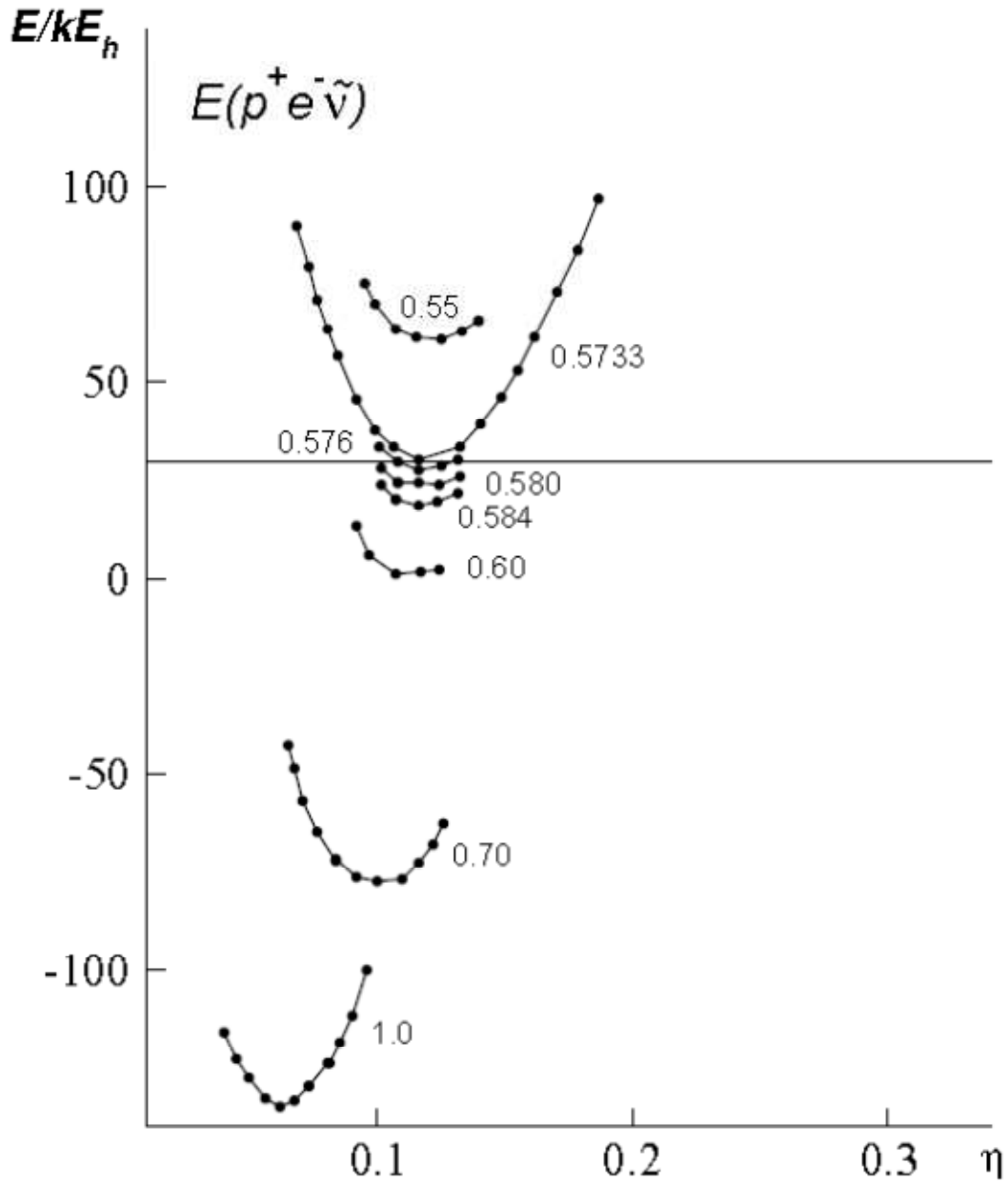
FIG. 11. Variation of the computed total energy (in khartree) of the $p^+ e^- \bar{V}$ (neutron) system as a function of the 2s,2p basis set scaling factor η in the XBPS treatment for various values of the antineutrino charge-to-rest-mass ratio q/m_o. The horizontal line at the center of the diagram corresponds to the experimental total energy of the neutron at rest. The value of the exponential damping constant A assumed in each case (1.054 e^{-1}) is taken from the results of the analogous $e^+ e^-$ calculations in the same basis.
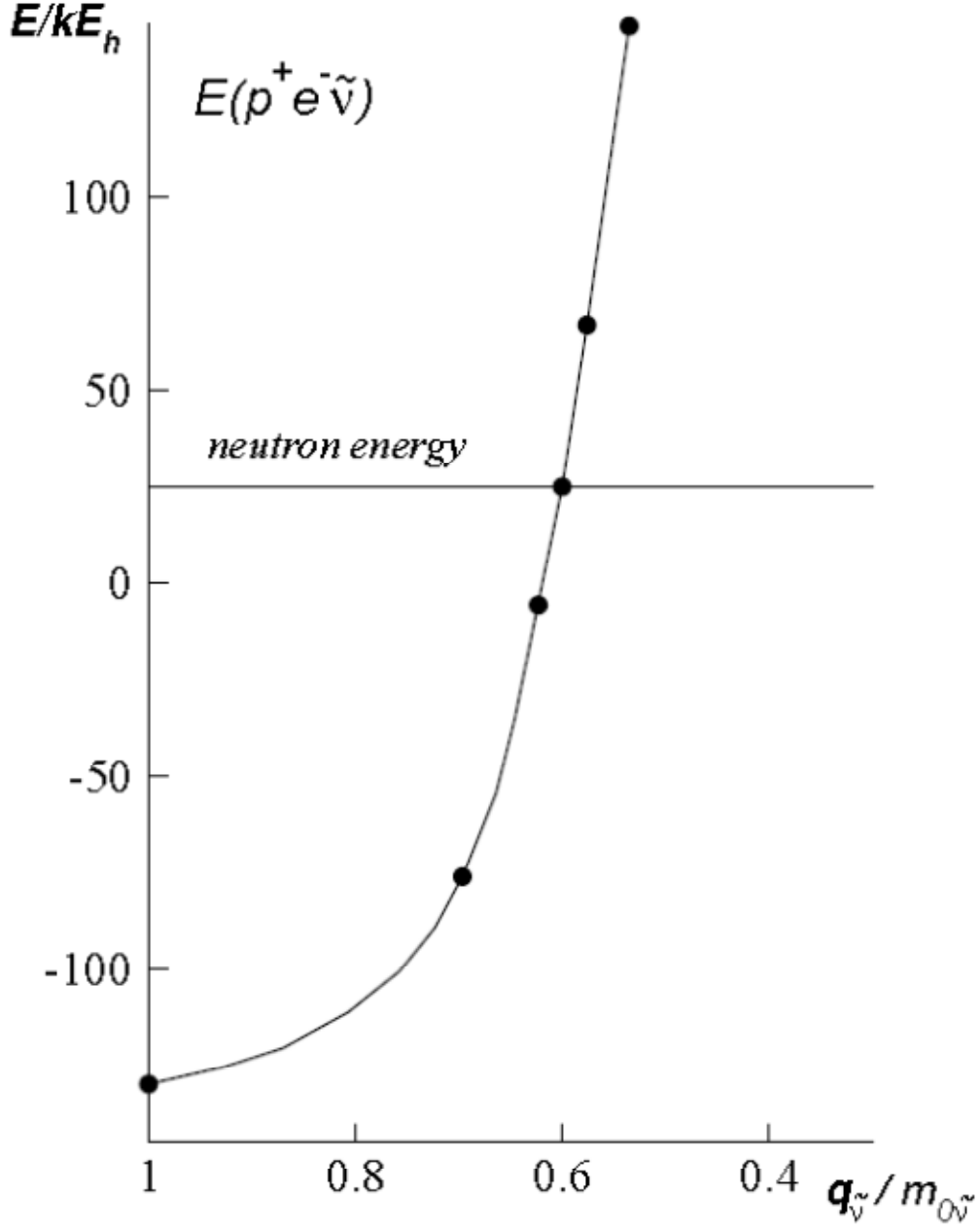
FIG. 12. Variation of the minimal $p^+ e^- \bar{V}$ system's computed total energy (taken from Fig. 11, 2s,2p basis) as a function of the antineutrino charge-to-rest-mass ratio q/m_o. The horizontal line at the center of the diagram corresponds to the experimental total energy of the neutron at rest.

Although the above calculations are fairly crude because of the small number of functions in the one-particle basis employed, they nonetheless illustrate a number of features of the present

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theoretical model which can be expected to be retained as the level of computational treatment is improved. To begin with, the fact that a minimum in total energy is always found demonstrates that the short-range potential employed is capable of binding the above three particles together within a relatively small volume. The computed binding energy is relatively sensitive to the choice of the $\bar{\nu}$ q/m_o, and indicates that for values close to +1.0 a.u, the $p^+e^-\bar{\nu}$ system would be more stable than the neutron actually observed experimentally. The corresponding optimal η values are indicated in parentheses in Fig. 12, and are seen to increase rather quickly as q/m_o decreases. There is thus a tendency for the constituent particles to draw more closely together at the same time that the total energy is increasing during the same variation. This is very similar to the experience noted for the $e^-\bar{\nu}$ system discussed in Sect. VI. J (see Fig. 10), which in turn can be anticipated based on the coordinate-scaling arguments of Sect. VI. H. It is also clear from comparing Figs. 10 and 12 that the rate of increase in total energy caused by lowering the antineutrino's q/m_o value is somewhat greater for the $e^-\bar{\nu}$ binary system than for tri-atomic $p^+e^-\bar{\nu}$.

The wavefunction corresponding to the experimental neutron binding energy is shown for the present basis in Table IX (SCF orbitals and full CI coefficients). It is relatively easy to analyze these results because the proton is seen to occupy a single s-type orbital almost exclusively in the small basis employed. For each product of three spatial orbitals, there are two doublet states possible, constructed from the three spin products ($M_J = +1/2$): $\alpha\alpha\text{ß}$, $\alpha\text{ß}\alpha$ and $\text{ß}\alpha\alpha$ (spins given in order of $p^+e^-\bar{\nu}$, α for $m_J = 1/2$ and ß for $m_j = -1/2$ for both $s_{1/2}$ and $p_{1/2}$ spin-orbitals. In the calculations the two doublets are represented by the linear combinations: $\chi_1 = (2/3)^{1/2} \alpha\text{ß}\alpha\text{ß} -6^{-1/2} (\alpha\text{ß}\alpha + \text{ß}\alpha\alpha)$ and $\chi_2 = 2^{-1/2} (\alpha\text{ß}\alpha - \text{ß}\alpha\alpha)$. In this basis the preferred spin combination is $(3/4)^{1/2} \chi_1 + (1/2) \chi_2$, which thus reduces to $\chi_{opt}= 2^{-1/2} (\alpha\alpha\text{ß} - \alpha\text{ß}\alpha)$. In essence the proton therefore almost always has α spin, while the $e^-\bar{\nu}$ pair forms a singlet ($\alpha\text{ß} - \text{ß}\alpha$) combination which is very reminiscent of the 0^- state preferred by the e^+e^- system and the other particle-antiparticle binaries studied, as well as $e^-\bar{\nu}$ itself. The $p_{1/2}$ $s_{1/2}$ configurations are again preferred for the $e^-\bar{\nu}$ complex, just as in the absence of the proton, but the corresponding $s_{1/2}$ $p_{1/2}$ product also makes a substantial contribution in each case. This type of polarization of the wavefunction is greater for the $p^+ e^- \bar{\nu}$ system, however. Clearly the $M_J = -1/2$ component of the

1/2⁻ state can be obtained by inverting all α (and ß . spins, which in effect means that the proton with ß. spin is then bound to the same 0^- e^- $\bar{\nu}$ structure as before.

TABLE IX. Self-consistent field (a) and selected CI (b) coefficients for the XBPS calculation (order 176) of the $1/2^-$ ground state of the $p^+e\bar{\nu}$ neutron system employing a 2s,2p basis ($\alpha_1 = 0.36 \times 10^8 \, a_0^{-2}$ and $\alpha_2 = 0.18 \times 10^8 \, a_0^{-2}$) with scale factor $\eta=0.18$, exponential damping constant $A=1.054 \, e^{-1}$ and $\bar{\nu} \, q/m_o$ value of 0.5733 a.u.

a)

Orbital	SCF Coefficients					
	p+		e^-		$\bar{\nu}$	
	α_1	α_2	α_1	α_2	α_1	α_2
$1s_{1/2}$	1.62047	-1.88386	-1.23308	2.15739	-0.01823	-2.48485
$2s_{1/2}$	-0.72534	2.37670	1.99702	-1.47876	1.01666	2.26743
$1p_{1/2}$	1.52029	-1.26858	-0.47744	-1.92162	-0.55928	1.89942
$2p_{1/2}$	-0.67147	1.86271	1.38257	1.41742	1.44199	-1.35692
$1p_{3/2}$	1.52029	-1.26858	-0.76754	1.82523	-0.23080	-1.96655
$2p_{3/2}$	-0.67147	1.86271	1.58427	-1.18771	1.19238	1.58076

b)

Configuration (orbital occupations)			CI ($1/2^-$) Coefficients *	
p^+	e^-	$\bar{\nu}$		
$1s_{1/2}$	$1p_{1/2}$	$1s_{1/2}$	0.572975	0.331858
$1s_{1/2}$	$2p_{1/2}$	$1s_{1/2}$	-0.211011	-0.122144
$2s_{1/2}$	$1p_{1/2}$	$1s_{1/2}$	0.082410	0.047579
$1s_{1/2}$	$1p_{1/2}$	$2s_{1/2}$	-0.242427	-0.140535
$1s_{1/2}$	$1s_{1/2}$	$1p_{1/2}$	0.274353	0.158440
$1s_{1/2}$	$1s_{1/2}$	$2p_{1/2}$	0.141213	0.081532
$1s_{1/2}$	$2s_{1/2}$	$1p_{1/2}$	0.255488	0.147615
$1s_{1/2}$	$2s_{1/2}$	$2p_{1/2}$	0.279461	0.161382
$1s_{1/2}$	$2p_{1/2}$	$2s_{1/2}$	0.196547	0.113694
$1p_{1/2}$	$1p_{1/2}$	$1p_{1/2}$	-0.023543	-0.040888
$2s_{1/2}$	$1s_{1/2}$	$1p_{1/2}$	0.058872	0.033968
$2s_{1/2}$	$2s_{1/2}$	$1p_{1/2}$	0.061997	0.035741
$2s_{1/2}$	$2s_{1/2}$	$2p_{1/2}$	0.055680	0.032099
$1p_{3/2}$	$1s_{1/2}$	$1s_{1/2}$	-0.054191	
$1p_{3/2}$	$2s_{1/2}$	$1s_{1/2}$	-0.057859	
$1p_{3/2}$	$1p_{1/2}$	$1p_{1/2}$	-0.067169	
$1p_{3/2}$	$1p_{1/2}$	$2p_{1/2}$	-0.060622	

*Two configuration-state functions are needed to span the spaces corresponding to (1/2) (1/2) (1/2) occupations (as defined in text), whereas only one such function is needed for the corresponding (3/2) (1/2) (1/2) species; all configurations with at least one CI coefficient whose absolute value exceeds 0.04 are listed.

B. THE MECHANISM FOR PROTON BONDING IN THE p+e-$\bar{\nu}$ SYSTEM

A comparison of Figs. 10 and 12 helps to illustrate the origin of the proton binding process in the present model. The energy difference results for e⁻$\bar{\nu}$ and p⁺e⁻$\bar{\nu}$ in the same (2s,2p) basis are plotted against the assumed q/m_o value for $\bar{\nu}$ in Fig. 13, and show that the proton binding energy increases as this charge-to-mass ratio is reduced from its original value of +1.0 a.u. Because of the small q/m_o value of the proton it is to be expected that only terms in the XBPS Hamiltonian (Table I) which do not involve this quantity will be important. An analysis of the p⁺e$\bar{\nu}$ total energy for the $\bar{\nu}$ q/m_o values of 1.0 and 0.5733 a.u. is given in Tables X and XI respectively, inc1uding corresponding results for optimal treatments of the e⁻$\bar{\nu}$ system in its lowest-energy 0⁻ state. The proton's kinetic energy contribution is relatively small because of its large mass, but at the interpartic1e distances involved, this quantity still works effectively against net binding . For the $\bar{\nu}$ q/m_o value of 0.5733 a.u. which leads to the experimental neutron total energy for the p⁺e$\bar{\nu}$ system, it is found, for example, that the proton kinetic energy is 36401.887 hartree, compared with a value of the proton-electron Coulomb attraction contribution of -5189.462 hartree. The latter result corresponds to a mean electron-proton distance of 3.62 α^2 bohr (10.2 fermi). There are two other XBPS terms for the proton which do not involve its own q/m_o value, however, which play a decisive role in the binding process, namely the spin-same-orbit and Darwin terms involving only the squares of either the q/m_o values for the electron or antineutrino (Table I). Consequently, these terms are of the same order of magnitude as their counterparts in the e⁺e⁻ calculation (see Table III).

The magnitudes of these contributions to the p⁺e⁻$\bar{\nu}$ energy depend very much on the characteristics of the various occupied spin-orbitals, however. A strong contribution toward proton binding comes from its spin-same-orbit interaction with the electron when the latter occupies a $p_{1/2}$ orbital while the proton occupies $s_{1/2}$ (see Table IX). This attractive interaction is countered by the corresponding term involving $\bar{\nu}$, however, because the latter also occupies $p_{1/2}$ heavily and its q/m_o value is assumed to be of opposite sign to that of the electron. The Darwin term also is characterized by large contributions to the p⁺e$\bar{\nu}$ total energy in which the proton is directly involved, however. In this case it is the antineutrino which provides an attractive force for the proton, however, which is countered only partially by the corresponding repulsive p⁺e⁻ Darwin term contribution (Table I).
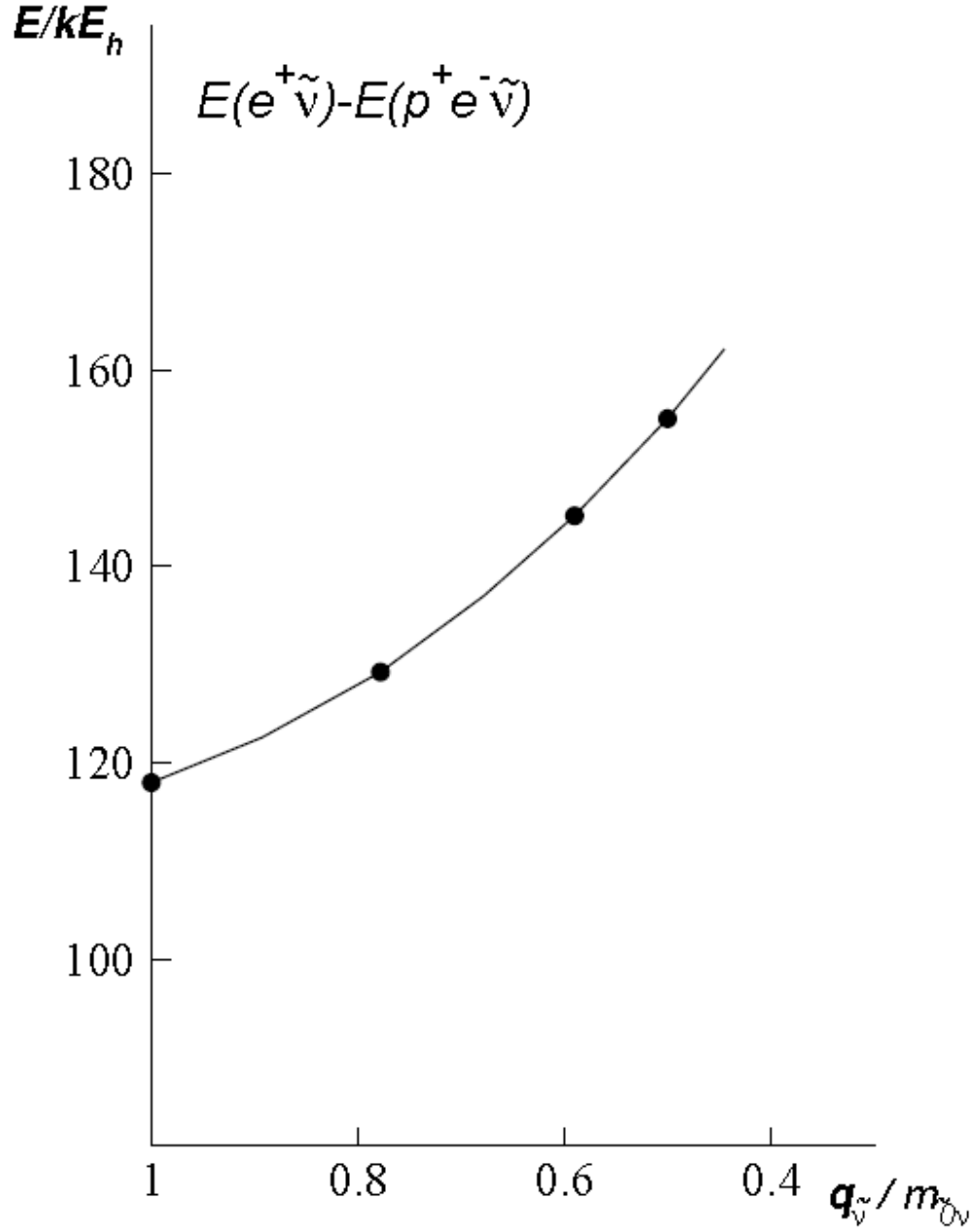
FIG. 13. Difference of the computed total energies (in khartree) of the $e^-\bar{\nu}$ and $p^+e^-\bar{\nu}$ systems (with separately optimized scale factors η) as a function of the antineutrino charge-to-rest-mass ratio q/m_o (2s,2p basis).

TABLE X. Energy contributions (in hartree) of various operators (see Table I for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+e^-\bar{\nu}$ neutron system and (b) the 0^- state of the isolated $e^-\bar{\nu}$ binary obtained by employing the 2s,2p basis with scale factor η = 0.115, exponential damping constant A = 1.054 e^{-l} and antineutrino q/m_o value of 1.0 a.u. for the XBPS Hamiltonian.

a)	Operator	p^+e^-	$p^+\bar{\nu}$	$e^-\bar{\nu}$	Total ($p^+e^-\bar{\nu}$)

	23276.671 (p^+)	1117472.413 (e^-)	1133826.572 ($\overline{\nu}$)	2274575.656
Kinetic Energy				
Coulomb	-4305.801	0.000	0.000	-4305.801
Spin-same-orbit	-172833.434	48470.883	-395254.117	-519616.667
Spin-other-orbit	-31.731	27.278	-875528.194	-875532.646
Darwin Term	92782.662	-228814.351	19549.016	-116482.672
Orbit-orbit	-44.084	129.057	-450397.290	-450312.316
Spin-spin	-0.550	-0.132	-447017.344	-447018.026
Spin-spin δ	-0.223	-1.938	7381.956	7379.795
Total Energy				-131312.679

b) Operator	Total ($e^-\overline{\nu}$)
Kinetic Energy	2247935.736
Coulomb	0.000
Spin-same-orbit	-411800.053
Spin-other-orbit	-916668.797
Darwin Term	8317.250
Orbit-orbit	-480109.984
Spin-spin	-461579.506
Spin-spin δ	0.000
Total Energy	-13905.355

There is thus a fairly complex system of interactions available among the Breit-Pauli terms, whose net energy contribution depends on two major factors: a)the character of the orbitals occupied respectively by the three particles and b) the assumed value of the antineutrino's charge-to-rest-mass ratio [since the corresponding spin-same-orbit and Darwin interactions with the proton depend on the square of this quantity (Table I)]. The easiest means of grasping the influence of these various factors in the proton binding energy is to first examine the situation for q/m_o $(\overline{\nu})$ = +1.0 a.u. If the orbital occupations of the e^- and $\overline{\nu}$ particles were exactly equivalent, all the proton spin-same-orbit and Darwin interactions would exactly cancel one another. That would effectively leave only the proton kinetic energy and proton-electron Coulomb terms to determine the binding energy, which would mean that the $p^+e^-\overline{\nu}$ system is decidedly unstable relative to $e^-\overline{\nu}$. By assuming a wavefunction in which the electron has more $p_{1/2}$ character than $\overline{\nu}$, it is possible to shift the balance toward a more favorab1e binding situation, however, by virtue of the fact that the attractive proton-electron spin-same-orbit term then outweighs the repulsive $p^+\overline{\nu}$ contribution of the analogous type.

TABLE XI. Energy contributions (in hartree) of various operators (see Table I for definitions) and particle combinations for (a) the $1/2^-$ ground state of the $p^+e^-\overline{\nu}$ neutron system and (b) the 0^- state of the isolated $e^-\overline{\nu}$ binary obtained by employing the 2s,2p basis with scale factor η = 0.18, exponential damping constant A = 1.054 e^{-1} and :antineutrino q/m_o value of 0.5733 a.u. for the XBPS Hamiltonian.

a) Operator	p^+e^-	$p^+\bar{\nu}$	$e^-\bar{\nu}$	Total ($p^+e^-\bar{\nu}$)
Kinetic Energy	36401.887 (p^+)	1346815.079 (e^-)	1453755.882 ($\bar{\nu}$)	2836972.849
Coulomb	-5189.461	0.000	0.000	-5189.461
Spin-same-orbit	208544.881	50249.141	-465129.910	-623425.649
Spin-other-orbit	-58.267	54.775	-1007066.189	-1007069.681
Darwin Term	107014.110	-262604.473	17161.765	-138428.597
Orbit-orbit	200.486	-58.064	-523108.311	-522965.888
Spin-spin	-1.096	-0.248	-516918.459	-516919.802
Spin-spin δ	-0.387	-4.109	5928.285	5923.790
Total Energy				28897.559

b) Operator	Total ($e^-\bar{\nu}$)
Kinetic Energy	2789426.035
Coulomb	0.000
Spin-same-orbit	-481697.661
Spin-other-orbit	-1063806.525
Darwin Term	11500.864
Orbit-orbit	-558243.900
Spin-spin	-531802.164
Spin-spin δ	1127.425
Total Energy	166504.076

Examination of Table X shows clearly that heavy net attractive contributions actually arise for both the spin-same-orbit and Darwin terms as a result of such a polarization. As a result, the total binding energy of the proton to the $e^-\bar{\nu}$ complex is computed to be 117400 hartree. By comparison, if only the proton kinetic energy and proton-electron Coulomb contributions were counted, a negative proton binding energy would result. There is another key factor in the $p^+e^-\bar{\nu}$ binding process, however, which is also important in the formation of tri-atomic systems in the realm of molecular physics. Some adaptation of the charge distribution of a diatomic component of the molecule is almost always crucial to the production of a stable tri-atomic system[109]. Generally speaking, the more stable the diatomic system, the higher the price to be paid for altering its charge distribution to accommodate bonding with a third atom. In the present context, it is important to recall that a quite small AO basis has been employed in these exploratory calculations, however, and so it can be anticipated that at least some of the $e^-\bar{\nu}$ system's affinity to polarize its charge distribution is a consequence of its relatively poor representation at this level of theoretical treatment.

When the q/m_o value is decreased from unity for the antineutrino, several additional effects emerge. The $e^-\bar{\nu}$ complex gradually loses its stability in the process, and it thus becomes easier for the proton to bind to it as a direct consequence. The cancellation of the Breit-Pauli p^+e^- and

$p^+\bar{\nu}$ interactions is no longer perfect even if both e^- and $\bar{\nu}$ have equivalent orbital occupations because the key $(q/m_o)^2$ weighting factors now favor the electron. In addition, polarization effects are less likely to destabilize the $e^-\bar{\nu}$ complex, so that the types of changes in the isolated system's wavefunction which maximize binding with the proton are accomplished with less resistance than before. For the q/m_o value of 0.5733 a.u. required to obtain the experimental neutron rest mass for the $p^+e^-\bar{\nu}$ calculations employing the 2s,2p basis, it is found that the proton binding energy increases to 137600 hartree, over 20000 hartree greater than the value which results when a unit value for this quantity is employed.

The results of Table XI show further that both the spin-same-orbit and Darwin interactions again produce a net attraction for the proton, namely 158296 and 155590 hartree, respectively (obtained by adding the corresponding p^+e^- and $p^+\bar{\nu}$ results). These results are accomplished to a large extent by polarization of the $e^-\bar{\nu}$ complex, as can be seen from the following comparison. For this q/m_o value the minimal total energy obtained for the isolated $e^-\bar{\nu}$ system is 166504 hartree, whereas the corresponding energy value obtained employing the polarized $p^+e^-\bar{\nu}$ wavefunction is 311438 hartree (obtained by adding the e^- kinetic energy of 1346815.079 hartree to the $e^-\bar{\nu}$ results including the $\bar{\nu}$ kinetic energy), an increase of 144934 hartree. Again, while it is very likely that such polarization effects are greatly exaggerated by the use of such a small one-particle basis set in the present calculations, it at least seems conceivable that the combination of changes of this nature in the $e^-\bar{\nu}$ charge distribution along with the use of a $|q/m_o|$ value for the antineutrino which is smaller than that of the electron could lead to sufficient proton binding to produce a meta-stable $p^+e^-\bar{\nu}$ complex with the properties of the experimentally observed neutron. The effect of this polarization increases with the $\bar{\nu}$ charge-to-mass ratio, so these considerations make it at least qualitatively understandable why the proton binding energy increases as q/m_o decreases, as seen clearly from the result of Fig. 13.

There is another important aspect of the proton binding process yet to be discussed, however, namely the influence of the exponential damping factors in the XBPS Hamiltonian (Table I). These factors play a decisive role in obtaining an energy minimum for the $e^-\bar{\nu}$ binary (Figs. 10-11), but they also have a subtle influence on the proton's interactions. As we have already seen, the Breit-Pauli terms of greatest importance for the proton are the spin-same-orbit and Darwin interactions with e^- and $\bar{\nu}$ which are multiplied by the $(q/m_o)^2$ factors of the latter particles only. In these cases the corresponding damping factors are totally independent of the

proton's momentum (Table 1). By decreasing the size of the proton's orbital it is therefore possible to increase the magnitude of the un-damped Breit-Pauli expectation values, with their net attractive influence, *without producing a corresponding decrease in the magnitude of the related exponential damping factors.* As long as the charge distributions of the electron and antineutrino do not change at the same time, such an increase in the proton momentum thus generally leads to a net increase in the attractive contribution of the damped Breit- Pauli interactions to the proton-bonded system.

The possibilities are not unlimited for the proton, however, because its kinetic energy also increases as its charge distribution is made more compact (see Fig. 14).
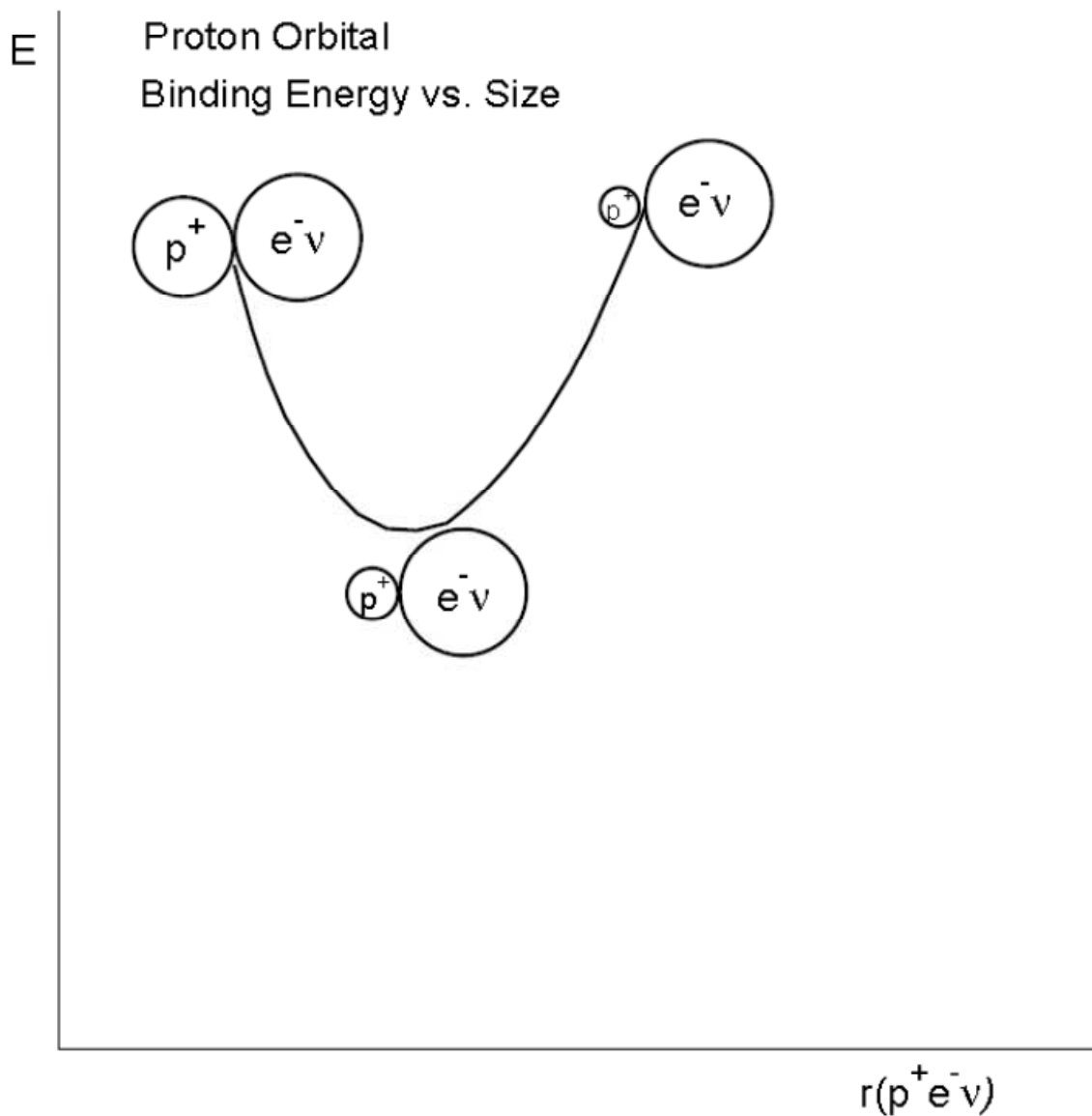
FIG. 14. Schematic diagram showing the relationship between the size of the proton orbital and the stability of the p+e⁻ $\overline{\nu}$ neutron system. The large mass of the proton *vis-a-vis* the e-$\overline{\nu}$ species allows it to assume a relatively contracted charge distribution which helps to maximize the effect of its attractive short-range interactions with the lighter particles without greatly increasing. its own kinetic energy, thus leading to the total energy minimum shown.

Because the Breit-Pauli terms vary as r^{-3} before damping effects are considered, while the proton kinetic energy increases only as $p^2 \approx r^{-2}$, it follows that in the interparticle distance range of interest ($r \approx \alpha^2$), the total binding energy at first increases with the proton's momentum. Eventually a point of diminishing returns is reached, however, because as the proton orbital shrinks in size the values of the Breit-Pauli integrals for fixed e⁻ and $\overline{\nu}$ probability distributions begin to change more and more slowly, ultimately approaching the point-charge limit commonly employed in atomic calculations using the Breit-Pauli method. The proton kinetic energy continues to increase at roughly the same rate, however, and so at some point an optimum proton size is reached. Nonetheless, this effect represents a distinct advantage for the proton *vis-a-vis* lighter particles, and will be seen to play an increasingly more important role in the theory of nuclear binding which emerges from consideration of the properties of the XBPS Hamiltonian.

To illustrate the proton's tendency to assume a relatively more compact charge distribution than the electron and antineutrino, an additional series of calculations has been carried out in which two s-type Gaussian functions have been added to the original 2s,2p one-particle basis employed above. Optimization of the new exponents (α_1 and α_2) while holding those of the other functions fixed at their optimal values for the e⁻ $\overline{\nu}$ wavefunction shows that the proton prefers relatively high values for these quantities (2.56×10^8 and 8.0×10^7 a_0^{-2}). These values are 8.0 and 2.5 times greater than that of the largest exponent in the optimal 2s,2p e⁻ $\overline{\nu}$ basis, indicating a significant contraction of the proton wavefunction, as expected. The new basis functions succeed in lowering the total p⁺e⁻ $\overline{\nu}$ energy by nearly 60000 hartree relative to the 2s,2p result given in Table XI. In order to be consistent with the general calibration procedures adopted earlier, it is necessary to readjust the respective value of the exponential damping constant A and the antineutrino q/m_o value so as to produce minimal energy results for the e⁺e⁻ and p⁺e⁻ $\overline{\nu}$ systems of the desired values. On this basis A is increased only slightly to a value of 1.0568 e⁻¹ while the $\overline{\nu}$ q/m_o result is adjusted downward to 0.5375 a.u. A summary of the energy contributions for the p⁺e⁻ $\overline{\nu}$ 1/2⁻ state found in this basis set are given in Table XII for comparison with the corresponding 2s,2p data discussed first (Table XI). The shrinking of the proton charge

distribution is most easily recognized from the magnitude of the kinetic energy obtained in the two treatments. This quantity increases by 79% as a result of the basis set

TABLE XII. Energy contributions (in hartree) of various operators (see Table I for definitions) and particle combinations for (a) the 1/2⁻ ground state of the $p^+ e^- \bar{\nu}$ neutron system and (b) the 0⁻ state of the isolated $e^- \bar{\nu}$ binary obtained by employing the 4s,2p basis with scale factor $\eta = 0.16$, exponential damping constant $A = 1.0568\ e^{-1}$ and antineutrino q/m_o value of 0.5375 a.u. for the XBPS Hamiltonian.

a) Operator	$p^+ e^-$	$p^+ \bar{\nu}$	$e^- \bar{\nu}$	Total ($p^+ e^- \bar{\nu}$)
Kinetic Energy	65280.208 (p^+)	1339330.432(e^-)	1478062.82 t(\j)	2882673.461
Coulomb	-5517.636	0.000	0.000	-5517.636
Spin-same-orbit	-250929.141	70545.115	-463958.811	-644342.837
Spin-other-orbit	-67.206	61. 902	-984257.224	-984262.528
Darwin Term	117676.768	-352977.701	15413.212	-219887.721
Orbit-orbit	-66.116	228.315	-492670.535	-492508.336
Spin-spin	-2.195	-0.350	-511881.100	-511883.645
Spin-spin δ	-0.474	-10.788	5302.617	5291.355
Total Energy				29562.111

b) Operator	Total ($e^- \bar{\nu}$)
Kinetic Energy	2759022.924
Coulomb	0.000
Spin-same-orbit	-475143.463
Spin-other-orbit	-1042436.569
Darwin Term	11692.897
Orbit-orbit	-536596.507
Spin-spin	-520943.516
Spin-spin δ	1286.444
Total Energy	196882.208

expansion, as compared to relatively small changes in the corresponding values for the electron and antineutrino. The net attractions of the proton spin-same-orbit and Darwin terms are also increased to 180384 and 235301 hartree respectively, representing a total of 101794 hartree additional binding to help offset the increase in proton kinetic energy. By construction, the total energies in both calculations are nearly identical, from which it is clear that the shrinking of the proton charge distribution is also paid for at the price of additional destabilization of the $e^-\bar{\nu}$ complex.

Another key point to consider in the present $p^+e^-\bar{\nu}$ calculations is the way in which the translational energy is treated. Because of the heavy mass of the proton, it follows that the relativistic translational energy for a given $p^+e^-\bar{\nu}$ wavefunction is much smaller than that for an equivalent e^+e^- species. We have seen in Sect. VI. E that the use of $<E_o> = <H-T>$ as criterion for the basis set optimizations leads to a. notably higher damping constant A (1.7725 e^{-1} in the 5s,5p basis) than when only $<H>$ is used (1.0775 e^{-1}), If the higher of these A values is used in the $p^+e^-\bar{\nu}$ calculations (or the corresponding value for the 2s,2p basis), it is impossible to obtain a low enough total energy for this system to satisfy the requirement that its rest mass be equal to that of the neutron. This situation is an artifact of the small (2s,2p) basis employed, however, because the exact solutions of the XBPS must be eigenfunctions of both H and T and the lowest total energy eigenvalue must correspond to a vanishing translational energy. Under the latter condition there would be no need to distinguish between the two different optimization procedures employed above, since they would necessarily lead to identical results ($E_{min} = E_o$), At this stage of development, however, it is necessary to deal with the fact that the approximate wavefunctions generated at the present level of treatment invariably have large expectation values of the translational momentum operator for the system at hand, at least when such compact charge distributions are involved as are known to characterize the internal structure of nuclei.

Under the circumstances, the decision to focus on the expectation value of the total energy in comparing the stabilities of systems containing different numbers of protons must carry with it the recognition that this choice favors heavier particles over lighter ones. Another way of seeing this is to recall that the value of the exponential damping constant A must be significantly smaller than variational arguments indicate it should be in order to obtain total energies for the particle-antiparticle binary systems which correspond to binding energies of $2m_oc^2$. As the

quality of the basis set improves, the effective value of A must increase to maintain this condition until it ultimately assumes the ideal result corresponding to the exact solution of the XBPS for the translationless binary systems. To illustrate this tendency calculations have been carried out for the $e^+ e^-$ system employing a 3s,2p,2d basis. They lead to a value for A of 1.2647 e^{-1}, as compared to that of 1.0775 e^{-1} obtained with the 5s,5p basis. Since the proton binding energy in the $p^+ e^- \bar{\nu}$ calculations considered thus far has been shown to arise primarily from terms which contain the corresponding damping factors in which the constant A appears, it seems highly likely that the degree of binding will be strongly influenced by such developments.

As usual, the binding energy of the $e^+ e^-$ system is held at -37557.773 hartree in the new basis, for example, but the degree to which it or its $e^- \bar{\nu}$ analog can bind a proton can be expected to decrease as the value of the damping factor increases. This is tantamount to concluding that the computed proton binding energy for a given system will generally decrease as the level of theoretical treatment is improved within the XBPS model. Moreover, this expectation is also consistent with the discussion given earlier regarding the relationship between the stability of the $e^- \bar{\nu}$ complex and its susceptibility to polarization by neighboring protons. In other words, as the basis set is improved there is a growing tendency for the $e^- \bar{\nu}$ system to more strongly resist having its charge distribution altered relative to its isolated state.

There is a competing factor which tends to insure that the minimal $p^+ e^- \bar{\nu}$ total energy can always be adjusted to the value corresponding to the neutron at rest, however, namely the dependence of this quantity on the q/m_o value assumed for the antineutrino as the quality of the one-particle basis is improved. In the present case it is found, for example, that it is necessary to increase this value to 0.63 a.u. to obtain the desired energy result to a suitable approximation. The corresponding energy contributions are given in Table XIII, similarly as for the other basis sets discussed earlier. Since the Gaussian exponents for the 3s,2p,2d basis have been optimized for the $e^+ e^-$ system, the results of these calculations are most meaningfully compared to those employing the 2s,2p basis (Table XI), i.e. without the benefit of specially optimized proton s functions. As expected, the use of a significantly larger damping constant leads to a reduction in the net binding associated with the proton spin-same-orbit and Darwin terms (74427 and 70107 hartree respectively), 46% of the previous total. The proton's kinetic energy is correspondingly smaller in the 3s,2p,2d computations as well, 56% of the former value. The proton-electron

Coulomb energy is also smaller, reflecting a general tendency to keep these two particles further apart on the average as a result of the addition of d functions to the basis set.

TABLE XIII. Energy contributions (in hartree) of various operators (see Table I for definitions) and particle combinations for the $1/2^-$ ground state of the $p^+e^-\bar{\nu}$ neutron system obtained by employing the 3s,2p,2d basis with scale factor $\eta = 0.111$, exponential damping constant A = 1.2648 e^{-1} and aritineutrino q/m_o value of 0.63 a.u. for the XBPS Hamiltonian.

Operator	p^+e^-	$p^+\bar{\nu}$	$e^-\bar{\nu}$	Total ($p^+ e^- \bar{\nu}$)
Kinetic Energy	20354.545(p^+)	1068402.098(e^-)	1140129.945($\bar{\nu}$)	2228886.588
Coulomb	-3856.027	0.000	0.000	-3856.027
Spin-same-orbit	-94307.843	19879.681	-363207.789	-437635.951
Spin-other-orbit	-23.085	19.111	-757175.358	-757179.332
Darwin Term	48531. 942	-118639.460	8357.925	-61749.593
Orbit-orbit	-16.656	145.966	-559254.673	-559125.363
Spin-spin	-0.370	-0.176	-383654.253	-383654.799
Spin-spin δ	-0.179	-1.489	2345.886	2344.218
Total Energy				28029.737

Particularly when one considers the capacity of the proton to assume a more contracted charge distribution when extra functions are included in the basis which can be specifically optimized for it, it appears feasible to construct a suitably quantitative theory of nuclear binding on the basis of the model under discussion. At least these results show that it is far from proven that the "nuclear electrons" which Fermi spoke of in introducing his theory of beta decay[52] actually cannot exist within the confines of a bound nucleus. Before discussing further computations employing the XBPS Hamiltonian, however, it is well to consider other experimental information regarding the neutron which has been claimed to lend support to the hypothesis of the disappearing electron.

C. COMPARISON OF THE PROPERTIES OF THE $p^+e^-\bar{\nu}$ SYSTEM WITH THOSE KNOWN FOR THE NEUTRON

The calculations discussed above have been suggested by the model of a neutron as a composite system formed by its known decay elements. The lowest-energy $p^+e^-\bar{\nu}$ system is found to be a doublet, consistent with what is known for the neutron. As a product of three fermions, it is a fermionic system itself, in agreement with Pauli's original interpretation of beta decay processes[37]. The charge distribution found to be optimal for the $p^+e^-\bar{\nu}$ system corresponds to nuclear dimensions, in fulfillment of another obvious requirement, but more extensive calculations are highly desirable in order to be more quantitative on this point. A three-way partnership is suggested in forming the neutron which is reminiscent of the excimer concept in molecular physics. The fact that the system corresponds to a local energy minimum which lies above that of its separated particles is clearly consistent with the known meta-stability of the neutron. This aspect of nuclear binding (weak interaction) will be taken up in more detail in .the following chapter.

A key to the binding of the three particles together is the assumption of a positive q/m_o value for the antineutrino, but one with a smaller absolute value than that of the electron. One has the picture of an $e^-\bar{\nu}$ system which is an imperfect copy of the e^+e^- massless binary first discussed. Its relative instability makes it more attractive to the proton than e^+e^- itself, and in effect the proton binding that results can be looked upon as an attempt to compensate for what is otherwise missing in the e-$\bar{\nu}$ bond *vis-a-vis* either of its e^+e^- or $\nu\bar{\nu}$ counterparts. The electrical neutrality of the antineutrino guarantees that the $p^+e^-\bar{\nu}$ system has no net charge, but this brings us to a far more delicate matter. The neutron possesses a magnetic dipole moment which is negative and of the order of the nuclear Bohr magneton[37,51]. As such it is far smaller in absolute magnitude than what one would expect for a system containing an electron.

To examine this point it is helpful to consider an experiment in which the hypothetical $p^+e^-\bar{\nu}$ system is subjected to a magnetic field in order to determine this quantity. If we assume that the magnetic moment of the combined system is equal to the algebraic sum of the individual moments of its constituents, the measured value must be expected to be nearly equal to that of the electron alone. This assumption works quite well in dealing with molecular properties and has also been found to be acceptable for nuclei, as for example in the comparison of the

deuteron's magnetic moment with those of its constituent proton and neutron[159]. If we look more closely at the way magnetic moments are measured, however, we note that for this additivity principle to hold in the $p^+e^-\bar{\nu}$ case, it is essential that the electron be just as apt to rotate in response to the torque applied by a magnetic field in this tri-atomic system as it is in its free state, or alternatively when it is weakly bound to a positive atomic or molecular ion.

Especially since the present calculations indicate that the binding process requires a particular spin orientation between all three particles in the optimum $p^+e^-\bar{\nu}$ resonance state, it seems far from obvious that the above condition is fulfilled in the present instance. In order for the electron spin to change its orientation, it seems. much more likely that the system as a whole must rotate, and that would mean that a much larger mass is involved than for a tightly bound electron as in an outer shell of an atom. A similar situation is well known in the study of the Compton effect[44], for example. The modified wave observed in x-ray radiation from atoms *is observed only when an essentially free electron is involved in the interaction,* in which case the change in wavelength $\Delta\lambda$ is found to be inversely proportional to the electronic mass. When an inner-shell electron does the scattering, however, it is as if the whole atom is involved in the interaction and consequently the Compton scattering law indicates the production of an essentially unmodified wave, i.e. $\Delta\lambda \sim 0$. The calculations discussed above suggest that the electron in the $p^+e^-\bar{\nu}$ resonance system is very tightly bound to both the proton and the antineutrino constituents, and thus that the model of an inner-shell electron in the preceding analogy is far more appropriate.

On this basis one is led to expect that the $p^+e^-\bar{\nu}$ system has a magnetic moment of an extremely heavy electron, i.e. negative in sign, but of the order of a nuclear bohr magneton, which is at least approximately what is observed for the neutron[51,93]. The additivity principle should not work at all well in this case because the interaction of the particle spins is intimately involved in the internal bonding process, unlike the case when predominantly Coulomb interactions are involved. In Chapter IX we will come back to this point when we discuss the muon magnetic moment, which is almost exactly what one would expect for an electron having this particle's rest mass. The overall situation is complicated for the proton and neutron, however. because of the pion cloud known to surround each of them[88,91,94,160], as also considered below. The observed neutron magnetic moment is roughly double[51,93,94] that which one would expect from the above "clamped-electron" model, but such a discrepancy is at least potentially

understandable in terms of the environment otherwise assumed for the neutron. Moreover, the fact that high-energy electron scattering experiments161 have indicated elastic form factors which correspond to a small but not necessarily vanishing neutron charge radius are also seemingly consistent with the present model's assumed tri-atomic composition for this neutral system.

D. THE BINDING ENERGY OF THE DEUTERON IN THE XBPS MODEL

The simplest nucleus consists of a single neutron and proton which are assumed to be bound together by a force generally referred to as the strong or hadronic interaction. By analogy to the quantum electrodynamics description of the Coulomb force as involving the exchange of a virtual photon, Yukawa[88] suggested in 1935 that the exchange of a heavier particle was responsible for the force joining any two nucleons together. According to the present model, the reaction of a proton and a neutron to form the deuteron product ^{2}H involves four separate particles, three of which are found in the neutron itself. The question that will be taken up next is whether the forces which have been proposed above to explain how three separate particles can be bound together to form a neutron might not also be involved in the nuclear binding process as well. Having computed a value for the charge-to-mass ratio of the antineutrino on the basis of the results of the $p^+e^-\bar{\nu}$ XBPS treatment in a given one-particle basis, it is thus proposed to examine the effects of adding a second proton to this system, while otherwise proceeding in an equivalent manner as in the calculations already discussed.

On the basis of the treatment of the helium atom in the standard quantum mechanical theory, one might expect the ground state of this two-proton system to correspond to double occupation of the same s orbital found to be optimal for the $p^+e^-\bar{\nu}$ system. Experiment tells us that the resulting singlet state is not favored by the deuteron, however, but rather one of triplet multiplicity[162]. Moreover, no other bound state is known to exist for the deuteron[163]. The fact that the triplet multiplicity is so favored by the deuteron has given rise to the well-known characterization of the nuclear force as being spin-dependent[164]. The XBPS calculations for the $p^+e^-\bar{\nu}$ system have indicated quite strongly that the $e^-\bar{\nu}$ complex itself retains the 0^- character preferred in its isolated state, and thus it is reasonable to expect that such relatively light particles themselves do not make a significant contribution to the total angular momentum of this system.

Especially since the traditional description of this nucleus is in terms of a proton $S_{1/2}$ and a neutron $S_{1/2}$ orbital[80,83], it seems clear that the only way to obtain a satisfactory explanation for the deuteron ground state's triplet multiplicity in the present model is therefore to assume that *there is a second* $s_{1/2}$ *proton orbital with nearly the same energy as the first, but orthogonal to it.* For simplicity, let us refer to these two orbitals as φ_a and φ_b, whereby it can be anticipated that one of them will be quite similar to that found to be most strongly occupied in the $p^+e^-\bar{\nu}$ calculations discussed in Sects. VII.A-B.

In view of the above experimental findings one is led to expect that neither of the closed shell $\varphi_a{}^2$ or $\varphi_b{}^2$ configurations leads to a bound deuteron state, i.e. the energies of the related singlet states must lie above that of the separated proton and neutron, or some 29000 hartree higher than that of the constituent protons, electron and antineutrino separated to infinity. According to the convention employed earlier, the deuteron ground state itself has a total energy of -52972.813 hartree, corresponding to the measured binding energy of 2.22452 MeV relative to the separated proton and neutron. To see how this general picture is reflected in actual calculations, we will begin by employing the simplest one-particle basis considered above, with 2s and 2p functions for each particle type. The same values for the exponential damping constant A and the $\bar{\nu}$ charge-to-mass ratio are assumed as in the analogous p^+e^- calculations which employ the same AO basis. After optimization of the scaling factor η for this basis, it is found that the lowest-energy state of the $p^2e^-\bar{\nu}$ system has 0^- symmetry, with both protons occupying the φ_a $s_{1/2}$ orbital with opposite spins. As expected, the electron and antineutrino form a singlet unit similar to that favored by the e^+e^- system, and this part of the wavefunction is responsible for the negative parity of the four-particle state as a whole. The total energy obtained for the 0^- state is -254512.252 hartree. The lowest-lying 1^- state with a $s_{1/2}$ $s_{1/2}$ $\varphi_a\varphi_b$ proton configuration has a much higher energy of +21955.705 hartree, but is still bound relative to the $p^+e^-\bar{\nu}$ system plus an isolated proton by 6941.856 hartree, or 0.189 MeV (see Table XI).

In Sect. VII. B it was found that the proton of the $p^+e^-\bar{\nu}$ system preferred a considerably more contracted charge distribution than either the electron or antineutrino. In the next series of $p^2e^-\bar{\nu}$ calculations the same 4s,2p basis is employed which demonstrated this effect in the first case. The additional two s functions ($\alpha_1 = 2.56 \times 10^8$ and $\alpha_2 = 0.80 \times 10^8$ a_0^{-2}) are found to have a much greater effect on the total energies of the $p^2e^-\bar{\nu}$ states than before for the corresponding one-proton system. For A=1.0567 e^{-1} and q/m$_o$ ($\bar{\nu}$)=0.5375 a.u., the values of these parameters

which lead to the desired binding energies of the e^+e^- and $p+e^-\bar{\nu}$ systems in this basis, results of -401335.635 and -86856.876 hartree are obtained for the 0^- and 1^- $p^2e^-\bar{\nu}$ states, respectively. The corresponding computed binding energies are thus 430897.744 hartree (11.72 MeV) and 116418.985 hartree (3.17 MeV), respectively, considerably larger in each case than for the 2s,2p basis set not containing specially optimized $s_{1/2}$ functions for the protons.

TABLE XIV. Energy contributions (in hartree) of various operators (see Table I for definitions and abbreviations) and particle combinations for the 0^- state of the $p^2e^-\bar{\nu}$ deuteron system obtained by employing the 4s,2p basis with scale factor $\eta = 0.16$, exponential damping constant $A = 1.0568\ e^{-1}$ and antineutrino q/m_o value of 0.527 a.u. for the XBPS Hamiltonian.

Op	p^+p^+	p^+e^-	$p^+\bar{\nu}$	$e-\bar{\nu}$	Total
KE	286751.843(p^+)	1389897.489(e^-)	1629685.689($\bar{\nu}$)		3306335.021
C	15221.838	-11234.341	0.000	0.000	3987.496
SsO	0.000	-669587.021	105016.667	-447338.511	-1011908.865
SoO	0.000	-93.245	94.858	-893545.158	-893543.546
D	-23.567	160901.391	-1103708.083	28456.498	-914373.757
OO	0.662	-122.703	414.328	-394442.299	-394150.012
SS	0.000	-0.352	-0.205	-499287.703	-499288.260
SSδ	47.127	-0.357	-4.749	15026.997	15069.018
					-387872.905
SδI*	392749.721				392749.721
TE**					4876.816

*Strong spin-spin S increment for proton-proton interaction (see text).

**Total energy

The energy contributions for the various one- and two-particle interactions for the above two states in the 4s,2p basis are given in Table XIV (0^-) and Table XV (1^-) respectively. These results are obtained with a slightly lower $q/m_o(\bar{\nu})$ value (0.527 a.u.) than in the $p^2e^-\bar{\nu}$ calculations mentioned first, but correspond to very nearly the same binding energies relative to the $p^+e^-\bar{\nu}$ system of the same charge-to-mass ratios as those given above (the total energy of the latter is 44067.869 hartree, or 14505.760 hartree higher than in the other calculation detailed in

Table XII). This second series of $p^2e^-\bar{\nu}$ calculations indicates that the $e^-\bar{\nu}$ complex is capable of providing a strong attraction for protons. As usual, by far the largest contributions to net binding for the system are the proton-electron and proton-antineutrino spin-same-orbit and Darwin terms. For the 0^- state the spin-same-orbit proton terms produce a net binding of 564570 hartree, while the corresponding Darwin term result is 942807 hartree. The corresponding results for the 1^- state are 441275 and 536870 hartree respectively (Tables XIV and XV). These values are 2.5 to 4.0 times greater than found in the corresponding $p^+e^-\bar{\nu}$ calculations in Sect. VII.B (Table XII), again indicating that the addition of a second proton is marked by a significant contraction of the charge distribution of the system as a whole (i.e. the $p^2e^-\bar{\nu}$ results are more than double those for the system with a single proton).

TABLE XV. Energy contributions (in hartree) of various operators (see Table I for definitions and abbreviations) and particle combinations for the 1^- state of the $p^2e^-\bar{\nu}$ deuteron system obtained by employing the 4s,2p basis with scale factor $\eta = 0.16$, exponential damping constant $A = 1.0568\ e^{-1}$ and antineutrino q/m_o value of 0.527 a.u. for the XBPS Hamiltonian.

Op	p^+p^+	p^+e^-	$p^+\bar{\nu}$	$e^-\bar{\nu}$	Total
KE	290643.137(p^+)	1341557.196(e^-)	1512125.933($\bar{\nu}$)		3144326.266
C	7726.523	. -10467.859	0.000	0.000	-2741.336
SsO	0.000	-530003.791	88729.000	-442961.158	-884235.949
SoO	0.000	-57.868	39.310	-906688.972	-906707.530
D	0.000	178980.181	-715853.570	22920.853	-513952.535
OO	0.277	-84.456	207.748	-428116.693	-427993.124
SS	0.000	-6.087	-0.388	-491176.127	-491182.602
SSδ	0.000	-0.091	-29.955	11380.624	11350.578
					-71136.232
SδI*					3.123
TE**					-71133.109

* Strong spin-spin δ increment for proton-proton interaction (see text).

**Total   energy.

The fact that the total binding energies obtained for the $p^2e^-\bar{\nu}$ system are considerably larger than observed for the deuteron experimentally, particularly when reference is made to the 0^- state's results, is a clear deficiency of the present theoretical treatment, but on the basis of the $p^+e^-\bar{\nu}$ computations discussed in Sect. VII. B, it can be expected that the addition of d functions to the basis employed will probably lead to a decrease in these values. Nonetheless, a more obvious deficiency in the present results seems far less likely to be an artifact of the computational treatment considered thus far, namely that the ground state of the $p^2e^-\bar{\nu}$ system, which is being proposed as having the composition of the deuteron, prefers singlet multiplicity by a wide margin. This result seems unavoidable so long as the interactions involving the protons are predominantly the spin-same-orbit and Darwin terms involving either the electron or antineutrino as second particle. Only the Coulomb repulsion between the two protons provides a counter-example for this type of behavior, and although this interaction does favor a triplet spin function for a two-open-shell proton configuration, it is not enough to override the decided preference of the $p^2e^-\bar{\nu}$ system for a closed-shell structure in which only the most stable proton $s_{1/2}$ orbital is strongly occupied. The situation is thus analogous to that encountered in the electronic structure of the helium atom, which also favors a singlet ground state, and as a result there seems no question that the XBPS Hamiltonian in the form given in Table I *is not capable of a suitable description of the forces which bind the proton and neutron together in the ^2H or other nuclear systems.*

E. THE SPIN-DEPENDENCE OF THE INTERACTION BETWEEN NUCLEONS

The failure of the above calculations to account for certain basic aspects of the structure of the deuteron is most easily overcome by looking for additional interactions which would not be expected to play a role in either the particle-antiparticle binary computations discussed in Chapter VI or those for the $p^+e^-\bar{\nu}$ resonance associated with the neutron in Sects. VII. A-B. This approach is certainly not inconsistent with the accepted theory of nuclear structure, which has long emphasized that a non-electromagnetic interaction must be assumed between any two nucleons in order to provide a plausible explanation for observed characteristics of the structure of nuclei[80,164]. Although it is generally believed that such forces involve spin-orbit coupling akin

to the terms in the Breit-Pauli approximation[102], it has also been shown[165] that the coupling constants associated with the interactions between nucleons must be far greater than would be expected based on consideration of electromagnetic effects alone. As an example from experiment, one may consider the energy splittings between the ground and excited states of the ^{11}B and ^{11}C nuclei. According to the nuclear-shell model[95,96], these quantities should give an accurate reflection of the $p_{3/2}$-$p_{1/2}$ spin-orbit splittings of a nucleon in these systems, but their magnitudes (2.14 and 1.85 MeV, respectively) are far larger[166] than can be explained on the basis of Breit-Pauli interactions[102]. Observations of the scattering of polarized nucleons from ^{4}He and ^{12}C nuclei lead to much the same conclusions[167].

There have been a number of concrete proposals for the form of the nucleonic spin-orbit coupling operator[164,168,169], but there is unfortunately no consensus on this point. If one goes through the list of Breit-Pauli terms with the goal of finding an interaction which is capable of influencing the relative stability of the singlet and triplet deuteron states, one discovers that most of them will have no effect on this situation by virtue of the fact that the proton orbitals which are strongly occupied in the corresponding wavefunctions are exclusively of $s_{1/2}$ type. Neither of the spin-orbit terms, nor the spin-spin tensor force or the orbit-orbit interaction has any non-vanishing matrix elements when only such spin-orbitals are involved. Moreover, the Darwin term is completely independent of spin[102] and is therefore also totally ineffective for this task. A clear exception to this pattern still remains, however, namely the spin-spin δ-function term. It is both spin-dependent and capable of producing a non-vanishing interaction between particles occupying $s_{1/2}$ orbitals. In order to obtain a suitably large effect from such an operator when applied to a pair of protons (nucleons), it is clearly necessary to change the value of the coupling constant relative to those of Table I, however, as can be seen from the results of Tables XIV and XV.

It is thus proposed to augment the XBPS Hamiltonian with a term of spin-spin δ-function type in which the q/m_o values for the proton particles are replaced by much larger coupling constants of the order of their electronic counterparts; for concreteness, a value of unity will be taken for this purpose. Otherwise, the previous description of the XBPS Hamiltonian is simply retained and full CI calculations are carried out as in the last section for both the 0^- and 1^- $p^2e^-\bar{\nu}$ states, employing the same values for the damping constant A (1.0567 e^{-1}), the q/m_o ratio for the antineutrino (0.527) and exponents for the Gaussian basis functions as in the treatment whose

results are given in Tables XIV and XV. It is found that the 1⁻ state is virtually unaffected by this change, with a total energy of -71133.109 hartree (-1.936 MeV) having been obtained. This result is only 3.123 hartree above the value shown in Table XV without including the spin-spin δ-function term. On the other hand, the total energy of the $p^2e^-\bar{v}$ singlet (0⁻) state is changed dramatically to +4876.816 hartree, an increase of 392749.721 hartree relative to the treatment without the new spin-spin δ-function term for proton-proton interactions. As a result the lowest state of the $p^2e^-\bar{v}$ system has triplet multiplicity, with a binding energy relative to the proton and $p^+e^-\bar{v}$ system of 2.718 MeV = 71133 + 28781 hartree (compared to the experimental value of 2.22452 MeV).

Examination of the above findings shows that the anti-symmetry of the proton wavefunction is a crucial factor in obtaining this result. The simplest representation of the triplet wavefunction is:

$$\Psi_T = 2^{-1/2} [s_1\alpha(1)s_2\alpha(2) - s_1\alpha(2) s_2\alpha(1)] \hspace{2cm} \text{VII.1}$$

The expectation value of the spin-spin δ-function term for this function has the familiar form of the difference of a Coulomb and exchange integral, but since the δ-function has equal values for these two quantities, unlike the case for the electrostatic interaction, the result is of vanishing magnitude. On the other hand, the simplest representation of the 0⁻ state whose energy contributions are listed in Table XIV is:

$$\Psi_S = 2^{-1/2} [s_1\alpha(1)s_2\beta(2) - s_1\alpha(2) s_1\alpha(1)] \hspace{2cm} \text{VII.2}$$

The expectation value in this case is:

$$<\Psi_S| -\frac{8\pi}{3} \alpha^2\delta (1,2) s_1 s_2 |\Psi_S> = 2\alpha^2\pi <s_1(1) s_1(2) \delta(1,2) s_1(1) s_2(2)>, \hspace{1cm} \text{VII.3}$$

a large positive energy contribution. Adding this term to the Hamiltonian produces a destabilizing effect on the closed-shell configuration of eq. (VII.2). A significantly different wavefunction is obtained for the lowest 0⁻ root from the corresponding 4s,2p full CI treatment

than with the original XBPS Hamiltonian, with much more emphasis on the open-shell configuration analogous to that dominating the triplet wavefunction.

Another encouraging result of the computations with the augmented XBPS Hamiltonian is the finding that the next three most stable states after the 1^- species are nearly degenerate and correspond to the symmetries expected for the translationally excited counterpart of the above ground state. Such a translational function would be expected to have 1^- symmetry itself and thus exchanging it for the 0^+ species of lowest energy as a factor multiplying the 1^- internal wavefunction would lead to new solutions of 0^+, 1^+ and 2^+ symmetry, with these expected to be very nearly equal in energy to one another. This expectation is fulfilled, with total energies of -31260.220 (0^+), -31265.499 (1^+) and -31265.715 (2^+) hartree being found for the most stable states of each of these three symmetries. This example serves as a reminder that the XBPS Hamiltonian of Table I contains translational energy contributions as well, as discussed in detail in Sect. V.B. As a consequence the excited states obtained with this method may differ from the corresponding ground state in the amounts of either their internal or translational energies or both. This introduces an additional complication into the present theoretical treatment which is clearly not present when the center-of-mass motion is factored out (or when the $\mathbf{P}= 0$ condition is imposed), but the present example shows how symmetry characteristics can be employed to distinguish between the two distinct types of excited states. In essence one obtains a discrete representation of the translational continuum in the present treatment of nuclear motion, whereby the density of states obtained is clearly dependent on the number and type of basis functions employed in explicit calculations.

One can summarize the present results for the ground state of the $p^2e^-\bar{\nu}$ system as follows. With the addition of a spin-dependent term for the proton-proton interaction having a coupling constant of the order of the electronic Bohr magneton, it is possible to obtain a state of lowest energy from the XBPS treatment in the 4s,2p basis which has triplet multiplicity and a binding energy relative to the proton-$p^+e^-\bar{\nu}$ dissociation products which is at least on the order of the experimental deuteron value. The attractive central potential which is responsible for this result is provided by the $e^-\bar{\nu}$ complex of 0^- symmetry which is analogous to the e^+e^- ground state associated with the photon in the present model. The spin-dependent proton-proton adjunct to the XBPS Hamiltonian has virtually no influence on the total energy eigenvalue and associated eigenfunction for the triplet ground state itself, but plays a key role in destabilizing the

corresponding singlet spin combination of the protons, particularly the helium-like closed-shell configuration which otherwise is so favored when this term is absent.

These results are obtained with relatively small basis sets and thus it is difficult to make a more quantitative assessment of the accuracy of the present model at this time, but there at least seems justification for pursuing this approach further, both at higher levels of computational accuracy and also for the description of other nuclear systems. There is another comparison with experimental data which is also encouraging, namely for the mean radius of the two nucleons in the deuteron system, estimated[170] to be 4.32 fm or 1.5 α^2 bohr on the basis of scattering observations. The value of the Coulomb proton-proton repulsion of 7726.523 hartree (Table XV) allows a straightforward estimate of this radius for the 1⁻ state, namely 2.43 α^2 bohr. While the agreement between these two results can hardly be described as quantitative, it at least shows that the range of the forces described by the XBPS Harniltonian is physically reasonable. Especially since the calculations are carried out without the *ad hoc* introduction of parametric inter-nucleon potentials, it can be argued that their results speak in favor of the major assumptions underlying the present model, especially the insistence upon treating the electron and antineutrino as being physically present in the deuteron nucleus. Again the results of Table XV appear reasonable on this point, showing kinetic energies for the electron and antineutrino of 36.49 and 41.14 MeV. Under the assumption of a deuteron radius of 1.5 α^2 bohr, an estimate based on the approximation $p \cong r^{-1}$ can be made of 46.68 MeV (pc $=1.5^{-1}\alpha^{-2}c = 1.5^{-1}\alpha^{-3}$ hartree) for each of these quantities.

It also should be noted that the addition of d functions to the basis set employed in the XBPS calculations should also favor the triplet state of the deuteron over that of the singlet. This is because a ds proton configuration can have a J = 1 multiplet but not one with J = 0. It has long been thought[164] that the deuteron ground state contains a small d-function component because of the observation that its magnetic moment deviates by a significant amount (2.6%) from the sum of the proton and neutron moment values. The small but non-zero value of the deuteron's electric quadrupole moment is also consistent with this conclusion. The extent of the present 4s,2p calculations for the $p^2e^-\bar{\nu}$ system precludes the addition of d functions to the basis set at the present time, so it has not been possible to verify this effect explicitly, but the above arguments at least make it seem plausible that such a result would occur.

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Before concluding this section it is well to return to a key point which has wide-ranging consequences regarding the qualitative interpretation of nuclear interactions in terms of neutrons and protons. To obtain a sound basis on which to discuss heavier nuclei it seems inescapable that proton orbitals appear in pairs of nearly equal energy, one set to be associated with neutrons in the conventional model, the other with the corresponding protons themselves. Such a development seems unlikely when reference is made to the electronic structure of atoms and molecules, according to which the lowest two s orbitals are of greatly different energy, for example. The form of the potential employed in the XBPS Hamiltonian suggests that the situation may well be different in the present case, however. Unlike the Coulomb potential, whose r-dependence is monotonic in character, the damped Breit-Pauli counterparts possess an extremum similar to that shown in Fig. 6, which is essential in the present model to avoid variational collapse and hence to suitably describe bound nucleons. Consequently there are equi-potential points on either side of the minima in the variation of the damped Breit-Pauli terms with inter-particle distance which conceivably could lead to orthogonal orbitals of contrasting radial dependence but very nearly equal stability. Once this possibility is accepted, it is not difficult to imagine how the results of calculations for larger nuclei could be formulated in terms of the nuclear-shell model of Goeppert-Mayer[95] and Jensen[96], or even more simply in terms of distinct neutrons and protons serving as the nuclear constituents.

F. STRUCTURE OF THE LIGHTEST NUCLEI

It was recognized by Wigner[171] in 1933 that the binding energies of the simplest nuclides increase very rapidly with atomic mass number, and that this phenomenon is very strong evidence for the short range of the forces involved. Even if one allows for the quadratic increase in the number of nuclear bonds as nucleons are added, one still finds that the binding energy trends are notably different than one observes in the study of atomic and molecular structure. In the present model, in which the electron and antineutrino are treated explicitly in the theoretical calculations, the next simplest system after the deuteron is the ^{3}He nucleus, with a total of five elementary constituents. The experimental total energy of the ^{3}He nucleus relative to that of its separated protons, electron and antineutrino is -254965.67 hartree, which corresponds to a

binding energy of 201992.86 hartree relative to the stable deuteron plus proton fragments. In view of the $\varphi_a\varphi_b$ configuration assumed for the deuteron in the last section, the simplest assumption is that the additional proton occupies one of these two orbitals with opposite spin. Since the potential in Fig. 6 has only one extremum, it seems plausible that a third orbital φ_c would not be sufficiently stabilized by the $e^-\bar{\nu}$ complex to make a proton configuration with three open shells advantageous. Because of the 0^- character of the $e^-\bar{\nu}$ unit the overall symmetry of such a $\varphi_a^2\varphi_b$ ground state would be $1/2^-$.

In order to investigate these possibilities a series of calculations has been carried out for the $p^3e^-\bar{\nu}$ system employing the 4s,2p basis introduced in Sect. VII.B. It was not possible to solve the secular problem corresponding to a full CI treatment in this case, however. Instead a multiple reference[172] CI common in molecular calculations has been employed, which should be capable of approximating the corresponding full CI eigenvalues and wavefunction to a satisfactory approximation. The CI space considered is generated by taking all possible single and double excitations relative to a series of 42 reference configurations chosen on the basis of the magnitude of their coefficients in the final eigenvectors. The Hamiltonian employed is again that of Table I augmented with the spin-spin δ-function term for the proton-proton interaction discussed in Sect. VII.E. The same q/m_o value (0.5375 a.u) for $\bar{\nu}$ is assumed as yields the experimental neutron (negative) binding energy for the $p^+e^-\bar{\nu}$ system (Sect. VII.B). The same value of the exponential damping constant A (1.0567 e^{-1}) is also employed as in the latter treatment. The exponents of the most compact two s-type functions were optimized specifically for the $p^3e^-\bar{\nu}$ system in the $1/2^-$ state, and it was found that these values are about 30% larger than those which produce minimal energy for the $p^2e^-\bar{\nu}$ system, *i.e.* $\alpha_1 = 0.338 \times 10^9$ a_o^{-2} and $\alpha_2 = 0.8 \times 108$ a_o^2).

The total energy obtained in the optimum 4s,2p treatment is found to be -212334.852 hartree. Optimization of the above exponents from their original $p^2e^-\bar{\nu}$ values brought an energy lowering of 14562 hartree, indicating a substantial contraction of the charge distribution of the protons in the larger $p^3e^-\bar{\nu}$ system. The resulting total energy is thus 42631 hartree (1.16 MeV) higher than the experimentally deduced value for the ^{3}He nucleus mentioned above. The computed binding energy relative to the $p^2e^-\bar{\nu}$ 1^- state (see Table XV) and a free proton is 141202 hartree. Although this is a substantial amount, it is still only 69.9% of the experimental energy difference between the ^{3}He nuc1eus and the deuteron plus proton system. The spin-spin

δ-function proton-proton interaction is quite important in this determination, having a value of 127952 hartree, as compared to virtually zero magnitude for the corresponding $p^2e^-\bar{\nu}$ expectation value in the 1^- state.

The most important configuration in the $p^3e^-\bar{\nu}$ wavefunction is of the $\varphi_a^2\varphi_b$ type, so that the simplest interpretation is that that the more compact s-type orbital corresponds to that occupied by the two ^{3}He protons in the conventional description. The magnitude of the Coulomb repulsion for the three protons is 35369.772 hartree, nearly five times the value found for the $p^2e^-\bar{\nu}$ 1^- system. Since there are three times as many pair-wise interactions in the heavier system, this result is also evidence for a general contraction of the $p^3e^-\bar{\nu}$ system relative to its two-proton counterpart (see Table XV). This result is understandable with reference to the nature of the XBPS Hamilonian employed in the present treatment. As a new proton is added to the $p^2e^-\bar{\nu}$ system, one has both an increase in the total kinetic energy of these heavy particles and an enhancement of their net attractive interactions for the $e^-\bar{\nu}$ complex of lighter elements, thereby disturbing the equilibrium prevailing for the original nuclide. Because the changes in the short-range attractive interactions are greater, this leads to a drawing of the entire system together and a greater increase in total binding energy than would be possible by simply occupying the original proton orbitals of the lighter system. Under these circumstances one must be wary of making a strict correspondence between the $s_{1/2}$ orbitals occupied in the deuteron and ^{3}He respectively. The fact that only one electron and antineutrino are present inevitably leads to the designation of only one of the nucleons as a neutron (open-shell occupation) and the remaining pair as protons (closed-shell).

This line of argumentation leads to an interesting question, however, namely what happens when still another proton is added to the system. On the basis of what has been said above, it is tempting to think that the resulting (^{4}Li) nuc1eus would be bound by a rather large margin, possessing a c1osed-shell $\varphi_a^2\varphi_b^2$ proton configuration. A Li isotope of this mass number is not stable, which raises the specter of a breakdown in the above model, or at least unusually high Coulomb repulsion effects that make such extrapolations very inaccurate. This need not be the case, however, because the ^{4}Li system is more likely unstable because it is prone to electron capture and the subsequent formation of the highly stable alpha particle ^{4}He (E=-982528.11 hartree), rather than because it is subject to spontaneous decomposition itself.

Calculations similar to those discussed above have thus also been carried out for a system with four protons and a single $e^-\bar{\nu}$ unit. The closed-shell $\varphi_a^2\varphi_b^2$ proton configuration mentioned above corresponds to a 0^- $p^4e^-\bar{\nu}$ state. The most stable state of this symmetry in the present treatment does not have a significant contribution from this configuration, however. Instead, proton p orbitals show

significant occupation, which according to the nuclear-shell model are the next available to the hypothetical ^{4}Li nucleus. The total energy of this 0^- state is -219926.758 hartree, some 7600 hartree lower than that of the $p^3e^-\bar{\nu}$ $1/2^-$ ground state. The lowest energy obtained for the four-proton system is for the 0^+ state (-313119.128 hartree). There is also a 1^+ state of only slightly higher energy (-310409.680 hartree) and a 1^- species lying just above it (-303091.067 hartree). In the first two cases the preferred configuration is $\varphi_a^2\varphi_b\varphi_c$, i.e. three-fold $s_{1/2}$ occupation corresponding to the $p^3e^-\bar{\nu}$ ground state discussed earlier and having the remaining proton located in the lowest p shell. Since there is no spin-orbit term of the type foreseen in the nuclear-shell model[95,96] in the Hamiltonian employed, there is essentially no splitting computed between the corresponding $p_{1/2}$ and $p_{3/2}$ sub-shells. Even with such a specific proton-proton interaction, however, it seems clear that the lowest of such states would be quite unstable with respect to the electron capture process required to form the ^{4}He nucleus. The experimental total energy of the latter's 0^+ ground state is still more than 18 MeV lower than any of the calculated results for the $p^4e^-\nu$ system associated with the ^{4}Li nucleus in the present model, and so it seems plausible that such states could not exist for sufficient time to be observed prior to their decay. Hence, no contradiction exists between the present computed results and experiment. The lifetime[173] of the ^{5}Li isotope is only 10^{-21} s and this species should be considerably more stable than the lighter ^{4}Li system, in agreement with this assessment.

As a result, it seems best to terminate the present discussion of systems formed by adding still more protons to a single $e^-\nu$ binary and turn instead to the far more interesting possibility that as the number of protons continues to increase in a nucleus, there is a strong attraction for additional electrons and antineutrinos. The fact that a $\bar{\nu}$ species is also required in electron capture is consistent with the inevitable appearance of a departing neutrino as well (see Sect. IV. B), both of which come from the same massless $\nu\bar{\nu}$ binary according to the present model. Such considerations lead one naturally into a discussion of the weak interaction in theoretical physics, but this is best deferred until the next chapter. Instead a second nuclear series will be considered which results from the addition of successive protons to a pair of $e^-\bar{\nu}$ units.

A system of two protons and two $e^-\bar{\nu}$ singlets can best be thought of in the present model in terms of a deuteron interacting with a single electron-antineutrino unit. The spatial orbitals which comprise the latter's wavefunction must be different than those employed for the first $e^-\bar{\nu}$ complex because according to the XBPS calculations of Chapter VI, every conceivable spin combination other than that for the lowest-lying 0^- state is quite unstable. It therefore follows that the next best two-particle $e^-\bar{\nu}$ function is probably somewhat less stable than the first. In essence the calculations show that one proton must make up for deficiencies related to the electron's pairing with a particle of a significantly smaller charge-to-mass ratio than that of its antiparticle. A proton has the capacity for

doing this because it has a net attraction for both the electron and antineutrino (see Tables X-XV) and it has relatively small kinetic energy in the required short inter-particle distance range. Nonetheless it can't quite achieve net binding with an $e^-\bar{\nu}$ unit on a one-to-one basis. This result amounts to concluding that a bound system of two neutrons does not exist, as is well known experimentally, even though the simplest view of nuclear structure holds that the nn interaction is of quite similar strength as that for pn.

The situation seems likely to improve when an excess of protons is available, especially because the inter-proton distances can become smaller because of the net increase in the spin-same-orbit and Darwin term attraction to the $e^-\bar{\nu}$ species. A system with three protons and two such binaries corresponds to one proton and two neutrons in the conventional accounting, i.e. the tritium nucleus ^{3}H. Experimentally this system is found to be only 0.4935 MeV (18136 hartree) less stable than the ^{3}He nucleus. Considering that their respective total energies are on the order of -250000 hartree relative to their separated (stable) particles, this is a comparatively small difference. As before with ^{3}He, two configurations come into question to describe the ground state, namely $\varphi_a^2\varphi_b$ and $\varphi_a\varphi_b^2$. Because the number of particles increases to seven in the ^{3}H calculations in the present model, it has not yet been possible to carry out explicit calculations to study this point. One can speculate that the relative stability of the φ_a and φ_b proton orbitals must depend fairly strongly on the number of $e^-\bar{\nu}$ units present in a given system, however. It may even be that the designation of each of these $s_{1/2}$ species as either of proton or neutron type is different for ^{3}H. so that the composition of the doubly occupied orbitals of the two systems are actually fairly similar to one another.

In any event the true ground-state wavefunctions of both ^{3}He and ^{3}H can be expected to consist of heavy mixtures of the above two configurations in a CI sense. What of the low-lying excited states of complementary structure which are indicated in each case, however? It seems conceivable that whenever either system occupies the state of lesser stability that it rapidly either loses or gains an $e^-\bar{\nu}$ unit to become the corresponding ground state of the other system without greatly altering the original occupation of its two most stable proton orbitals φ_a and φ_b. In fact, only one of the four possible configurations is truly stable, since the ^{3}H ground state is known to undergo beta decay with a half-life of 12.4 years to form the ^{3}He ground state[173]. In other words, ^{3}He is weakly repelled by a second $e^-\bar{\nu}$ species but this effect can be minimized by altering its proton charge distributions. The situation may also in some way be similar to a phenomenon which often occurs in molecular physics [174], according to which two iso-electronic systems have *mutually inverted ground and excited states* with correspondingly distinctive nuclear conformations. For example, ozone prefers to doubly occupy the $4b_2$ orbital which tends to give it an open-chain structure, whereas cyclopropane[175] prefers the $2b_1$ species instead, which in turn allows it to have a triangular ring conformation. Both molecules have

excited states with the opposite occupations, however, whose nuclear conformations are quite different as a result, corresponding to an open-chain form of cyclopropane and a ringed isomer of ozone. In this case the numbers of electrons are equal for the two systems but the nuclear environment is quite different. In the present comparison of ^{3}H and ^{3}He, the number of protons is equal according to the present model (corresponding to the mass number in standard usage) , but the two systems differ in the number of e$^-$ $\overline{\nu}$ units combined with them in each case.

Another important characteristic of the e$^-$ $\overline{\nu}$ complex which it shares with the e$^+$e$^-$ massless system is that the Darwin repulsion between the two constituent particles is very small (see Tables X-XV) As remarked in; Sect. VI.D, the expectation value for the un-damped Darwin term vanishes exactly for the e$^+$e$^-$ 0$^-$ prototype system. In both cases this result indicates that the particles avoid each other so completely that they never (or only rarely in the e$^-$ $\overline{\nu}$ case) reside in the same region of space (with or without the same spin). This result strongly implies that a definite region of space must be reserved to accommodate each electron-antineutrino pair within a given nucleus. Analysis of Table III shows that this situation arises primarily because it allows for maximum advantage to be taken of the attractive Breit-Pauli interactions between the two particles. In e$^+$e$^-$, for example, any change in the respective charge distributions which leads to a non-zero δ-function expectation value must result in a net increase in total energy because a variational minimum is involved. The situation is not quite as severe for e$^-$ $\overline{\nu}$ (Tables X-XV), but nonetheless the absolute magnitude of the Darwin term in this instance is only 5% of that of the corresponding spin-same-orbit expectation value. At the same time the calculations indicate that the proton and electron charge distributions are not similarly restricted. as a rather large Darwin interaction is invariably computed between them (178980.181 hartree in Table XV, for example).

These results are very reminiscent of the experimental observations mentioned in Sect. IV:C which indicate that the nuclear volume is essentially directly proportional to the number of its constituent nucleons. According to the present calculations, it appears somewhat more precise to say that the nuclear volume is proportional to the number of constituent e$^-$ $\overline{\nu}$ units, which arithmetically amounts to essentially the same relationship. In other words, because each electron must avoid each antineutrino, one can expect a fixed volume to be reserved for every such pair of light particles to insure maximum stability for the nucleus as a whole. The protons are much freer to move in a small volume because their rest masses are so much greater, but even as they shrink in orbital size with increased binding, the corresponding e$^-$ $\overline{\nu}$ charge distributions remain relatively unaffected. This observation is also consistent with the form of the exponential damping factors in the spin-same-orbit and Darwin terms (Table I), which as mentioned before allow a closer approach to the lighter particles to maximize the attractive forces as each proton is added to the system. The exponential operators are

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strongly dependent on the momentum (and consequently the orbital compositions) of the electron and antineutrino, however, so that the volume occupied by them is relatively independent of the number of protons in their environment. Only if the proton occupation drops below or rises above a critical level necessary to support a given number of $e^-\bar{\nu}$ units is a volume change advantageous, which is to say a beta interaction occurs to either decrease or augment the number of electron-antineutrino pairs within the confines of the nucleus.

To conclude this section it only remains to discuss the result of adding a fourth proton to the double $e^-\bar{\nu}$ complex. The product is clearly the relatively stable ^{4}He nucleus, originally named the alpha particle a century ago[48]. It has a closed-shell ground state corresponding to the $\varphi_a^2\varphi_b^2$ proton configuration and represents an optimal proportion between the number of protons and electrons for such light nuclei The experimental binding energy of the last proton (compared to ^{3}He) is 728212 hartree. Relative to two neutrons and protons this represents a binding energy of 1.04 megahartree, or in excess of 250000 hartree per proton {nucleon). The latter result is about 2.5 times larger than the corresponding value for ^{3}He and ^{3}H. The indication is that the proton φ_a, φ_b orbitals shrink markedly as the last proton is added, which is quite consistent with the closed-shell configuration characterizing this system. The fact that ^{4}He is the only stable nucleus with this mass number indicates that the number of $e^-\bar{\nu}$ units is similarly optimal, however, as already discussed in connection with the hypothetical ^{4}Li system.

G. NUCLEI HEAVIER THAN THE ALPHA PARTICLE: COMPARISON WITH THE NUCLEAR-SHELL MODEL

The discussion of the last section is seen to be largely consistent with the nuclear-shell model of Goeppert-Mayer[95] and Jensen[96], differing from it mainly in the concept of protons interacting with $e^-\bar{\nu}$ units rather than neutrons as elemental particles. The lowest-energy proton $s_{1/2}$ orbitals φ_a and φ_b simply replace the corresponding neutron and proton species employed in the shell model. A key element in the XBPS calculations which suggests this adjustment is the finding that the most stable state of the $e^-\bar{\nu}$ system is a singlet. Since neither the electron nor the antineutrino can exist inside the nucleus without the other, it follows that pairs of these fermions can be added indefinitely without directly affecting the magnitude of the angular momentum of the constituent protons of a given system.

The possibility exists that the hierarchy of spin-orbitals for proton-neutron shells in the original model can be obtained from calculations based on an augmented XBPS Hamiltonian similar to that employed above in Sects. VII.E-F, in which case the resulting proton shells are expected to occur in pairs of the same symmetry, in analogy to the φ_a and φ_b $s_{1/2}$ species already considered. It is likely that

the $p_{1/2}$ $s_{1/2}$ configuration for $e^-\bar{\nu}$ is not the only possibility, however. Any configuration of two orbitals of the same j but differing by one unit in l contains a single 0^- multiplet, and thus different $e^-\bar{\nu}$ units could vary widely with respect to their l-type orbital composition while still retaining their singlet characteristic. One can conceive of a series of concentric spherical units, each differing in volume from their neighbor of smaller radius by a constant amount, consistent with the arguments of the last section based on the Darwin term. The exact nature of these relationships is clearly a matter to be settled by explicit calculations for larger nuclei.

The negative parity of the $e^-\bar{\nu}$ units must have an effect on the overall nuclear symmetry, but it is generally accepted that there is no way to determine absolute parities of nuclear states experimentally[154]. In the case of nuclei it should be noted that parity is always inferred indirectly on the basis of angular momentum changes which occur upon scattering relatively light systems such as deuterons off heavier nuclei. Analysis of the resulting proton or neutron capture reactions[176] allows a determination of the l value of the incoming nucleon from which the corresponding parity change is deduced. Since the $e^-\bar{\nu}$ complex has J = 0, it is impossible to learn anything about its parity from such investigations, so that existing assignments of nucleon quantum numbers necessarily only refer to the proton orbitals which are occupied in the calculations of the present model. By assuming that the $e^-\bar{\nu}$ units always have 0^- symmetry, it is always a simple matter to convert the total J values obtained on the basis of the present model to those deduced from the nucleon capture angular momentum distributions. For example, since the deuteron has a single such unit, it follows that the 1^- symmetry computed for its ground state is tantamount to predicting that the results of such experiments will lead to a 1^+ assignment, as actually observed. Nonetheless, although the intrinsic negative parity of the $e^-\bar{\nu}$ unit defies detection in the traditional nuclear physics experiments described above, it will be seen to have interesting consequences regarding the interpretation of other pivotal observations in elementary particles physics, as will be discussed in succeeding chapters.

It is well to emphasize that the XBPS treatment does not assume fixed positions for the various nucleons, in contrast to the practice in the Born-Oppenheimer approximation commonly used in molecular calculations. As a result, the symmetry of the corresponding Hamiltonian is that of the full rotation group with inversion, exactly as assumed in the nuclear-shell model. This feature does not really distinguish molecules from nuclei, however, because the rotational states of the former also transform rigorously as irreducible representations of the same point group[177]. This seems surprising in view of the fact that molecules are said to have electric dipole moments, but a closer analysis shows that such properties only result as a consequence of the symmetry reduction produced by the introduction of an external electric field[178]. The electric dipole moments of molecules are thus seen to result because perturbations of this nature cause rotational states of different parity to mix with one

another. In the last analysis it is the relatively small energy separation of rotational states which distinguishes molecules from nuclei in this respect. In summary, the rotational symmetry of the XBPS wavefunctions results because no comparable "clamped-nuclei" assumption to that of the Born-Oppenheimer approximation is made, rather than because a convenient simplification is introduced *ad hoc* in order to maintain identification with the principles of the nuclear-shell model. The calculations discussed above have the disadvantage of simultaneously including translational effects as well, but the difficulties brought about by this characteristic become somewhat less critical for systems containing relatively heavy particles such as protons.

After the $1s_{1/2}$ shells have been filled, the shell-model leads us to expect that the next most stable orbital available to succeeding protons is $1p_{3/2}$, closely followed by $1p_{1/2}$. Since the present model foresees pairs of such proton orbitals, it might be expected that the same symmetry is not always involved for both partners. Because the two orbitals are assumed to occupy essentially equi-potential locations (see Sect. VII.E and Fig. 6) relative to a given $e^-\bar{\nu}$ unit, however, it seems at least plausible that they differ only in their respective radial distributions. The operators involved in the XBPS model are of the spin-orbit and related type, and as this choice has been suggested in part by the results of the nuclear-shell model, it is thus clearly consistent with this approach. If the traditional q/m_o factors of the Breit-Pauli Hamiltonian[102] (Table I) are employed exclusively, it is seen that the only interactions of this nature which are of great importance for protons are the spin-same-orbit and Darwin terms, whereas the $e^-\bar{\nu}$ units participate in all such interactions (except the Darwin term for the most part). A spin-orbit term which can account for the large $p_{3/2}$- $p_{1/2}$ and related splittings deduced from the locations of nuclear energy levels[95.96.164.166] is also required, however, which has far larger coupling constants than those found in the conventional Breit-Pauli interactions[102] between protons.

A spin-spin δ-function operator employing a unit (electronic) q/m_o value for the proton-proton interactions has been found to be effective in explaining the triplet multiplicity of the deuteron's ground state (Sect. VII.F), but it is clear that such a term cannot account for the observed splittings of j levels belonging to the same l quantum number. It needs to be recognized that such spin-orbit interactions cannot be assumed to occur between a proton and an antiproton without upsetting the arguments of Sect. V.D. They predict that the e^+e^- and p^+p^- systems have wavefunctions which are strictly related by a scaling operation. It is also clear that some type of long-range damping is required, similar to that originally suggested by Yukawa[87.88], to account for the fact that the influence of the strong interaction does not appear to extend beyond nuclear dimensions.

The search for such spin-dependent proton-proton interactions is clearly far easier when an accurate description of the central field attracting these particles is available. It is in this context that the present XBPS Hamiltonian can play a crucial role. In this model the $e^-\bar{\nu}$ units provide a strong

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attraction for protons while having no direct effect on the angular momentum of the nucleus as a whole by virtue of their 0^- symmetry. In this view the bond between a proton and a neutron comes about to a large extent because two protons are strongly attracted to the same electron and antineutrino. Since the protons in question occupy different orbitals (Sect. VII. E), their individual relationships to the lighter particles can still be quite different from one another, thereby making it appear that they are associated with two fundamentally different particles, *i.e.* a neutron on the one hand and a proton on the other, exactly as foreseen in the nuclear-shell model.

H. THE ISOSPIN PROPERTY AND ITS RELATION TO THE XBPS MODEL

Before concluding this comparison between the results of the preceding calculations and the concepts which form the basis of the theory of nuclear physics, it is important to give close consideration to another fundamental quantity, the isospin property[84,85]. The present model employs a Hamiltonian which depends only on the spatial and spin coordinates of the constituent particles of a given system, and on this basis the corresponding energies and other expectation values are computed without making any prior assumptions whatsoever regarding the isospin property. It is thus important to see how the fundamental relationships underlying the isospin theory might be inferred from the results of such calculations. The idea which led to the introduction of isospin can be traced back to the suggestion of Heisenberg[83] in 1932 that a neutron and a proton can be looked upon as two different states of the same system (nucleon). As such, these fundamental particles are referred to as an isospin doublet, differing only in the z component of the vectorial quantity **I**. The latter's transformation properties are assumed to be identical to those of orbital and spin angular momentum, and so a well-defined mathematical framework for the theoretical treatment of isospin is immediately at hand.

The most important physical conclusion regarding such isospin multiplets is that the various component systems would be perfectly degenerate if only the hadronic force were active, and not the electromagnetic (gravitational forces can be safely ignored because of the nature of the problem). The theory asserts that under these circumstances the rest masses of the proton and neutron would be of identical magnitude. In the present model emphasis is placed instead on the possibility of converting a proton into a neutron by the addition of an electron and an antineutrino in the form of an $e^-\nu$ binary unit. In this sense, elimination of the electromagnetic force can be equated with placing bare protons in an environment which is absolutely free of electrons and antineutrinos. Because of the high density of massless e^+e^- and $\nu\bar{\nu}$ binary systems, so the present model goes, this state of affairs is impossible to achieve by any experimental means, similarly as it is not really feasible to turn the electromagnetic force off and on at one's volition.

Explicit calculations with the XBPS Hamiltonian show that a system of two protons forms neither. bound states nor resonances, and hence in this sense the pp interaction is purely repulsive. The reason that roughly one hundred years of scattering experiments have been interpreted differently is understandable in terms of the premise that whenever two protons approach each other at very close range, they have a sufficiently strong attraction for an electron and an antineutrino to overcome the dissociation barriers of the ever-present e^+e^- and $\nu\bar{\nu}$ binary systems, thus making these particles available, i.e. "creating" them in the conventional terminology. The fact that the partner positrons and neutrinos are always set free in such nuclear formation processes is consistent with this interpretation, although the evidence is clearly less than totally unambiguous. To settle the matter definitively would require experimental proof that particles can be created or destroyed with appropriate application of energy, which amounts to observing the unobservable. Suffice it to say that the charge independence of nuclear reactions, which is at the root of the isospin concept, is accounted for in the XBPS model. Thus to the extent that a suitable Hamiltonian can be found and the associated Schrödinger equation solved to a satisfactory degree of accuracy, one can expect the key results of isospin theory to emerge from such treatments without having to make additional assumptions of an ad *hoc* nature.

One of the greatest successes of the isospin theory in the realm of nuclear physics is the elucidation of trends in the rest masses of isobaric nuclides[179], i.e. groups of systems with the same mass number. One of the clearest examples of this type is the isospin triplet ^{14}C, ^{14}N and ^{14}O in their respective 0^+ states. In such cases it is found that the differences in the rest masses of these systems can be estimated to a good approximation by comparing the respective magnitudes of the Coulomb repulsion of their constituent protons and correcting for the promotion energy required to convert protons into neutrons. In the present model isobaric nuclides differ from one another mainly in the number of their constituent $e^-\nu$ units, each of which changes the atomic number of a given nucleus without changing its mass number. Assuming that the various proton shells correspond to pairs of orbitals φ_a, φ_b which can be associated on a one-to-one basis with the neutron and proton one-particle functions of the nuclear-shell model, it is only necessary in the present view that the occupation of the outermost shells (in the above case of $p_{1/2}$ symmetry) change in concert with the addition or loss of the $e^-\bar{\nu}$ units.

The relatively good agreement achieved with experimental data through the above approximation suggests that proton orbitals can be distinguished in a reasonably unambiguous manner on the basis of their spatial relationship to the $e^-\bar{\nu}$ unit lying closest to it. Again the existence of pairs of equipotential points in the exponential damped Breit-Pauli terms, as indicated in the schematic diagram of Fig. 6, make such an identification at least plausible. More significant in this respect is perhaps the spin-dependent force which exists between the protons, however. The (partial)

concretization of this interaction discussed in Sect. VII.E prevents the development of atomic-like trends in which shells tend to be completely filled with electrons before the next most stable species is to be occupied. By providing a strong repulsive interaction between protons of opposite spin, this term minimizes the importance of stability differences between two neighboring shells and causes them to be occupied alternately in a manner akin to the way houses and hotels are added to properties of the same color in the parlor game Monopoly. Moreover, the $p^4 e^- \bar{\nu}$ calculations of Sect. VII.F indicate that the gain in stability possible by further occupying one of the "neutron" orbitals is very much dependent on the number of neighboring $e^- \bar{\nu}$ units in the system.

Another key aspect of the isospin formulation is its assumption of nuclear wavefunctions which are anti-symmetric with respect to any exchange of a proton and a neutron[84]. This specification is referred to as the generalized Pauli principle[180] to call attention to the fact that in the original work[134] an anti-symmetric form was only required for indistinguishable fermions. In the present model no such generalization is necessary because protons are already assumed to satisfy the Pauli anti-symmetry principle, and accordingly it is actually two of these particles which are being permuted when one speaks in the conventional model of an exchange of a proton and a neutron. The corresponding electron and antineutrino of the "neutronic" proton are totally separate particles in this view, and thus are not affected by such a permutation. In retrospect, it would be more difficult to bring the present model in line with the isospin formulation if the latter insisted that its nuclear wavefunction *did not have to satisfy* a particular permutation symmetry for the exchange of a proton and a neutron. This eventuality would force an exception to the original Pauli principle[134] to be allowed in the present model according to which the exchange of two such indistinguishable protons would not lead to a sign change in the total wavefunctions under these circumstances. Instead the realization of the consequences of beta decay of neutrons left no other choice in the formulation of the isospin theory[84,85] to broaden the definition of "indistinguishable" particles in the Pauli principle to include the proton and the neutron in the same class despite their apparently distinctive characteristics.

In this discussion it is also important to recognize that the predictions of the isospin theory in nuclear physics are not of a generally quantitative nature. The comparisons of the energies of isobaric nuclides mentioned at the beginning of this section do often work remarkably well for ground states, but the experience with excited states is less sanguine. In their book Goeppert-Mayer and Jensen[80] point out, for example, that although excited levels of a nucleus with $I_z = 1$ should be closely related to some of those of the corresponding isobar with $I_z = 0$, "seldom has it been possible to make an unambiguous assignment of this kind"[166,181]. As a result these authors prefer to distinguish between the "charge symmetry" of nuclear forces, involving either two protons or two neutrons $I_z = \pm 1$), and

"charge independence" when a neutron-proton interaction is present, concluding that the experimental evidence for the latter property is not nearly as strong as for the former. Furthermore, it is well known that correspondingly simple relationships in either isotopic or isotonic series are all but non-existent, as illustrated by the deuteron-^{3}He-alpha particle series[171] discussed in Sect. VII.F. There is thus a danger of oversimplifying the theory of nuclear binding by rigidly assuming that all nucleons in equivalent shells have the same properties except for electromagnetic effects.

As an additional remark on the relation of the isospin concept to the present model, it seems inescapable that at some point the number of $e^- \bar{\nu}$ units reaches a critical value, at which point it is no longer possible to form even a meta-stable nucleus with the original number of nucleons. In isospin theory such theoretical isobaric members are simply assumed to be non-existent, whereas in an *ab initio* approach they correspond to virtual states whose mass and other properties can be computed in principle, but which in all likelihood are quite unstable to either beta decay or electron capture processes. A similar situation is not uncommon in atomic and molecular physics, whereby states which correspond to bound species for other (isoelectronic) systems are found to lie in some continuum region for the system of immediate interest. Sometimes a resonance of the expected character can be located, but more often than not such corresponding (diabatic) states are totally absent in the computed spectrum, or more precisely, they are apportioned among the wavefunctions of continuum states lying in the appropriate energy range.

In summary, the present model can be expected to be at least indirectly relevant to the isospin concept by virtue of its (hoped for) capacity to successfully predict the masses and properties of a variety of related nuclear systems. To the extent that the solutions of the corresponding Schrödinger equations can be obtained quantitatively, and in the process be shown to reproduce the pertinent experimental data to a satisfactory degree of approximation, such results will inevitably be found to be in agreement with the conclusions otherwise reached independently through application of the theory of isospin. Such an attitude toward a concept which has become so well established over the years as to have been referred to by one author[182] as an "industry" might well be perceived by some as unacceptably *leger*. Opinions have varied widely on this issue, however, as illustrated by a passing reference made in 1950 by Goldstein[183] in his survey of the principles of classical mechanics in which he wrote, "indeed for nuclear forces we don't have any theory worth speaking about." We shall return to the subject of isospin in Sect. IX.J when the relationship of the present model to the quark theory of elementary particles is taken up.

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I. NUCLEAR REACTIONS

The discussion of nuclear forces in terms of the XBPS model has thus far being confined to the study of structure alone. The possibility of carrying out calculations for such systems on an *ab initio* basis also carries with it the prospect of studying nuclear reactions in a new light. One can order these processes most conveniently in terms of the types of particles which are produced by them. These include α, β and γ rays in the original language used to describe this field[48]. Later on neutrons were also discovered as decay products[14], and the general possibilities of nuclear fusion and fission became known. When studying reactions it is necessary to know something about energy barriers separating products from reactants, and not just relative energies in their respective equilibrium states. Since the Born-Oppenheimer approximation is not used in the XBPS treatment, there are no potential curves as such from which to deduce the magnitudes of such barrier heights. For many purposes the variation of the total energy with scale factor η for the associated one-particle basis set provides equivalent information, however, as demonstrated in the e^+e^- and $\nu\bar{\nu}$ treatments (Figs. 7 and 9). As η is increased from small values the energy always rises until a certain point when the short-range interactions begin to overtake the effects of kinetic energy (Fig. 5). The heights of such computed barriers are necessarily basis-set dependent, since for a complete set the corresponding results must be independent of scale factor. For relatively large basis sets this degree of saturation still remains an unattainable ideal, however, and one can hope that data such as shown in the above figures can be successfully analyzed in the suggested manner.

Because of the extensive computations required in the present model for the treatment of systems consisting of a few particles, it is likely that such investigations will not be feasible for heavy nuclei, thus making the study of nuclear fission and alpha-particle decays accessible only if suitable approximations can be found which allow inert shells to be treated in a simplified manner. The subject of beta decay will be taken up in detail in the next chapter, however, and the treatment of photon emissions from light nuclei also appears to be a practical goal of *ab initio* calculations of this type. In the latter case it is necessary to develop methods which clearly distinguish pairs of states differing mainly in translational energy from those corresponding to distinct internal states between which spectroscopically observable transitions are expected to occur.

The most attractive reaction type that might be accessible to treatment by such computational methods is nuclear fusion, since only relatively small nuclei in their respective ground states are involved in the most interesting cases. There has recently been renewed interest in this subject because of reports on electrochemical studies on heavy water with platinum and palladium electrodes[184,185]. The

results of these experiments have led to speculation regarding the identity of the reactions which might be involved, although there is mounting evidence that some of the original observations[186] can be explained to at least a large extent on the basis of purely chemical transformations. Traditional high-temperature fusion experiments have been based largely on the reaction of tritium and deuterium to give ^{4}He plus a neutron[187]. The exothermicity of the above tritium reaction is 646458.62 hartree (17.6 MeV), which compares favorably with that of the analogous fusion process
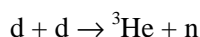
$$d + d \rightarrow {}^3He + n \qquad\qquad\qquad\qquad VII.4$$

whose exothermicity is only 120238.74 hartree (3.3 MeV). Despite the fact that no radioactive substances are required to drive the latter reaction, the higher heat output of the tritium process makes it technically more appealing. The reaction of ^{3}He plus a neutron is itself quite exothermic, however, since it leads to the formation of the highly stable alpha particle. If this product were formed directly in the d + d reaction, the corresponding exothermicity would increase to 876582.49 hartree (23.8 MeV), 6.2 MeV more than in the

t + d counterpart preferred in the high-temperature fusion investigations[187]. Moreover, both reactants and product of the former process are thermodynamically stable.

The reason that ^{4}He is not formed with sufficient regularity from d + d collisions is understandable in terms of the laws of conservation of energy and momentum. A second product is indispensible for sharing the energy potentially released in this reaction, and decomposition into ^{3}He plus a neutron achieves this purpose with relatively high probability under typical reaction conditions. A photon could carry away most of the available 23.8 MeV energy if the intermediate ^{4}He state reached were of non-zero multiplicity, however, so as to make a subsequent emission process to the J=0 ground state allowed. Since deuterons have zero isospin such an excited ^{4}He state must also be expected to be an isospin singlet, but this in itself would not seem to rule out the formation of an intermediate with angular momentum J = 1 or 2 because of the triplet multiplicity of each deuteron. Once the pertinent ^{4}He wavefunctions became available, it would be possible to compute the rate for radiative decay to the ground state as well as for decomposition into the ^{3}He-plus-neutron products. The practicality of such calculations is something which remains open to question at the present stage of development, but this example at least illustrates how the above quantum mechanical approach might be used to shed light on the mechanisms governing the reactions of small nuclei.

VIII. WEAK INTERACTION IN THE XBPS MODEL: PARITY CONSERVATION AND SOLAR NEUTRINOS

In the preceding chapter the strong interaction responsible for nuclear binding has been represented in large part by means of an exponentially-damped, momentum-dependent potential which has as its low-velocity limit the spin-orbit and related operators of the Breit-Pauli Hamiltonian[102]. The key assumption in the accompanying theoretical model is that the antineutrino is capable of undergoing an attractive short-range interaction with other particles because it possesses a non-zero charge-to-rest-mass ratio (its charge and rest mass are each assumed to be zero). Under these conditions it is shown that the two-body potential defined above (XBPS Hamiltonian) is capable of binding protons to $e^-\bar{\nu}$ binary systems with sufficient force to allow identification of the stable products with experimentally observed nuclei. Instead of assuming that the electron and antineutrino decay particles of a neutron are created at the time of its decomposition, it is demonstrated that, contrary to past argumentation, a potential does exist which is sufficiently attractive to confine such light particles within a volume of typical nuclear dimensions. In this view the electron and antineutrino retain their existence within the bound nucleus, and they are simply set free in beta decay processes.

The question that obviously arises as a result is whether such a model can be used in a consistent manner to describe the weak interaction itself, i.e. the force which is believed to be responsible for the beta decay of nuclides. The accepted view in theoretical physics today is that the strong and weak interactions are basically different in nature, although the possibility has long been sought of unifying them in some way with each other as well as with at least the electromagnetic interaction. In more recent times a theory which succeeds in combining the electromagnetic and the weak interactions has been formulated[188], but the hoped-for union with the strong nuclear force has yet to be achieved. Once it has been assumed that the binding of nuclei can be described in terms of interactions involving three different constituents, the proton, electron and antineutrino, however, it is straightforward to assume that the same types of effects must be involved when such systems undergo spontaneous decomposition. This line of approach is investigated in the present chapter.

A. QUANTUM MECHANICAL THEORY OF RADIATIONLESS TRANSITIONS

It has been mentioned in Sect. IV.A that the neutron can be looked upon as an excited state of a system whose ground state is unbound, a state of affairs which is analogous to that existing for excimers in molecular physics. The beta decay of a neutron involves a radiationless transition in the

language of that field, since photons are not emitted in the process. The most direct way of computing the lifetime of any radiationless decay is to solve the corresponding Schrödinger equation for the excited state in question (more commonly referred to as a resonance). The eigenvalues associated with such meta-stable states are complex ($E_r - iE_i$) and the line width Γ associated with its decay is equal to twice the imaginary component

E_i[189]. The same Hamiltonian is employed for this purpose as for the computation of bound states. The distinction between stability and meta-stability of a given state arises naturally out of the solution of the corresponding Schrödinger equation, namely in terms of the magnitude of E_i (zero for stable, non-zero for unstable or resonance states). Upon carrying over this formally simple approach to the present discussion, one must expect that the neutron's energy eigenvalue is actually complex and corresponds to a line width

$\Gamma = \hbar/\tau$, where the lifetime τ is observed experimentally to be 918 s.

Because the neutron's lifetime is quite long in comparison with those of typical atomic or molecular radiationless processes, it is quite impractical to compute its value in the above manner, however. The imaginary part of the corresponding energy eigenvalue is only in the order of 10^{-18} eV, as compared with a total (real) energy of 29000 hartree, so the accuracy required in obtaining a useful result in this way is prohibitively high. Instead, it is far better in such cases to resort to an approximate method closely related to it, namely the Fermi golden rule[190], which can be derived from the time-dependent perturbation formalism introduced by Dirac[191]. In essence low-order perturbation theory is employed to obtain the desired result, and because the effect of interest is very small the accuracy expected from this level of approximation is correspondingly high. The usual procedure is to divide the total Hamiltonian into two parts, H_0 and H', and first obtain solutions for the former operator's Schrödinger equation which can subsequently be used as a basis in which to apply the golden-rule formalism. In a typical application a spin-free Hamiltonian (H_0) is diagonalized, while the spin-orbit operator serves as H'. In the present example, it is very difficult to know how to divide up the XBPS Hamiltonian, however, especially to identify a small interaction in it which is responsible for inducing the transition from initial to final. Such a division of the total interactions is not essential, however, as can be judged from the complex eigenvalue approach first discussed, in which only the full Hamiltonian is actually needed. Because the golden-rule formalism deals with off-diagonal matrix elements of H' in a basis of functions which diagonalize H_0 it follows that the corresponding full Hamiltonian matrix elements have the same magnitude as those for the perturbing term alone.

On this basis it can be recognized that the main purpose of dividing up the full Hamiltonian into H' and H_0 components is to provide a suitable definition of the initial and final states of the problem. The difficulty in applying the golden-rule formalism for the XBPS Hamiltonian in the description of

neutron decay thus comes down to identifying realistic initial and final states which represent the physical situation satisfactorily. In a spin-orbit pre-dissociation of a molecular energy level[192], for example, one chooses pure spin states of different multiplicity for this purpose. In another typical example, Born-Oppenheimer solutions are taken as initial and final states and allowed to interact via the nuclear kinetic energy operator (non-adiabatic effects). A completely equivalent way of proceeding in both cases, however, consists of first diagonalizing the full Hamiltonian and identifying solutions which correspond to mixtures of the initial and final states normally employed in the golden-rule approach. A unitary (diabatic) transformation can then be defined which connects the diagonal representation of the full Hamiltonian to one which is non-diagonal, based on the physically appropriate golden-rule states. The off-diagonal matrix elements of the latter Hamiltonian matrix are then recognized as the quantities to be substituted in the golden-rule formula for the desired lifetime computation.

When described in this way the above procedure sounds a bit arbitrary, which in a certain sense it is, but no less so than in the alternative method of dividing up the full Hamiltonian into two parts. The correct lifetime can only be obtained if the choice of initial and final states conforms to the experimental conditions at hand. That is why the complex eigenvalue method is more acceptable in principle because no comparable choice is needed in this case. Instead, it is only necessary to identify the Schrödinger equation solution which corresponds to the physical state of interest. In summary, to describe the weak interaction in terms of the XBPS Hamiltonian it is not necessary to first devise a new interaction, but rather to identify a suitable diabatic transformation which leads to the physically meaningful initial and final states involved in the decay processes of interest.

B. SPIN-FLIP MECHANISM FOR NEUTRON DECAY

In Sect. VII.A it was noted that the most stable state of the $p^+e^-\bar{\nu}$ system to be associated with the neutron has $1/2^-$ symmetry and that the $M_s = 1/2$ component can be described primarily in terms of the spin configuration $p\alpha$ ($e^-\alpha\,\bar{\nu}\,\beta - e^-\,\bar{\nu}\,\beta\alpha$), that is, the $e^-\,\bar{\nu}\,0^-$ complex discussed above is bound meta-stably to the proton in this model. The spin of the neutron is thus determined almost entirely by that of the proton. The spins of the three particles emitted after neutron decay have been studied experimentally by Burgy et al.[72]. These authors found that of the three $M_J = 1/2$ spin combinations possible for the $p^+e^-\bar{\nu}$ system, one is missing almost entirely, namely $\alpha\alpha\beta$. It is thus seen that the antineutrino carries away the original total spin of the neutron to a large extent, and this implies that the proton and electron form a singlet complex once decomposition takes place. Since the beta decay process involves a transition between two states of the same system, we can combine the above

theoretical and experimental results to arrive at the following interpretation. In the initial (meta-stable) state the spins of the electron and antineutrino are opposite and constantly alternating. At the time of decay a *spin flip* occurs so that the proton and electron spins become opposite and alternating and the $\overline{\nu}$ spin consequently must remain fixed. The system is expected to be quite unstable in the resulting spin configuration, so that decomposition occurs as a consequence.

A key result of the above $p^+e^-\overline{\nu}$ calculations based on the XBPS Hamiltonian is that the absence of the ααß spin configuration is not complete. The CI coefficients of Table IX show that the ideal ratio of $\sqrt{3} : 1$ of the two doublet spin eigenfunctions (see Sect. VII.A) corresponding to the same spatial orbital occupation is never quite attained. Discrepancies of 1- 2 % in the square of this ratio are generally observed.

With reference to the general discussion of lifetime computations given in the preceding section, it seems reasonable to assume that this admixture of another $1/2^-$ spin eigenfunction is at least partially responsible for the neutron's instability. Because of this impurity in the $p^+e^-\overline{\nu}$ wavefunction (Ψ_1) there is a small probability that the system foregoes the 0^- arrangement preferred by the $e^-\overline{\nu}$ complex, thereby causing the system as a whole to break apart. A diabatic transformation can then be defined along the lines discussed above which removes this impurity by mixing in a second state (Ψ_2) which consists primarily of the other (undesirable) spin configuration. The $p^+e^-\overline{\nu}$ diabatic state Ψ_i^D with the pure $e^-\overline{\nu}$ 0^- component is then defined as:

$$\Psi_i^D = \cos\Theta\,\Psi_1 + \sin\Theta\,\Psi_2 \qquad\qquad\qquad \text{VIII.1}$$

A second diabatic state corresponding to the orthonormal $1/2^-$ spin function is similarly defined as:

$$\Psi_2^D = -\sin\Theta\,\Psi_1 + \cos\Theta\,\Psi_2 \qquad\qquad\qquad \text{VIII.2}$$

Normally several such excited states are required to allow for a thorough deperturbation of the desired initial diabatic state, in which case a unitary transformation of higher dimension is required.

In order to use the results of the $p^+e^-\overline{\nu}$ calculations discussed in Chapter VII to obtain an estimate of the corresponding decay lifetime, it is necessary to compute matrix elements over the full XBPS Hamiltonian between the diabatic states defined in the last section. This is a simple matter if the eigenvalues corresponding to the adiabatic excited states Ψ_n needed to define the required unitary transformation are known, involving a reverse transformation of the diagonal Hamiltonian matrix in the original eigenvector basis. This brings us to an important qualitative point, however. The decay process must involve a final state of the same total energy as that of Ψ_1^D which rules out any of the other diabatic

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species described above. All such states must be far less stable, corresponding to a highly undesirable change in the spin orientations of the three component particles relative to those in the initial state. Instead the true final state Ψ_c^D is a member of a continuum corresponding to the free proton, electron and antineutrino system of the same energy as Ψ_i^D. More precisely there are an infinite number of such states which correspond to the continuous range of antineutrino and electron energies whose distribution was successfully predicted by Fermi.[50,52,193] in his original theory of beta decay. The other diabatic species mentioned above are thus to be looked upon as virtual states which only act as intermediates in the decay process.

The decay is accordingly expected to be predominantly of second order, and properly described by the golden rule formula[190]:

$$<\Psi_1^D \, H\Psi_c^D> = \sum_n{}' \; <\Psi_1^D \, H\Psi_n^D> <\Psi_n^D \, H\Psi_c^D> (E_n^D - E_1^D)^{-1} \qquad\qquad \text{VIII.3}$$

and $\tau^{-1} = 2\pi |<\Psi_1^D \, H\Psi_c^D>|^2 \, dN/dE,$ $\qquad\qquad\qquad\qquad\qquad$ VIII.4

where τ is the lifetime in atomic units ($\hbar a_0/e^2$) and dN/dE is the density of states of the continuum[193]. From these equations it is relatively easy to imagine why the neutron's decay lifetime is so long. First, the energy denominators $E_n^D - E_1^D$ are expected to be quite large, as already mentioned. Secondly, each of the continuum functions Ψ_c^D is a free-particle state and as such cannot be expected to have very large Hamiltonian matrix elements with the very compact diabatic functions Ψ_n^D which are confined for all practical purposes to within nuclear dimensions. In this connection it is well to note that the matrix element $<\Psi_1^D \, H\Psi_c^D>$ is not computed *ab initio* in the original theory[193], but rather is inferred from the measured lifetime of the decay process.

Thus far we have only emphasized the effect of different spin orientations on the lifetime of the $p^+e^-\bar{\nu}$ system, but it also seems clear that the $s_{1/2}$-$p_{1/2}$ (or $p_{3/2}$-$d_{3/2}$) polarization effects discussed in Sect. VII.A also should be an important factor. One of the major inducements for the binding of a proton in the XBPS calculations is its attractive spin-same-orbit interaction with the electron, which for an $s_{1/2}$ proton orbital requires that the electron occupy a $p_{1/2}$ species as far as possible. The antineutrino resists any polarization of the electronic charge distribution from $s_{1/2}$ to $p_{1/2}$ because such a change decreases its own attraction for the electron. When a relatively small number of protons is present in a nuc1eus as compared to the number of $e^-\bar{\nu}$ units, the competition for the electron's favor is expected to be intense, and this in turn increases the possibility of a spin flip of the type discussed above. When more protons are added, however, the electron can more readily adjust its orbital character to satisfy them at the expense of the

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antineutrino without sacrificing the overall stability of the nucleus. It can thus be anticipated that nuclei of high binding energy are characterized by relatively highly polarized $e^-\bar{\nu}$ units relative to their respective proton-free states, while at the same time preserving the 0^- character of the light-particle binaries more thoroughly than has been computed above for the $p^+e^-\bar{\nu}$ neutron system.

Nevertheless, in view of the variety of Breit-Pauli terms which contribute to the stability of the $e^-\bar{\nu}$ units as opposed to those with a major influence on the nuclear protons (Tables XI-XIII), it still seems highly plausible that the electron and antineutrino remain indispensible to one another for the purposes of keeping them bound within the narrow confines of any given nucleus. This characteristic of the XBPS Hamiltonian (Table I) thus ensures adherence to the well-known fact that beta decay always involves an addition or loss of the same number of electrons as antineutrinos (or positrons and neutrinos). In this way neutrons can be converted into protons and *vice-versa* as a consequence of beta decay. When an excess of protons is present the results of the XBPS calculations (Sect. VI.F) indicate that a relatively high binding energy is still possible. Under these circumstances, however, the nucleus becomes unstable to electron capture that would otherwise lead to the conversion of protons into neutrons via this mechanism in order to further enhance stability. Ultimately in the present model it is the high density of the e^+e^- and $\nu\bar{\nu}$ massless binaries throughout the universe (see Sect. II.D) which keeps systems such as ^{4}Li from having a stable existence.

C. LONGITUDINAL POLARIZATION AND PARITY CONSERVATION

In Sect. IV.B the phenomenon of longitudinal polarization of the emitted particles in beta-decay processes was briefly mentioned. In the period immediately following the first such experiments[57] physicists quickly had to come to grips with the surprising thesis that parity is not conserved in the weak interaction. This proposal had first been made by Lee and Yang[56] in 1956. The original motivation for their hypothesis was the observation[194] of two seemingly identical particles, then tentatively referred to as τ and Θ, which nonetheless were thought to decay differently from one another (see Sect. IX.C). As mentioned before, since the evidence from nuclear and atomic-molecular processes was consistent with the view that parity is conserved in all physical interactions, the assertion that this state of affairs does not hold generally was first met with considerable skepticism.

The experiments of Wu *et al.*[57], carried out in response to the suggestion of Yang and Lee and fulfilling their expectations in a spectacular manner, changed this view dramatically, however. The situation is perhaps best illustrated by a letter[66] written by Pauli shortly after being confronted with this experimental evidence. The theoretician who had prevented the discarding of the conservation of energy principle[49] in interpreting the results of beta decay by postulating instead the existence of a previously

unknown particle[37], the neutrino ν, was now put in the position of having to accept the fall of an equally cherished law of nature, the conservation of parity. When we examine the theoretical arguments which ultimately led to this development, however, we find that an important assumption of a different kind is also involved, one which has been at issue from the outset of the present study, namely the hypothesis of the creation and annihilation of matter. It is therefore interesting to review the key experimental observations with special emphasis on this aspect of the theoretical description.

The ^{60}Co beta decay studied by Wu *et al.*[57] involves the Gamov-Teller selection rule[59], $\Delta J = \pm 1$, which makes the experimental verification of the longitudinal polarization of the emitted particles somewhat easier to follow. Since the experiment is carried out at an initial temperature of only 0.01° K in the presence of a strong magnetic field, the J = 5, M_J = 5 state of ^{60}Co is populated almost exclusively prior to decay, from which the J = 4, M_J = 4 ^{60}Ni state must be formed in view of the above selection rules. Conservation of angular momentum thus requires that the emitted electron and antineutrino both have α spin, *i.e.* pointing in the same direction as that of the product nucleus. Since electrons are *observed to exit predominantly in the direction opposite to that of the external magnetic field,* the hypothesized polarization effect is clearly demonstrated by this experiment. The polarization is not complete, but it is found to be of the highest degree that can be expected when account is taken of the fact that the electron's velocity is less than the speed of light. As Wu recounted in a subsequent review article[195], "... viewed from the position of the emitted beta particles, the nuclei ^{60}Co appear to rotate clockwise; left can be distinguished from right, therefore, parity is *not* always conserved, as shown by this experiment; moreover, the asymmetry observed is as large as possible".

Subsequent experiments[196] on the electron capture of ^{152}Eu demonstrated that the emitted neutrinos are also left-handed, and by inference that the antineutrino is right-handed. Nuclear recoil in both Gamov-Teller ($\Delta J = \pm 1$) and Fermi ($\Delta J = 0$) decays consistently verify this property of neutrinos and antineutrinos[195,197,198]. In the ^{60}Co decay the recoil of the product nucleus is such, for example, that conservation of momentum forces the emitted antineutrinos to always exit in the direction of the applied magnetic field. This fact when combined with its α spin as deduced above leads to the conclusion that the antineutrino is characterized by positive (right-handed) helicity.

The thrust of the parity non-conservation argument is thus as follows. Since neutron decay takes place in free space, one must expect on the basis of assuming that parity is conserved in all interactions, beta decay in particular, that there should be no preferred direction of motion relative to the nuclear spin. *An applied magnetic field can have no effect on this relationship,* because as a pseudo-vector it is unchanged by a coordinate inversion. If a neutron is a particle without internal structure prior to its decomposition, the observed longitudinal polarization of its decay particles is therefore incompatible with parity conservation. If the original system consists of *three particles instead of one,* however, the situation

changes in a fundamental manner. Particularly if the relative orientation of the spins of these particles plays a major role in determining the stability of the initial and final systems, *it is no longer logically compelling that the longitudinal polarization observed in all beta decays is inconsistent with parity conservation.*

The calculations for the $p^+ e^- \bar{\nu}$ system discussed above indicate, for example, that the $e^- \bar{\nu}$ complex greatly prefers to be in a singlet state, and that the spontaneous decay comes about primarily because of a spin flip which destroys this relationship between these two light particles. The terms in the XBPS Hamiltonian which are responsible for this result are the short-range Breit-Pauli interactions which at typical atomic inter-particle separations correspond to magnetic effects. The Hamiltonian itself commutes with each of the parity, charge-conjugation and time-reversal operators as well as any combination thereof, including the CPT product of the Schwinger-Pauli-Luder's theorem[69-71], so it cannot be claimed that there is any inherent property in the present model which could lead to a non-conservation of any of the above quantities. Calculations with this Hamiltonian find a strong binding for the proton, electron and antineutrino in the lowest $1/2^-$ state of the combined system, which it has been argued (Sect.. VII.C) would correspond to a (meta-stable) particle with a magnetic moment of the order of a nuclear Bohr magneton. After a spin flip occurs of the type discussed above, it can be expected that a quite different situation results, however, *in which each particle is strongly repelled by the same forces which hold the system together in its original meta-stable configuration.* That there might be a high correlation between the spin of each particle and its direction of motion under such circumstances is not at all implausible, in decided contrast to what must be concluded if it is simply assumed that *the electron and antineutrino first come into existence at the time the decay process begins and not before it.*

If we assume that strong magnetic-like fields are set loose at the time of beta decay, in accord with the arguments based on the present calculations, it follows that the spins of the individual particles (and their magnetic moments) determine the strength of these forces. There is no law of conservation of magnetic fields, but the net force on the three constituent particles must be unchanged by virtue of Newton's Third Law as a result of such an internal change in the structure of the decaying system, and the same holds true for the system's total angular momentum. Since no particle is affected by a self-generated magnetic field, it follows that the resulting forces are different for each of the decaying species. For example, the antineutrino is only acted upon by the fields of the proton and electron, while the electron experiences the fields of the antineutrino and proton only. The only requirement is that the vector sum of the individual forces vanishes.

Since the exponentially-damped Breit-Pauli interactions are mainly responsible for such forces according to the present calculations, it follows that the charge-to-rest-mass ratios which are the coupling constants determining the magnitudes of these effects are crucial quantities for understanding the

dynamics of beta decay. In the $p^+e^-\bar{\nu}$ system the absolute magnitude of the q/m_o values decreases in the order: electron, antineutrino, proton. A positive q/m_o value only about 60-70% as great in absolute magnitude as that of the electron is required to obtain the experimental neutron energy in the above calculations. It will be recalled from Sect. VI.I that this choice still allows for a consistent explanation of the non-ionizing properties of neutrinos (Fig. 8) on the basis of the exclusively long-range repulsive character of the corresponding interactions which result from it, while at the same time providing for short-range attractive forces of the strength required to form a suitable $e^-\bar{\nu}$ complex with which to bind protons in nuclei.

On the basis of these considerations an interesting series of correlations can be identified between various theoretical and experimental properties related to the longitudinal polarization phenomenon. For example, the q/m_o values for e^+ and $\bar{\nu}$ are positive, just as are their helicities. Charge conjugation changes the sign of these q/m_o ratios, giving them negative values for e^- and ν, which is also the sign of their helicities. In terms of the Breit-Pauli interactions the correlation becomes even more striking, namely *the magnetic moments of each of the four particles above point in the same direction as their momentum vector in beta-decay.* After taking account of the v/c factor in the polarization function[195,198] one can summarize all known experiments dealing with this phenomenon as follows (Fig. 15): *the emitted electrons and neutrinos and/or their antiparticles always show a strong preference for orienting their magnetic moments parallel to and in the same direction as the forces acting on them after beta decay occurs.*

In any conventional system subjected to a strong magnetic field, it can be expected that the constituents with the largest magnetic moments will show the strongest reaction in order to minimize the total energy, so the direction of the accompanying force can be deduced most simply by observing their behavior. The above correlation is perfectly consistent with such a pattern. The lighter components of the neutron or other beta-decaying system orient themselves with a definite preference. The importance of the v/c ratio also fits in with such observations, since the velocity of the emitted electrons is clearly a good indicator of the strength of the forces to which they are subjected, and their tendency to orient themselves in the magnetic field depends very much on this factor. Since the net force on the nucleus must be zero as a whole because no external interactions are present, it is also clear that not every constituent is free to alter its spin direction so as to have its magnetic moment parallel to the magnetic field.

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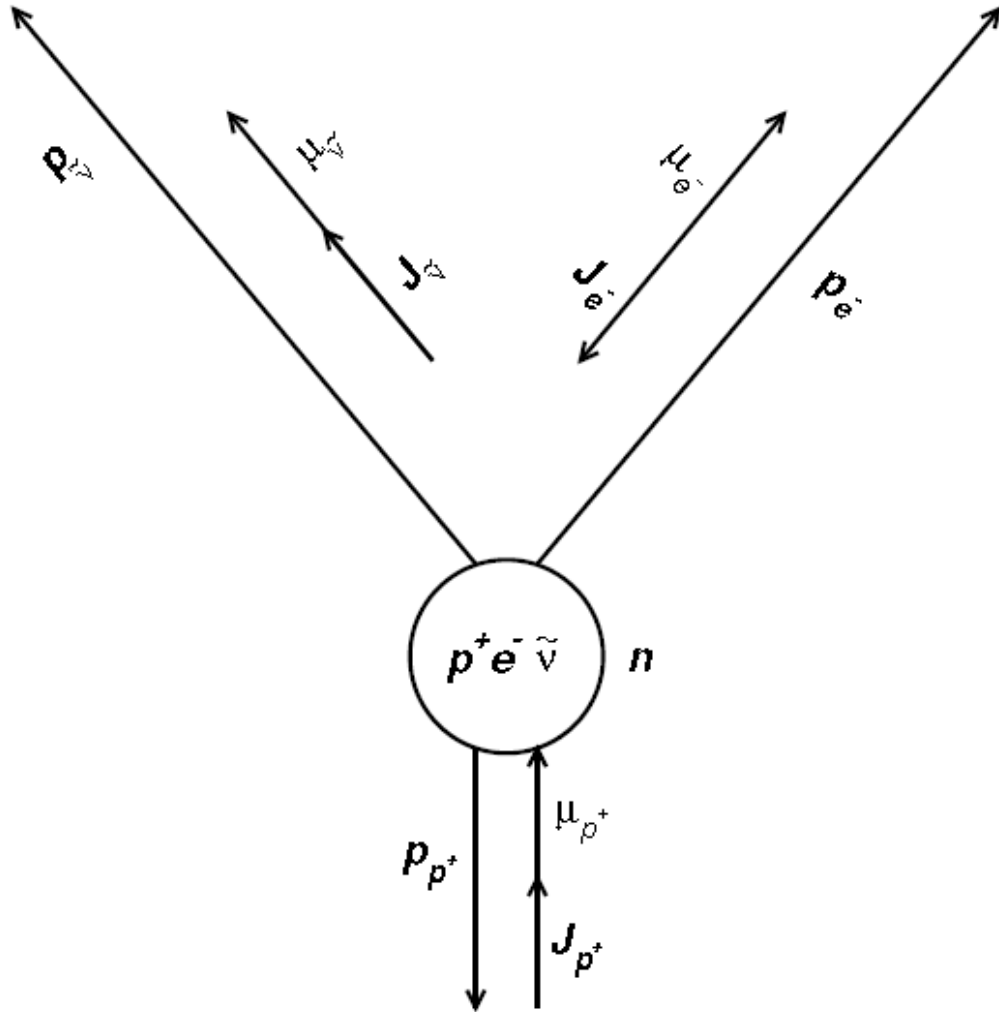
FIG. 15. Schematic diagram showing the relationship between the linear p_i and total angular J_i momenta of the particles emitted upon the beta decay of a neutron. The magnetic moment $\mu_i = (q_i/2m_{oi})\,J_i$ of the electron and antineutrino is observed to point in the same direction as its momentum vector p_i, thus giving rise to the phenomenon of longitudinal polarization. The corresponding proton quantities (with by far the smallest $|q/m_o|$ value of the three decay particles) must point in opposite directions in order to satisfy the conservation laws for linear and angular momentum for the system as a whole. If all three particles are assumed to be present in the neutron prior to decay, it is possible to explain this behavior in terms of inhomogeneous magnetic-like fields which arise as a result of a non-radiative transition (spin flip), rather than as a violation of the law of parity conservation.

As a result, the behavior of the heavier particles in the weak interaction is inevitably more determined by momentum conservation laws than by any magnetic preferences of its own. Thus, the positive muon must leave a decaying pion with the opposite spin and velocity direction as the associated

neutrino[73.74]. Consequently, since a neutrino has negative helicity, so must the positive muon as well. We thus have a system (μ^+) which has a magnetic moment with a sign opposite to that of its helicity. This behavior is exactly what must be expected when one realizes the $|q/m_o|$ value of the neutrino is much larger than that of its heavier counterpart. It therefore has more to gain by orienting its spin in a particular direction than does the muon. Similarly, the proton also exhibits a negative helicity[72.198] in beta decay, i.e. of opposite sign to its magnetic moment. This is clearly seen in Fermi ($\Delta J = 0$) interactions for which both electron and antineutrino must depart in the same direction with opposing spins. Under these circumstances the proton must recoil in the opposite direction. The experimental results indicate that the $\overline{\nu}$ spin is almost always parallel to that of the decaying neutron, so the proton spin must be opposite to that of the electron (and thus parallel to that of the antineutrino) to ensure doublet multiplicity for the system as a whole. Again, the proton can be looked upon as simply reacting to the magnetic behavior of the lighter particles, and therefore foregoing its tendency in the isolated state to align its magnetic moment in the opposite direction. In summary, the lighter particles behave in the weak interaction as if they strongly prefer to orient their magnetic moments in the direction of the respective field acting on each of them (Fig. 15), while the behavior of all other (heavier) particles in the same processes can be understood on the basis of the consequences of the conservation laws for linear and angular momentum.

There is one other key point which needs to be taken into account in attempting to rationalize the observed correlation between magnetic moments of particles and the direction of their motion. A homogenous magnetic field does not impart a net force on a magnet. Instead only a torque is applied which tends to orient the magnetic moment of the system in the direction of the field. Since the particles in beta decay depart with high energy, it is clear that the corresponding fields acting upon each of them cannot be homogenous, but such a property is hardly surprising in view of the spin-flip mechanism in the above model, which causes a rapid reorganization of the magnetic forces. .In the Stern-Gerlach experiment an inhomogeneous magnetic field imparts a net force on an electron or other particle which deflects it in a given direction, depending on the sign of the magnetic moment μ and that of the field gradient. If the **B** field is in the z direction, $E = -\mu_z B_z$ and the component of force in the same direction is given by $F_z = \mu_z \, \partial B_z/\partial z$. This means that if the gradient is positive in the direction of the magnetic field, i.e. $\partial B_z/\partial z > 0$, the direction of the force must be the same as that of the magnetic moment μ_z.

Since electrons and neutrinos prefer to move with their magnetic moments in the same direction as their momenta in beta decay, the above arguments indicate that the field gradient would have to be positive in the field direction for each particle at the time immediately following the radiationless transition. While it is difficult to be certain about such details, the above set of circumstances does make such an arrangement seem at least plausible. If it were possible to observe one of the lighter particles from down field at the start of the decay, it is reasonable to expect that the field strength acting on it would

increase monotonically as it is approached more closely. For the gradient to be positive at the location of the particle itself would mean that there is a type of inertial effect according to which the field's build-up would not stop until some later point in time. The larger this effect, the greater the force that would be imparted to the particle, and the greater the tendency for a system with a large $|q/m_o|$ ratio to orient its magnetic moment along the field direction in which it departs. That would mean positive helicity for positrons and antineutrinos, and negative helicity for electrons and neutrinos, exactly as observed.

The point of this argument is not to insist that specific conditions caused by damped Breit-Pauli. interactions must occur exactly as speculated above. Rather, it is to show that once one assumes that the neutron is a system with a tri-atomic composition ($p^+e^-\bar{\nu}$) *prior to its decay,* it is no longer necessary to assume that the pertinent Hamiltonian must fail to commute with the parity and/or charge-conjugation operations to produce the type of correlations observed experimentally between the respective spin and momentum directions of beta-decay fragments. In other words, *there is no reason to be certain that parity is not conserved in the weak interaction once one gives up the idea that only a single particle with no internal components is present prior to decay.* The same observations can be taken as strong evidence that: a) the electron and antineutrino do exist as components of the neutron or of an associated bound nucleus, and b) the forces which bind them together (and overcome the high kinetic energies required by such close confinement) are very much dependent on the spin orientations of the individual particles both before and after the decay process occurs. Again, it can be noted that the analysis of multiplet structure in the nuclear-shell model of Goeppert-Mayer[95] and Jensen[96] is quite consistent with the second of these points, indicating that strong spin-orbit and related effects of Breit-Pauli type are needed to explain the observed systematics in electromagnetic and other properties of such systems.

Viewed in this manner it is possible to draw a parallel between the longitudinal polarization phenomenon and another effect which proved to be crucial in the development of the theory of molecular structure. For many years there was considerable uncertainty over the question of whether an electrolyte such as NaCl is best looked upon as a neutral diatomic molecule or as a pair of oppositely charged atomic ions. It was known that melts of sodium chloride and other salts could be electrolyzed to give the respective positive and negative ions, but since a substance such as water was thought to consist of molecules even though it can be decomposed by electrolysis into ions as well, this characteristic was not deemed sufficient in itself to settle the issue. The study of the colligative properties of solvents (vapor pressure, melting and boiling points and osmotic pressure) ultimately did produce clarity[199], however, since a system such as NaCl was found to exhibit nearly double the effect per gram-mole of solute as any of a large series of non-electrolytes. This result showed that the individual ions in NaCl enjoy nearly the same freedom of movement in the condensed phase as do single molecules of non-polar substances.

By analogy, the question raised in the present analysis is whether a neutron always behaves as a single unit up to the point at which it undergoes decay. The only way to answer this question affirmatively based on the longitudinal polarization results is to discard the law of parity conservation in the weak interaction. As soon as one leaves open the possibility that the neutron is a meta-stable complex of three different elemental particles, however, such a conclusion is no longer the only alternative. Instead, one can turn the argument around and say, *because parity must be conserved,* there is no recourse but to assume that the neutron does contain several component particles before decay, and therefore that *they are not merely created at the time the decomposition process begins.* The construction of a Hamiltonian which is capable of describing the binding of protons, electrons and antineutrinos within the small dimensions of a nucleus clearly gives credence to the latter interpretation, and thereby raises the possibility that parity, charge conjugation and time reversal , just as energy, linear and angular momentum, are all perfectly conserved in beta decay, as well as in all other physical processes which have yet to be observed.

D. THE FATE OF SOLAR NEUTRINOS

The neutrino plays a potentially crucial role in the field of astronomy because of its ability to penetrate through dense material such as that contained in stars. It is well established that nuclear reactions of various types dominate in processes binding stellar material together, and details regarding their rates and relative occurrence in the sun's core are well understood[200]. As mentioned in Sect. IV. B, however, careful measurements of the solar flux reaching the earth[77] have led to the conclusion that less than half of the neutrinos expected on the basis of the above theoretical conclusions are actually observed. A number of different reactions are involved, but the overall production of neutrinos on the sun is governed by the elemental process in which two protons combine with a single electron to produce a deuteron plus one neutrino. Although vast amounts of matter are involved and the cross section for neutrino detection is extremely small, it can still be argued strongly that the discrepancy between theory and experiment cannot simply be explained away on the basis of faulty observation. On the other hand, the number of protons undergoing fusion on the sun would appear to be equally well established, so that no legitimate doubt can be raised on this basis either.

Especially since the solar neutrino puzzle has been with us for half a century[77], it seems highly possible that a solution exists which does not cast doubt on either the accuracy of the data employed to explain the sun's changing composition with time or the reliability of the pertinent experimental neutrino detection techniques. The neutrino-producing reaction referred to above is closely related to eqs. (IV.ii-v) in Sect. IV.A. Accordingly, it is necessary *that a neutrino be created at the time of the fusion reaction.* In the previous discussion we have consistently employed an alternative interpretation based on the

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corresponding eqs. (IV.ii'-v'). In this view, a $\nu\,\overline{\nu}$ binary species present at the start of the reaction *is dissociated* to produce both the departing neutrino and the antineutrino required for neutron formation. Elemental balance is restored to the fusion reaction and no particles need to be either created or destroyed in the process.

Calculations with the XBPS Hamiltonian lead to a relatively deep energy minimum for the $\nu\,\overline{\nu}$ system (Fig. 9) as a function of basis set scale factor, which can be interpreted in terms of a massless bound state with a relatively small interparticle distance ($r \leq \alpha^2$). In this model the $\nu\,\overline{\nu}$ system can be looked upon as a diatomic system undergoing a reaction in which the bond holding the two component particles together first needs to be broken. If a molecule such as N_2 reacts with a heavy atom according to the equation, $N_2 + X \rightarrow NX + N$, it is generally necessary to add an amount of energy equal to the $N_2\ D_0$ value of 9.92 eV to break the strong N-N bond. If the molecule exists in one of its high-lying ro-vibrational states, a much smaller amount of energy is needed for this purpose, however, and the probability of inducing a reaction is likely to be enhanced as a result. More generally, one speaks of an activation barrier[201,202] to such reactions, with the reaction rate increasing as the higher vibrational levels of the reactant complex become more densely populated.

The $\nu\,\overline{\nu}$ binary system discussed in Sect. VI.H (Fig. 8) does not have a series of ro-vibrational levels comparable to those of a diatomic molecule, but it does have a definite energy barrier for dissociation, the value of which could be a key factor in understanding the results of the solar neutrino observations[77]. It seems at least conceivable that at the high temperatures present in the solar environment (for which kT is on the order of a keV), the equilibrium distribution between bound and separated $\nu, \overline{\nu}$ pairs,
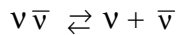
$$\nu\,\overline{\nu} \rightleftarrows \nu + \overline{\nu} \hspace{6cm} \text{VIII.5}$$

would be shifted significantly toward the side of free neutrinos and antineutrinos compared to what is observed under typical laboratory conditions. If, as seems likely, the probability of (essentially) free antineutrinos participating in the key solar fusion processes is considerably greater than for their counterparts found in tightly bound $\nu\,\overline{\nu}$ binary systems, a relatively small change in the position of the above equilibrium could lead to a significant variation in the energy profile of the neutrinos produced in such reactions.

The simplest fusion reaction,

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$$p^+ + p^+ + e^+ e^- + \nu\,\bar{\nu} \rightarrow d^+ + e^+ + \nu \qquad\qquad \text{VIII.6}$$

would now be replaced in a large number of cases by its "neutrinoless" counterpart,

$$p^+ + p^+ + e^+ e^- + \bar{\nu} \rightarrow d^+ + e^+. \qquad\qquad \text{VIII.7}$$

In the latter process the positron is emitted with a mono-energetic spectrum, rather than the continuous range of values expected from eq. (VIII.6), and none of the excess energy is transmitted to a neutrino. Similarly, the other most commonly occurring solar fusion reaction,

$$^7Be + e^- + \nu\,\bar{\nu} \rightarrow {}^7Li + \nu \qquad\qquad \text{VIII.8}$$

in which mono-energetic neutrinos are produced (0.86 Me V), becomes

$$^7Be + e^- + \bar{\nu} \rightarrow {}^7Li \qquad\qquad \text{VIII.9}$$

when free antineutrinos are available. Again, no neutrino is expected to result from this reaction, at least as long as the excess energy can be carried away by another particle such as a photon. In both cases the effect would be a substantial reduction in the number of neutrinos otherwise expected to reach the earth's surface with sufficient energy to undergo the capture reaction employed in the experimental detection scheme of Davis and coworkers[77].

If one assumes that the neutrinos produced in fusion reactions are simply created from nothing as a means of carrying off the energy expended in such processes, there is seemingly no need to consider initial reaction conditions in attempting to predict the energy distribution of the emitted particles. The situation changes dramatically, however, if one recognizes the possibility that the antineutrinos and neutrinos normally observed are actually produced by the dissociation of binary systems in which these particles are strongly bound together. In this case, questions about the equilibrium concentration of bound and unbound species under a given set of experimental conditions, as well as their comparative re-activities, need to be answered in order to obtain a reliable prediction of the outcome of such processes.

IX. ELEMENTARY PARTICLE THEORY WITHOUT CREATION AND ANNIHILATION OF MATTER

The concept of the creation and annihilation of matter is such an integral part of modern theoretical physics that any attempt to modify it requires a wide-ranging survey of many types of phenomena. It is not enough to show that the hypothesis can be avoided in dealing with one group of experimental findings, only to conclude that it is nevertheless essential for a consistent interpretation of other classes of

observations. In the present chapter we will give consideration to the properties of elementary particles observed in high-energy interactions other than those directly involved in conventional chemical and nuclear reactions.

The broad lines of the approach we shall take to this subject have already been laid down in the previous sections dealing with the structure of the neutron. If one makes the counter- hypothesis that nothing appears and disappears from existence in the course of physical processes *and that an essential particle balance is always maintained as a consequence,* it becomes necessary to assume that the neutron is not an indivisible entity, but rather that it is a compound of the three elemental particles into which it decays after a relatively short lifetime. Consistency requires that the same assumption apply to the many (meta-stable) members of the lepton, meson and baryon families which have been identified since high-energy processes became accessible in laboratory investigations. At the same time, it is important to consider what relation such an approach has to existing theories on this subject, particularly those based on the quark model of elementary particles. As mentioned in Sect. IV.D. the possibility that the building blocks employed in a comprehensive theory of elementary particles are themselves composed of still simpler forms of matter does not necessarily invalidate such an alternative model, particularly if useful results can be obtained from it which are either unavailable from the competing theories or can only be extracted from them with significantly more difficulty.

In the present approach, we will continue to replace the conventional assumption of the creation and destruction of particle-antiparticle pairs with that of the existence of binary systems of zero rest mass, specifically e^+e^-, p^+p^- and $\nu\bar{\nu}$. In one sense this substitution has already been programmed into the theory because most elementary particle processes as yet observed can be interpreted in terms of effectively balanced equations obtained by the heuristic addition or subtraction of appropriate numbers of particle-antiparticle pairs. The reactions of eqs. (IV.i-v) and/or eqs. (IV.i'-v') mentioned in Sect. IV.A offer a simple illustration of this general procedure. In many ways it is easier to deal theoretically with particles which simply pass to and from existence, because otherwise one has to be concerned with the detailed representation of states of matter which carry neither energy nor momentum at a given stage of a reaction. The calculations of Chapter VI have attempted to address this problem with the help of a Schrödinger-type formalism, but there remain many details of specific experimental processes which need to receive careful consideration in order to properly assess the viability of the massless binary hypothesis.

A. THE STRUCTURE OF MUONS AND PIONS IN THE XBPS MODEL

The lightest meta-stable elementary particle with a measurable lifetime is the muon $\mu^{\pm}$, a fermion of unit electric charge, existing in particle and antiparticle forms. As before with the neutron, one can

proceed on the supposition that the identity of is decay products is an important indicator of its elemental composition. A significant distinction can be noted, however, namely the muon's decay products are not unique, although this is nearly the case. By far the main decay mode leads to the production of three particles[203.204], $e^{\pm}$, ν and $\overline{\nu}$. In general we will refrain from discussing how the identification of the decay products is established unless there is some important question regarding it. There are three other reactions for which upper limits for fractional occurrence are reported, however. Of these the most likely corresponds to the products $e^{\pm}$ and two photons ($\gamma\gamma$), which are thought to occur in less than 1.6×10^{-3} % of all muon decays. Two other even rarer sets of products (by roughly three orders of magnitude) are respectively 3e ($e^{\pm}$, e^{+}, e^{-}) and $e\gamma$. Under the circumstances there is really no choice in the present approach but to assume that the muon composition is $e^{\pm}$, ν and $\overline{\nu}$, i.e. the major products, but it is useful to consider how the other possible products might arise.

The almost negligible occurrence of the $e\gamma$ decay branch is one of the factors which has led to the conclusion[78] that muon neutrinos are not identical with those occurring in nuclear beta decay, as will be discussed in the fol1owing section. In the present model one can at least rationalize the existence of such a possible decay branch by simply assuming that the component ν and $\overline{\nu}$ species might combine to form a massless $\nu\overline{\nu}$ binary which does not carry away any energy and is thus undetectable. Since the remaining electron product would violate the conservation laws of energy and momentum by taking up all the decay energy, something else would have to be involved, however. Just as with the conventional radiative processes discussed in Chapters II and III, it can be assumed that some energy could be carried away by a photon. Instead of the latter simply being created, however, it is consistent with the present model to assume that an $e^{+}e^{-}$ system which is found in the massless state prior to the initiation of the decay process might undergo an interaction with the muon.

In much the same way, more than one photon might conceivably become involved. At the same time, the decay energy of 104 MeV is more than sufficient to cause an $e^{+}e^{-}$ binary to be broken into its two component elements, thereby producing yet another set of products (3e). The only point that should be emphasized at this juncture is that just because a balanced equation can be written down does not at all mean that the corresponding reaction will occur with significant probability. The potential advantage of an *ab initio* model such as the present one is that in principle it allows not only the computation of the energy and wavefunction of particles such as the muon, but also its relevant reaction rates and/or transition probabilities. The experimental evidence shows that each of the three minor decay modes is rarely if ever observed and one would hope that calculations can eventually be carried out to satisfactorily explain why this is the case.

For the present the immediate goal is to treat the $e\nu\bar{\nu}$ system with the help of the XBPS Hamiltionian in an analogous manner as for $p^+e^-\bar{\nu}$ in Sects. VII.A-B. The simplest way of comparing these two systems is to replace the $0^-e^-\bar{\nu}$ complex of the neutron by the corresponding $\nu\bar{\nu}$ state and let a positron play the role of the proton relative to the earlier calculations. The resulting wavefunction would seemingly be consistent with the fact that the muon's magnetic moment is measured to be exactly what one would expect for an electron with a correspondingly greater rest mass $(207\ m_e)$[205]. Any magnetic properties of ν and $\bar{\nu}$ should be expected to be effectively cancelled, but the binding of an electron to them in a resonance-type state would keep it from behaving as a free particle in the presence of an applied magnetic field. A quite similar argument has been given in Sect.VII.C to rationalize the fact that the neutron's magnetic moment is only on the order of a nuclear Bohr magneton, even though it is also assumed to have an electron as one of its constituents. The energy of the $e^+\nu\bar{\nu}$ system relative to its separated products can be expected to be much higher than for $p^+e^-\bar{\nu}$ because a) a positron must have much higher kinetic energy than a proton in a tight-binding state and b) even though it has a much larger q/m_o value than p, it can make little use of this advantage because of the essential neutrality of the $\nu\bar{\nu}$ component.

To investigate how the XBPS Hamiltonian would describe such an $e^+\nu\bar{\nu}$ system, calculations have been carried out by employing two different basis sets which are analogous to those discussed in Chapter VII for the $p^+e^-\bar{\nu}$ complex. The same neutrino (antineutrino) $|q/m_o|$ values are assumed as before, namely 0.5733 and 0.630 a.u. for the smaller (2s,2p) and larger (3s,2p,2d) basis respectively. It will be recalled that these values were chosen in order to fulfill the condition of obtaining the experimental neutron rest energy of 28781 hartree as lowest eigenvalue in the corresponding full CI treatment. For consistency the previous values of the damping constant A have been employed for each basis (1.054 e^{-1} for 2s,2p and 1.26475 e^{-1} for 3s,2p,2d), these having been chosen in a similar manner to obtain the $2mc^2$ binding energies for the various particle-antiparticle binary systems.

The total energy values obtained from these calculations are shown as a function of the respective basis scale factors in Fig. 16. The small basis leads to a very low minimal energy of -277630 hartree, which would indicate that the $e^+\nu\bar{\nu}$ system is actually bound with respect to the energy of the constituent particles separated to infinity. When the d functions are included the picture changes significantly, however, with the minimal energy of the system increasing to a positive value of 17954 hartree, an increase of nearly 300000 hartree (8.0 MeV) relative to the first result. By contrast, the corresponding $p^+e^-\bar{\nu}$ minimal energies in the analogous treatments are computed (Fig. 17) to be the same in both treatments (as provided for by the above choices of the respective q/m_o antineutrino values in each basis).

The computed results thus indicate that the positron can easily polarize the $\nu\bar{\nu}$ system in

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the very small 2s,2p basis, producing significant binding, but that this process becomes rapidly less effective as polarization functions are added to the basis. The increased value of the damping constant A in the 3s,2p,2d basis greatly reduces the magnitude of the attractive r^{-3} operators in the Hamiltonian, and the corresponding reduction in the system's total kinetic energy brought about by the basis set improvement falls far short of making up for this effect, unlike the situation for its $p^+e^-\bar{\nu}$ counterpart. An analysis of the corresponding energy contributions is given in Table XVI for the larger basis which helps to understand the origin of the above effects. The most novel result in the table is that the usual pattern of signs of the various interactions is not adhered to for the $e^+\bar{\nu}$ pair. Normally one finds that particles with the same sign for their q/m_o values, as in this case, have positive energy contributions for each of the spin-orbit, orbit-orbit and spin-spin interactions, but a negative value for the Darwin term. Instead, for $e^+\bar{\nu}$ one obtains negative values for the spin-other-orbit, orbit-orbit and spin-spin contributions. The spin-spin δ-function term in the latter case has an expectation value which is actually more negative than either of its $e^+\nu$ and $\nu\bar{\nu}$ counterparts.

FIG. 16. Comparison of the variation of the $e^+\nu \overline{\nu}$ (μ^+) total energy (in khartree) a function of the scaling factor η in XBPS model calculations employing two different basis sets.
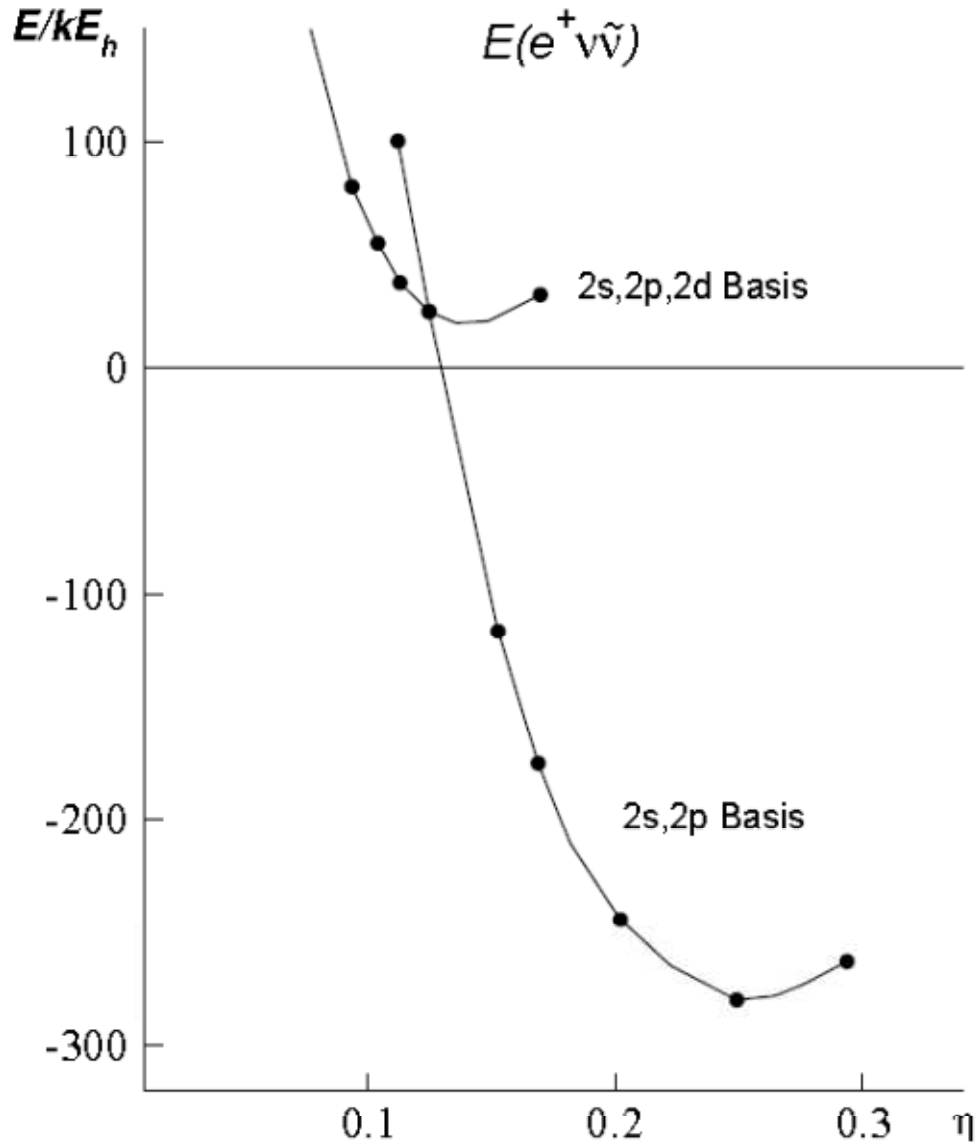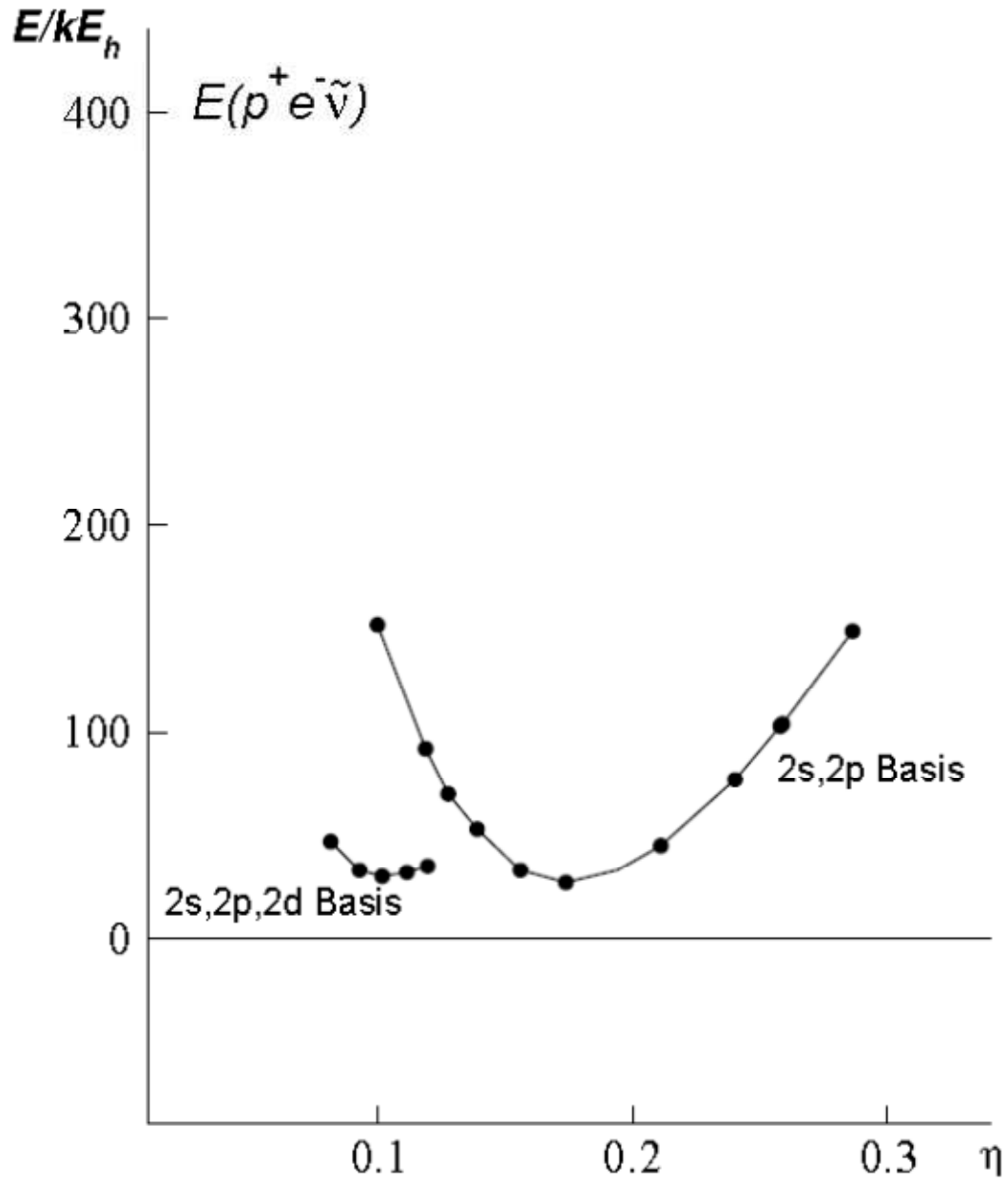
FIG. 17. Comparison of the variation of the $p^+e^-\bar{\nu}$ (n) total energy (in khartree) as a function of the scaling factor η in XBPS model calculations employing two different basis sets.

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TABLE XVI. Energy contributions (in hartree) of various operators (see Table I for definitions) and particle combinations for the $1/2^-$ ground state of the $e^+\nu\bar{\nu}$ muon (μ^+) system obtained by employing the 3s,2p,2d basis with scale factor $\eta = 0.14$, exponential damping constant $A = 1.2648\ e^{-1}$ and antineutrino q/m_o value of 0.63 a.u. for the XBPS Hamiltonian.   .

Operator	$e^+\nu$	$\nu\bar{\nu}$	$e+\bar{\nu}$	Total
Kinetic Energy	914699.446 (e^+)	1246689.492 (ν)	1366978.39 ($\bar{\nu}$)	3528367.329
Spin-same-orbit	-186658.825	-324413.699	37375.604	-473696.920
Spin-other-orbit	-377147.037	-655810.773	-92665.363	-1125623.173
Darwin Term	144372.056	99577.870	-260521.486	-16571.560
Orbit-orbit	-385608.003	-610417.712	-164475.609	-1160501.324
Spin-spin	-157191.708	-303279.608	-24061.778	-484533.094
Spin-spin δ	-57379.559	-42673.178	-149434.600	-249487.337
Total Energy.				17953.923

Comparison with the corresponding 2s,2p results shows, however, that the absolute magnitudes of these various $e^+\bar{\nu}$ terms are only about one-half as large for the 3s,2p,2d basis, and so there seems to be a definite trend to sharply curtail their influence on the overall binding process as the level of treatment improves. It is very difficult to say anything more quantitative about what the minimal $e^+\nu\bar{\nu}$ energy is in the limit of a complete basis being employed, but it seems at least likely that a) a reasonably deep potential well should still exist and b) that the corresponding energy at this point should be much higher than indicated in the 3s,2p,2d treatment. The experimental rest mass of the muon corresponds to a total energy of 3.864×10^6 hartree relative to that of its decay products, so the calculations carried out to date are still greatly deficient in this respect. It may also be that the resonance state most properly identified with the muon at the present level of treatment actually corresponds to one of the computed excited states lying at much higher energy rather than the most stable such species whose properties are shown in Table XVI and Fig. 16.

Qualitatively a rather consistent picture emerges from the computed results, however. The positron must come very close to the $\nu\bar{\nu}$ complex to overcome a large centrifugal barrier. For a narrow range of interparticle distance it might be sufficiently attracted to the neutrino (negative q/m_o value according to the $p^+e^-\bar{\nu}$ computations) to induce a polarization in the $\nu\bar{\nu}$ system. Comparison with the $p^+e^-\bar{\nu}$ results indicates that this is a relatively unstable situation, because the neutrino is more strongly attracted to e^+ than to $\bar{\nu}$ by virtue of the former's larger (positive) q/m_o value. In order to bind such light particles

together it has been found in the preceding chapters that they should have opposite spins. In the $p^+e^-\bar{\nu}$ system the antineutrino is sufficiently more attractive for the electron than the proton that there is relatively little question about which pair of particles should bear a singlet relationship to one another. One can imagine there is a much greater competition between e^+ and $\bar{\nu}$ to form a 0^- complex with a neutrino, however, and that this eventuality shortens the lifetime of this three-particle system considerably. Experimentally the muon decays 4.17×10^8 times faster than the neutron. The possibility that a muon neutrino (ν_μ) is fundamentally different from ν_e is an additional source of uncertainty in the above calculations, as considered in detail in the next section. One could vary. the q/m_o value for ν_μ so as to obtain the experimental muon binding energy, in which case it would be of interest to see how much the two charge-to-mass ratios differ in the $p^+e^-\nu_e$ and $e^+\nu_\mu\bar{\nu}_e$ calculations.

The above model for the structure of the muon also provides a ready explanation for the negligible probability of producing a photon in its decay processes. Since the stability of the $e\nu\bar{\nu}$ system requires the existence of a particular spin relationship between ν and $\bar{\nu}$, it seems quite likely that any bond between them is destroyed in the decay process, causing these two particles to go off in different directions rather than as a bound diatomic system. The decay energy is therefore divided up among all three component particles, with no need to involve an additional system in the neighborhood of the muon at the time of disintegration. The possibility that a muon is formed by the interaction of the same three particles, *i.e.* the reverse reaction, is almost never realized experimentally. Instead the most common mechanism for the formation of muons involves the decay of pions, as will be discussed below.

Since the $\pi^\pm$ mesons are observed to have a neutrino and a muon product in virtually all decay processes, it is consistent with the present model to propose that they are tetra-atomic systems, composed of an electron and three neutrinos. For π^+ it seems likely that two neutrinos are present, along with one e^+ and one $\bar{\nu}$. In this way two of the components would have positive q/m_o values, and two others negative. The μ^+ species has a positive magnetic moment, and thus would be much more likely to form a bond with a neutrino (negative q/m_o) at short range than with an antineutrino. Just as the two muons, the $\pi^\pm$ systems bear a particle-antiparticle relationship to one another, consistent with the above structure. The rest mass of the charged pions is 34 MeV or 1.25×10^6 hartree greater than for $\mu^\pm$. Thus the increase in energy required to attach a fourth particle to the system is about the same as the energy per particle needed to form the $e^+\nu\bar{\nu}$ complex.

The present model also helps to explain why pions are more likely to be formed in high-energy reactions than are muons. The latter are seen to be relatively unsaturated with respect to the addition of neutrinos, and these can be provided by massless $\nu\bar{\nu}$ binary systems. It is not possible to be more definite about such relationships without the benefit of quantitative calculations, but at least a picture of the muon-

pion relationship emerges which is qualitatively consistent with what has been found in the treatment of proton-containing systems. The pions are known to have singlet multiplicity and as such can be distinguished from the other tetra-atomic system treated above, the $p^2e^-\bar{\nu}$ deuteron analogue. The XBPS computations for the e^+e^- and $\nu\bar{\nu}$ systems show that by far the lowest energy results in such light particle-antiparticle pairs for singlet multiplicity. This finding strongly suggests that the most stable states of systems composed of more than two such particles will either be singlets for an even or doublets for an odd number of constituents.

The same argument has been used above (Chapter VII) to justify the well-known fact that the multiplicities of nuclei are determined solely by the nucleon spins, since electrons and antineutrinos must be expected to occur exclusively with paired spins to maintain the required stability within systems of such small radius. The relatively saturated nature of the charged pions also helps to explain the fact that they interact with nuclear matter much more readily than do muons. In this view, while the ingredients for muon formation, namely electrons and neutrinos of both helicities, are available in typical high-energy processes, the possibility of further reaction to form pions under the same conditions is relatively high. In cosmic rays it is found that when the translational energy drops below 100 MeV, however, the pions are much less strongly interacting, at which point their decay into muons becomes the favored process.

Five other minor decay modes are thought to occur for the charged pions, with fractions of 10^{-4}-10^{-8}. The corresponding products are $e\nu$, $\mu\nu\gamma$, $\pi^0e\nu$, $e\nu\gamma$ and $e\nu e^+e^-$. All of these results can be thought of as arising from the addition or loss of a $\nu\bar{\nu}$ and/or e^+e^- binary system. In the first case, for example, it is necessary to assume that a $\nu\bar{\nu}$ species is formed but that all the decay energy is carried away by the remaining electron and neutrino of the original pion. In the second case, an additional photon is observed as well as the usual muon and neutrino products, which is always at least a theoretical possibility in the present model. The π^0 species mentioned in the third minor process requires a separate discussion below, while the manner in which the $e\nu\gamma$ products could be formed is analogous to that in the process already discussed above for muon decay involving a photon. Consequently this is an equally (highly) improbable event, as is the related process in which the photon is replaced by its e^+ and e^- fragments.

The neutral pi meson π^0 has a rest mass only 3.4% smaller than that of $\pi^{\pm}$ and hence is thought to bear a close relationship to them, a feature which is of considerable importance in the theory of isospin. Yet the decay products are quite different for the neutral pion, and thus there is evidence that its composition is notably different than that of the charged species. In this case there are two relatively important decay modes[203,204], namely $\gamma\gamma$ and γe^+e^-, the former occurring 98.83% of the time. The assumed e^+e^- structure of the photon in the present model indicates that the π^0 neutral pion contains at least one

electron and positron. Even though no neutrinos have ever been observed among the decay products, it still seems probable that they are constituents of π^0 based on its close relationship to the charged pions. A conceivable structure would be $e^+e^-\nu\bar{\nu}$, for example, in which case it must be assumed that upon decomposition of the π^0 a $\nu\bar{\nu}$ binary is always formed in a massless state. This would suggest that the decay mechanism is significantly different than for the charged pions, which at least is consistent with the fact that π^0 has a much shorter lifetime (0.84 x 10^{-16} s vs. 2.6024 x 10^{-8} s) than the other two particles. In this case the decay would most plausibly be assumed to arise from a pairing of the $\nu, \bar{\nu}$ and e^+, e^- constituents which is quite different than in the metastable π^0 structure itself. The energy given off in the decomposition would then have to be taken up in part by a neighboring (initially massless) e^+e^- system, producing two observable photons as decay products, either as bound e^+e^- species in the main branch or as one bound (γ) and one unbound electron-positron pair in the other.

Since e and ν have similar kinetic energies in tightly bound systems, it is reasonable to expect that a π^0 structure of this nature would also be a singlet, with about the same energy relative to its separated products as the charged pions. The larger q/m_0 values of electrons *vis-a-vis* neutrinos, plus the possibility of a favorable Coulomb interaction (with suitable polarization of the respective charge distributions) is seemingly consistent with an increase in attractive contributions to the potential energy relative to systems with only one electron and three neutrinos, and this might explain why π^0 is 2.0 x 10^5 hartree more stable than $\pi^{\pm}$ (again compared to their respective separated products). The assumed π^0 structure also agrees with the fact that it is its own antiparticle. From this point of view an $e^+e^-e^+e^-$ composition would be equally suitable, but again consideration of characteristics of the various pion decay modes suggests that such a structure differs too greatly from that assumed for the charged pions to be consistent with the fact that all three of them have very similar rest masses. The $\pi^{\pm}$ decay to π^0 plus $e\nu$ mentioned above can be made into a balanced equation by imparting a portion of the released energy to a neighboring massless e^+e^- system, which then decomposes. The resulting electron would then effectively replace one of the charged pion's neutrinos to form π^0, while setting free the corresponding charged antiparticle. In general, once the available energy exceeds the rest masses of such meta-stable particles, there exists a certain theoretical possibility for their formation because of the relatively small binding energies of the $\nu\bar{\nu}$ and e^+e^- binaries in their massless ground states and the abundance expected for such species based on statistical considerations (Sect. II.D).

B. MUON NEUTRINOS

A key consideration in the foregoing section is whether the neutrinos involved in the decay of pions and muons are different from those first encountered in (nuclear) beta decay processes. In 1961 the

conclusion was reached[78] that they are not the same, and it is interesting to consider the experimental evidence supporting this view. It should be recalled that the XBPS Hamiltonian leaves open the possibility that any q/m_o value for a massless, chargeless particle is consistent with the requirement that its particle-antiparticle binary have zero rest mass (Sect. VI.H). The same damping constant A leads to the desired (vanishing) energy for any q/m_o value by virtue of the scaling theorem discussed earlier, so in this sense the present model would be compatible with the existence of more than one type of neutrino. It is necessary, however, to assume that the rest masses of each type of neutrino be exactly zero to obtain such a result in the XBPS model. Experimentally one is only able to give upper limits for the rest masses of ν_e and ν_μ but there is no proof that the true value might not be zero in both cases, so such a theoretical assumption is tenable. Originally the argument for the muon neutrino being distinct from that involved in nuclear beta decay was based solely on the absence (or at least vanishingly small occurrence rate) of the decay $\mu \rightarrow e\gamma$ (see previous section). A muon quantum number L_μ was introduced[206] to explain this observation (or lack of it). As usual this quantity is assumed to be conserved in allowed processes, and by construction this is not possible for the above photon-producing decay. Accordingly L_μ is ∓ 1 for $\mu^{\pm}$ but zero for e and γ. The pions are also assigned a value of $L_\mu = 0$, which means that the corresponding values for $\overline{\nu}_\mu$ and ν_μ are -1 and 1 respectively, which assignment renders pion decay into a muon and neutrino allowed (*i.e.* L_μ is conserved).

On the basis of L_μ conservation it can also be argued that the reaction of a muon neutrino with a neutron cannot produce an electron and a proton, since only the former particle has $L_\mu \neq 0$. The reaction would be allowed if a muon and a proton were the products, however. An experiment employing high-energy pions to produce muon neutrinos [78] was carried out to test this supposition and the results verified that muons are produced and not electrons, consistent with the L_μ conservation principle. On this basis it has generally been accepted that ν_μ and ν_e are two different entities, each having its own antiparticle and presumably differing in other properties from one another. The helicities measured for ν_μ and ν_e are the same (negative and both positive for the respective antineutrinos)[207,208].

One should be careful to note, however, that the reaction conditions under which pion neutrinos are scattered off neutrons are not the same[78] as for the corresponding processes first studied in nuclear beta decays. For example, the original Reines-Cowan experiment[55] involves antineutrinos of only 2.5 MeV energy, for which the corresponding cross section for proton-positron production is only $11 \pm 4 \times 10^{-44}$ cm^2. The Brookhaven experiments[78] by contrast employ neutrinos of much higher energies (in the GeV range), in which case muons are produced with a cross section as high as 10^{-38} cm^2, at least five orders of magnitude greater. Under the circumstances it cannot be argued with complete certainty that the distinctive reaction profiles of the two types of neutrinos prove that an intrinsic difference exists between

these particles unless it can be shown that such widely different experimental conditions have no effect on the outcomes of the respective experiments. In other words it would not be the first time that different products are found to result from ostensibly the same reaction performed under widely differing experimental conditions.

If one looks at the specifics of the neutrino scattering process, it is clear that a muon can only be produced if the neutrino energy in the center-of-mass framework is in excess of 105 MeV because of the relatively large muon rest mass. This condition is not fulfilled in the Reines-Cowan experiment[55]. On the other hand, it is conceivable that when much higher energies are involved, as in the Brookhaven experiments[78], the production of electrons is suppressed to the extent that only muons are formed initially (muons decay to electrons eventually anyway). The only way to really be certain that this is not the case is to run the two types of experiments under exactly the same conditions, but this is very difficult (impossible?) to arrange in practice. If muon neutrinos are really different from electron neutrinos, it could be demonstrated by somehow accelerating the latter to energies of 1 GeV or more and seeing how many muons and electrons are produced under these circumstances. As it stands now all we know for certain is that the cross section for muon production is relatively high when the reacting neutrinos have energies in excess of 1 GeV, while that for electron production is much lower for neutrino energies of a few MeV and essentially zero for energies in the GeV range.

Especially since the longitudinal polarization experiments for the two types of neutrinos indicate no difference in their helicity properties[207,208], the availability of such experimental evidence for high-energy ν_e scattering (E $\geq$ 500 MeV) would seem highly desirable, if not essential, to remove any doubt that ν_μ might not in fact be identical to ν_e. Based on what has been found in calculations with the XBPS Hamiltonian for the $p^+e^-\bar{\nu}$ system, which in the present model is associated with the neutron, it is not difficult to imagine that the probability of freeing an electron by scattering neutrinos off a meta-stable nucleus is very small. With relatively little kinetic energy it is very difficult for the neutrino to overcome the substantial centrifugal barrier (Figs.8-9), and thus come close enough to the system's electron to dislodge it. Increasing the energy to the GeV range decreases the neutrino's de Broglie radius by at least 100 times, which greatly increases the probability of decomposing the nucleus, as observed in the high-energy pion experiments[74]. At such close distances the neutrino is much more likely to make use of the short-range attractive potential needed to obtain binding in such systems according to the present model. In the process of removing an electron from the rest of the nucleus under such high-energy conditions, it therefore seems likely that the neutron's antineutrino could retain its attraction for the electron while at the same time forming a bond with the incoming neutrino. The result would be that the tri-atomic $e^-\nu\bar{\nu}$ complex is formed, which in the last section has been identified with the negative muon.

In other words, increasing the energy of the scattering neutrinos not only increases the probability of breaking up the nucleus but also can be expected to have a decided influence on the nature of the decay products as well. When the energy falls below the 105 MeV threshold, it is impossible to form a muon under any circumstances, so when a reaction does occur, the only possibility is for an electron to be set free. Above this energy the experiments might be indicating that the only way to free the electron from its position within the nucleus is to have it become attached to the incoming neutrino without losing its hold on an antineutrino also present in the scattered nucleus.

Without the benefit of quantitatively reliable calculations it is difficult to either prove or disprove the validity of such a model. If the neutrino q/m_o value required for the present XBPS Hamiltonian (retaining the original e^+e^- damping constant A) to yield the experimentally measured rest mass of the muon as the minimal energy for a (meta-stably) bound $e\nu\bar{\nu}$ system is the same as needed to obtain the neutron's rest mass for the $p^+e^-\bar{\nu}$ system, such evidence would seemingly favor the conclusion that ν_μ and ν_e are actually identical, as suggested by the arguments given above. If instead, one looks at such reactions as involving annihilation of incoming particles and creation of outgoing species in their place, it is reasonable to overlook such possibilities and simply conclude that at least two fundamentally different types (or flavors) of neutrinos must exist in order to explain the results of the Brookhaven high-energy pion experiments[78]. Once it is assumed that particles cannot be created and destroyed, however, it is necessary to consider the mechanism of the neutrino attachment process in more detail, and it becomes less obvious that such a conclusion is justified.

C. CHARGED KAON COMPOSITION AND DECAYS: SYSTEMATIC NOTATION

Since a general procedure has been developed in the previous chapters to describe the compositions of elementary particles, it becomes convenient to define some simplified notation which will help to make the subsequent presentation more compact. Especially after the discussion of the preceding section, it is at least tempting to restrict the class of *Aufbau* particles to include only the proton, electron and neutrino plus their respective antiparticles. It is helpful then to define a "composition vector" which contains an ordered set of occupation numbers for these particles in the order p^-, e^+, $\bar{\nu}$. p^+, e^-, ν, i.e. antiparticle occupations before the decimal point, and corresponding particle values after it. For example, a proton is 100 (omitting the decimal when no antiparticles are involved), an electron is 10 (omitting left-hand zeroes where possible), and a neutrino is simply 1 (without a decimal point). The corresponding antiparticles then are 100*.,* 10. and 1. respectively, suppressing right-hand zeroes when only antiparticles are present. The corresponding three particle-antiparticle binaries are then denoted by 100.100, 10.10 and 1.1 respectively, i.e. zeroes are only used as placeholders when absolutely necessary to avoid any ambiguities regarding which particles are present in a given system.

With the help of this notation a number of the most frequently occurring elementary particles are analyzed in terms of their known decay products in Table XVII. Since the thesis put forth in the above discussion is that different groups of decay products can only differ from the corresponding meta-stable particle's composition by an integral number of particle-antiparticle binaries, however, a further simplification can be introduced. Instead of listing the composition vectors for each collection of decay products, the numbers of such (differentiating) particle-antiparticle binaries is listed in the order: p^+p^-, e^+e^- and $\nu\bar{\nu}$. Net losses of such species are indicated with a bar over the corresponding number. As an example, consider the muon and its various decay products. Since its composition has been assumed above to include one electron (μ^-) one antineutrino and one neutrino, the corresponding vector is 1.11 (or 11.1 for μ^+). Since the major decay products are $e^-\nu\bar{\nu}$, no change in the corresponding composition vector is needed to represent this branch, as indicated by a zero in the right-hand part of Table XVII. The next set of decay products considered is $e\gamma\gamma$, which can be obtained relative to the original 1.11 composition by the addition of two e^+e^- binaries and the corresponding loss of one of $\nu\bar{\nu}$ type. Hence this decay is listed as $2\bar{1}$ (again left-hand zero occupations are suppressed in the notation, in this case indicating the lack of involvement of p^+p^- binaries). Note that the corresponding μ^+ decay also involves the $2\bar{1}$ differential occupation of binary systems, this time relative to a 11.1 composition. Charge conjugation is represented in the composition vector notation by interchanging parts of the vector lying on opposite sides of the decimal point, but no corresponding change in the net gain or loss of particle-antiparticle binaries is ever needed. The other two decays discussed in Sect. IX. A involve a $1\bar{1}$ change in binary systems in both cases, i.e. producing respectively three electrons ($e^\pm e^+e^-$) or $e^\pm\gamma$. In general no distinction is made between γ and e^+e^- in this notational scheme, consistent with the discussion in Chapters II and III in which the photon is assigned an electron-positron composition. The various conventions may be conveniently checked using the example of the pions and their decays, also as discussed in Sect. IX.A.

TABLE XVII. Classification of elementary particles by means of composition vectors as discussed in the text (rest masses in units of MeV /c^2). The composition vector is an ordered set of six occupation numbers abc.def, whereby. a, b and c are respectively the number of constituent antiprotons, positrons and; antineutrinos, and d, e and f are the corresponding numbers of protons, electrons and neutrinos. Left-hand zeroes are suppressed and the decimal point is given explicitly only when antiparticles are present. Also listed are the known (or in some cases, assumed) decay products of each particle and the fraction of each type of decay (see Refs. 203-204). The number of particle-antiparticle binary systems needed to balance the corresponding decay reactive equation (binary increment BI) according to the present model are listed in the last column by means of the ordered set of numbers ghi, where g, h and i are respectively the number of additional p^+p^-, e^+e^- and $\nu\bar{\nu}$ species required (a bar over any of the latter quantities indicates that the binary systems must be included as products rather than reactants to achieve

elemental balance; 0 denotes that no change in the corresponding number of binaries is needed relative to the reactants and decay fragments as written).

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
proton	P^+	938.2592±0.0052	100	stable		
electron	e^-	0.5110041±0.0000016	10	stable		
neutrino	ν	0.0	1	s table		
antiproton	p-	938.2592±0.0052	100.	stable		
positron	e+	0.5110041±0.000001.6	10.	Stable		
antineutrino	$\bar{\nu}$	0.0	1.	Stable		
prophoton	p^+p^-	0.0	100.100	Stable		
photon	$e^+e^-(\gamma)$	0.0	10.10	Stable		
photrino	$\nu\bar{\nu}$	0.0	1.1	Stable		
muon	μ^-	105.659±0.0003 (τ=2.1994 x 10^{-6}s)	1.11	$e^-\nu\bar{\nu}$	1.00	0
				$e^-\gamma\gamma$	< 1.6 x 10^{-5}	$2\bar{1}$
				3 e	<6 x 10^{-9}	$1\bar{1}$
				$e^-\gamma$	< 2.2 x 10^{-8}	$1\bar{1}$
charged pion	π^-	139.5688±0.0064 (τ = 2.6024 x 10^{-8}s)	2.11	$\mu^-\bar{\nu}$	1.00	0
				$e^-\bar{\nu}$	1.24 x 10-4	$\bar{1}$
				$\mu-\bar{\nu}\gamma$	1.24 x 10-4	10
				$\pi^0e^-\bar{\nu}$	1.02 x 10-8	10
				$e^-\bar{\nu}\gamma$	3.0 x 10-8	$1\bar{1}$
				$e^-\bar{\nu}e^+e^-$	< 3.4 x 10-8	$1\bar{1}$

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TABLE XVII continued

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
neutral pion	π^0	$134.9645 \pm 0.007\,4$ $(\tau = 8.4 \times 10^{-17}\text{s})$	11.11	$\gamma\gamma$	0.9883	$1\,\bar{1}$
				$\gamma\,e^+e^-$	0.0117	$1\,\bar{1}$
				$\gamma\gamma\gamma$	$<5.0 \times 10^{-6}$	$2\,\bar{1}$
				$e^+e^-e^+e^-$	3.47×10^{-5}	$1\,\bar{1}$
charged kaon K$^-$		493.715 ± 0.037 $(\tau = 1.2371 \times 10^{-8}\text{s})$	13.22	$\mu^-\,\bar{\nu}$	0.6352	$1\,\bar{1}$
				$\pi^-\pi^0$	0.2106	0
				$\pi^-\pi^-\pi^+$	0.0559	2
				$\pi^-\pi^0\pi^0$	0.0173	11
				$\mu^-\pi^0\,\bar{\nu}$	0.0324	0
				$e^-\pi^0\nu$	0.0485	$\bar{1}$
				$e^-\pi^0\pi^0\nu$	1.8×10^{-5}	10
				$\pi^-\pi^{\mp}e^{\pm}\nu$	3.7×10^{-5}	1
				$\pi^-\pi^{\pm}e^{\mp}\nu$	$<5.0 \times 10^{-7}$	1
				$\pi^-\pi^-\mu^+\nu$	0.9×10^{-5}	2
				$\pi^-\pi^+\mu^-\,\bar{\nu}$	$<3 \times 10^{-6}$	2
				$e^-\,\bar{\nu}$	1.38×10^{-5}	$\bar{1}\,\bar{2}$
				$e^-\,\bar{\nu}\,\gamma$	$<7 \times 10^{-5}$	$\bar{2}$
				$\pi^-\pi^0\gamma$	2.66×10^{-4}	10
				$\pi^-\pi^+\pi^-\gamma$	1×10^{-4}	12
				$\pi^0e^-\,\bar{\nu}\,\gamma$	3.7×10^{-4}	$1\,\bar{1}$
				$\pi^-e^+e^-$	$< 4.0 \times 10^{-7}$	$\bar{1}$
				πee	$< 1.5 \times 10^{-5}$	$\bar{1}$
				$\pi^-\mu^+\mu^-$	$< 2.4 \times 10^{-6}$	1
				$\pi^-\gamma\gamma$	$< 3.5 \times 10^{-5}$	$1\,\bar{1}$
				$\pi^-\gamma\gamma\gamma$	$<3 \times 10^{-4}$	$2\,\bar{1}$
				$\pi^-\nu\,\bar{\nu}$	$<1.4 \times 10^{-6}$	$\bar{1}\,0$
				$\pi^-\gamma$	$<4 \times 10^{-6}$	$\bar{1}$

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
				$\pi^- e^+ \mu^-$	$< 3 \times 10^{-8}$	0
				$\pi^- e^- \mu^+$	$< 1.4 \times 10^{-8}$	0
				$\mu^- \nu \bar{\nu} \bar{\nu}$	$< 7 \times 10^{-6}$	$\bar{1}\,0$
neutral kaon K_S^0		497.71 ± 0.13	13.13	$\pi^+ \pi^-$	0.6881	0
(short-lived)		(τ=8.82 x 1^{-11}s)		$\pi^0 \pi^0$	0.3119	$1\,\bar{1}$
				$\mu^+ \mu^-$	$< 7.0 \times 10^{-6}$	$\bar{1}$
				$E^+ e^-$	$< 3.5 \times 10^{-4}$	$\bar{3}$
				$\pi^+ \pi^- \gamma$	2.3×10^{-3}	10
				$\gamma\gamma$	$< 7.0 \times 10^{-4}$	$1\,\bar{3}$

TABLE XVII continued

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
Neutral kaon	K_L^0	497.71 ± 0.13	13.13	$\pi^0 \pi^0 \pi^0$	0.215	20
(long-lived)		(τ=5.181 x 10^{-8}s)		$\pi^+ \pi^- \pi^0$	0.126	11
				$\pi^+ \mu^- \bar{\nu}$	0.269	0
				$\pi^+ e^- \bar{\nu}$	0.388	$\bar{1}$
				$\pi^+ e^- \bar{\nu} \gamma$	0.013	$1\,\bar{1}$
				$\pi^+ \pi^-$	1.57×10^{-3}	0
				$\pi^0 \pi^0$	9.4×10^{-4}	$1\,\bar{1}$
				$\pi^+ \pi^- \gamma$	$< 4.0 \times 10^{-4}$	10
				$\pi^0 \gamma\gamma$	$< 2.4 \times 10^{-4}$	$2\,\bar{2}$
				$\gamma\gamma$	4.9×10^{-4}	$1\,\bar{3}$
				$e^\pm \mu^\mp$	$< 1.6 \times 10^{-9}$	$\bar{2}$
				$\mu^+ \mu^-$	$< 1.9 \times 10^{-9}$	$\bar{1}$
				$E^+ e^-$	$< 1.6 \times 10^{-9}$	$\bar{3}$
Eta	η	548.8 ± 0.6	22.22	$\gamma\gamma$	0.380	$\bar{2}$
		(τ=2.50 x 10^{-19}s)		$\pi^0 \gamma\gamma$	0.031	$1\,\bar{1}$
				$3\pi^0$	0.300	11
				$\pi^+ \pi^- \pi^0$	0.239	2

$\pi^+\pi^-\gamma$	0.050	1
$\pi^0 e^+ e^-$	$<4.0 \times 10^{-4}$	$\bar{1}$
$\pi^+\pi^- e^+ e^-$	1.0×10^{-3}	1
$\pi^+\pi^-\pi^0\gamma$	$<2.0 \times 10^{-3}$	12
$\pi^+\pi^-\gamma\gamma$	$<2.0 \times 10^{-3}$	11
$\mu^+\mu^-$	2.2×10^{-5}	$\bar{1}\,0$
$\mu^+\mu^-\pi^0$	$<5 \times 10^{-4}$	1

The next most massive mesons are the charged kaons $K^{\pm}$. Their decay energy is 3-4 times larger than for the muons and pions, and as a result it comes as no surprise that the number and variety of corresponding decay products grows accordingly. No less than six branches are observed with fractions in excess of 1%. In addition, many other less probable reactions are thought to occur. The first problem that arises as a result is that it is now less clear what elemental composition should be assigned to these particles prior to their decay. It is interesting to compare this situation with that faced by chemists when they are confronted with a previously unknown system which is found to participate in several different reactions under similar conditions. The new ingredient in the puzzle which clearly complicates matters in the present case, however, is that there is no way to be absolutely certain how many particle-antiparticle binaries are either consumed or generated during the course of high-energy elementary particle decays. Under the circumstances one can never really claim that an absolute determination of the structure of a given particle has been made. There is naturally a certain logic in choosing a composition which requires a minimal addition or loss of various particle-antiparticle binaries to achieve elemental balance in the totality of all decay modes for the system, and this approach has generally been taken in constructing the entries of Table XVII. In the future it can be hoped that calculations of the particle's rest mass along the lines discussed in Sect. IX.A and earlier in this work will help to verify whether a given assignment is realistic, but even if that situation comes to pass, it is still necessary to face up to the fact that there can never be any definitive means of proving that a particle's elementary composition has been correctly determined. The potential non-observability of the massless particle- antiparticle binaries which might be involved in a given decay reaction precludes this possibility.

With these preliminary remarks let us consider what composition can be assigned to the charged kaons based on the present model. One possible choice for the composition vector of K^- is 2.11, i.e. the same as for π^-. The possibility that two different elementary particles might have the same composition in terms of numbers of electrons and neutrinos cannot be ruled out, because their different properties may

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simply arise because of the way the constituent particles are bonded together in each system. Again, in terms of conventional phenomena known in chemistry and molecular physics, one can speak of either different isomers or different excited states of the same collection of atomic building blocks (elements). It is at least conceivable that a collection of one electron and three neutrinos can be bound together in ways that are sufficiently different to produce distinct meta-stable states differing from one another by a large amount of energy (rest mass). On the basis of the calculations already discussed, for example, it is clear that rearranging the respective spin orientations of the various pairs of constituent particles can have a profound effect on the resulting total energy of the state formed.

The principle of requiring a minimal number of particle-antiparticle binaries to account for the observed decay modes of a given system nevertheless suggests a different assignment for the K^- structure, namely 13.22. Such a choice keeps the number of e^+e^- and $\nu\bar{\nu}$ binaries to an average of essentially one each in the six most important K^- decay modes. The entries in Table XVII are thus determined on this basis. The fact that this assignment puts the number of constituent particles at eight, which is double that for each of the pions, whereas the corresponding $K^\pm/\pi^\pm$ rest mass ratios are on the order of three to one, would therefore suggest that the kinetic energy of each electron and neutrino must be significantly greater for the charged kaons than for the muon and pion systems first considered. Since it is expected that the electron constituents must travel with nearly the speed of light, i.e. only slightly more slowly than the neutrinos, because of the very small dimensions involved, it follows that the total kinetic energy increases nearly linearly with the reciprocal of the interparticle separation for both kaons and pions. Thus the 13.22 assignment for the K^- composition in Table XVII suggests that the radius of this system is somewhat smaller than that of the pions.

The main decay branch (63.52%) for $K^\pm$ leads to the formation of a neutrino and a muon of like charge. With an eight-particle assignment for the charged kaons it follows that both a $\nu\bar{\nu}$ and an e^+e^- massless binary must be formed in this decay and go undetected by virtue of their total lack of energy. Historically, the other major decays involving pion products have received the lion's share of interest, particularly since events producing both two and three such particles are observed. Because of this observation Yang and Lee were led to postulate[56] that parity is not conserved in these decays (τ-Θ puzzle), since each pion is assigned a negative parity and the appearence of both even and odd numbers of them had raised questions about how a single system can produce different sets of decay products of opposite parity. Further discussion of this point will be reserved until the next section, in which the neutral kaons are discussed. In the present context it is better to pursue the question of how balanced reactions can be written down which cover both sets of pion products.

As shown in Table XVII, no differential binary systems are needed in the present assignment to allow for the formation of $\pi^\pm$ and π^0 from $K^\pm$, while two additional $\nu\bar{\nu}$ species are needed to produce

$\pi^+\pi^-\pi^+$ products (a binary increment of 2 in the table). In the latter case, the large amount of decay energy released by the charged kaons must cause the decomposition of these neighboring particle-antiparticle systems, with subsequent rearrangement to give the various multiple-pion products. The other three relatively frequently occurring decay modes for this system require the addition of an e^+e^- and $\nu\bar{\nu}$ binary, no change, and the loss of a single $\nu\bar{\nu}$ species, respectively. Thus, in both the $\mu^-\bar{\nu}$ and $e^-\pi^0\bar{\nu}$ branches it is assumed that massless particle-antiparticle binary systems are formed in the decay of the negative kaon. In this connection it is well to recall that the discussion of the quantum characteristics of photons in Sect. III.C has led to the general conclusion that whenever a particle and its antiparticle form a tight-binding state of zero rest mass, the product should have no translational energy relative to the center of mass of the initial reactants. Any other state of translation for a product of vanishing rest mass must correspond to motion at the speed of light, which makes a transition to it from a stationary set of reactants highly improbable. By the same argument, whenever a photon with non-zero energy is observed as a decay product, it should correspond to an initially massless e^+e^- system which has simply gained energy as a result of the decay, rather than to one which has been formed from the elements of the original meta-stable species.

This situation implies that the possibility of any such binary system being formed in its massless state (and thus going undetected) must be left open on a general basis, i.e. just as for the various e^+e^- and $\nu\bar{\nu}$ species which are indicated in Table XVII to be present after a given reaction (as denoted by means of a bar over the appropriate entry for the number of differential binaries in each case}. At the same time, it will be assumed throughout that $\nu\bar{\nu}$ and p^+p^- massless systems are much less likely to absorb a portion of the decay energy in such processes than their e^+e^- counterparts because of the latter's larger radius (see Sects. VI.F-H) and greater affinity to participate in both long- and short-range interactions. There can be no question that the *unobservability* of massless particle-antiparticle species is a key assumption in the present discussion, because it ultimately allows one to forgo the creation-annihilation hypothesis in interpreting the results of elementary particle decays.

D. NEUTRAL KAONS AND MORE ABOUT PARITY CONSERVATION

The next heaviest elementary particles are the neutral kaons. Just as the charged kaons, they have played an important role in the development of the theory of this branch of theoretical physics, particularly in the parity non-conservation analysis. It is now believed that there are two distinct such particles[203,204], a short-lived K_S^0 and a relatively long-lived K_L^0. The K_S^0 species decays predominantly into two pions ($\tau=0.882 \times 10^{-10}$ s), while K_L^0 decay generally produces three pions ($\tau=0.5181 \times 10^{-7}$ s).

The rest masses of the two particles are nearly identical, but they are believed to differ by 2×10^{-5} eV. The K_L^0 species also undergoes a small fraction of two-pion decays[68], however, whose discovery led to some consternation about the then existing theoretical interpretation of such processes. Specifically, up to that time (1964) it was held that the product of the charge conjugation and parity operations CP is conserved in all interactions, but the two-pion decays of K_L^0 were taken as an unexplained violation of this rule. Since there is general agreement that the product TCP commutes with any conceivable Hamiltonian[69-71], this development could also be taken as proof that the time-reversal operation also is not conserved.

In Section VIII.C it has been argued that the creation-annihilation hypothesis is crucial to the logical argumentation which has led to the conclusion that parity is not conserved in the weak interaction. Once one insists that electrons and neutrinos which are observed to exhibit longitudinal polarization[72-74.195] *existed as components of meta-stable particles prior* to *their decay,* however, it is no longer possible to justify such a thesis unambiguously on the basis of such findings. When Yang and Lee proposed that parity might not be conserved[56], their main concern was to explain how the same particle $K^\pm$ (or τ - Θ in their notation) could decay both into two and three pions in different channels. This argument is based on the assumption that the pions all have negative parity, from which it follows that it is impossible for both decays to be allowed according to selection rules based on the conservation of this quantity. Since the three-component model of a neutron ($p^+ e^- \bar{\nu}$) suggested by the XBPS model offers a way of explaining the longitudinal polarization phenomenon while still retaining the possibility that parity does commute with the corresponding Hamiltonian, it is well to review the whole question of parity conservation from the time it was first called into question by Lee and Yang.

To do this we start at the point at which the parity concept for particles is usually introduced into the theory of elementary particles, namely to interpret the results of low energy π^- capture by deuterons[209]. This process leads to the production of two neutrons with or without an additional photon. Because a corresponding set of products $nn\pi^0$ is never observed, it has been argued that this process must be forbidden, from which the supposition follows that some selection rule must be violated in this case which is satisfied in both of the first two reactions[154]. In search of a suitable selection rule one was drawn to the consideration of parity. Since a deuteron is known to be a triplet while π has zero angular momentum, it can be safely concluded on the basis of conservation of angular momentum that the reaction products must form a triplet. In this connection it is important to note that it is also assumed that the pion has come to rest relative to the deuteron, and thus that there is no additional orbital angular momentum (or translation) to be taken into account in the present analysis.

It is by no means straightforward to deduce the total parity eigenvalue for this system, however, because there is no way to measure this quantity directly[154]. Moreover, it is not even clear that each particle must have a definite parity. An electron is not assigned a parity value, for example, nor is the

neutrino. It only becomes obvious that a given system should be characterized by a definite parity when it can be described in terms of a many-particle wavefunction, such as is the case for hydrogenic atom states in the most well-known example. Such an assumption is then implicit for each of the particles involved in the $d\pi^-$ interaction. With this background the argument can proceed as follows. Since the deuteron is known to correspond to a bound proton-neutron state, the parity of the overall reactants is assumed to be the product of the individual parities of the three particles, p, n, and π^-. At this stage of the argumentation one has no way of assigning a definite parity to any of these particles individually, nor is the value of the product of these quantities known. If parity is conserved, however, as one assumes for this process, the combined parity of the product system nn must be equal to the corresponding value for the $d\pi^-$ reactants.

Since the two neutrons are indistinguishable fermions, their total wavefunction must be anti-symmetric with respect to their exchange. This anti-symmetry can be looked upon as deriving from two factors, the spatial and spin parts of the two-neutron wavefunction, in analogy to the treatment of the two-electron He atom which ultimately led to the anti-symmetry principle for fermions[134]. On this basis it is argued that if the spin state of the neutron pair is a singlet, and therefore anti-symmetric with respect to such a permutation, the corresponding spatial state must be symmetric. Then it is asserted that a symmetric spatial state is only consistent with an even value of the total angular quantum number L of the system. If S = 0 this implies that J = L + S is also even, which is inconsistent with the J=1 value of the initial system $d\pi^-$. Consequently the value of S must be unity and the spin state must be symmetric, the spatial state anti-symmetric. Therefore, by the same logic, the spatial state must be of odd L, specifically L = 1, and the parity of the products as a whole must be negative.

It should be noted, however, that the purported connection between the permutation symmetry of the spatial part of the two-neutron wavefunction and the value of its associated orbital angular momentum quantum number is by no means certain once when one allows for the possibility that the neutron is not an elemental system, i.e. without internal structure. If the nn system contains pairs of protons, electrons and antineutrinos (making it a six-particle system), it is not possible to properly describe it in terms of a single set of internal coordinates r_{12}. This being the ease, one cannot be certain that the angular characteristics of the two-neutron state can be represented exclusively by spherical harmonic functions whose parity is always given by $(-1)^L$. In the helium atom, for example, states of S (L=O) and P (L=1) type are known for both triplets and singlets, and all of them satisfy the Pauli anti-symmetry principle[210].

Keeping this qualification in mind, we may follow the argument further. Assuming that L = 1 implies odd parity and that this quantity is conserved in the present reaction leads one to conclude that the parity of $d\pi^-$ must also be negative. If the parity of the proton is defined to be positive, *which amounts to an additional assumption in the argument,* it therefore follows that the combined parity of the $n\pi^-$ system is also negative, which means the neutron and pion must have opposite values for this quantity. Even if

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one accepts this conclusion, however, it is still not possible to say with certainty which of the latter two particles has negative parity, because a straightforward theoretical argument[154] shows that it is impossible to settle the matter by experimental means. A way out of this impasse can only be found with recourse to yet another theoretical assumption, as discussed below.

In Sect. VII.H the theory of isospin was reviewed. The key assumption in this development is that the proton and neutron form an isospin doublet. Although a careful parallel is drawn between the properties of isospin[83-85] and those of spatial angular momentum, no provision had been made for an associated parity operation. Since each component of a multiplet of a given angular momentum has the same parity value in conventional space, there is seemingly good reason to assume that the same relationship holds true for the members of an isospin multiplet. On this basis it was ultimately decided that the neutron's parity is the same as that of the proton, i.e. positive, and by implication that the pion's parity must be negative. In making the above assignment of parities of different components of the same isospin multiplet, however, one overlooks the possibility that the spatial parity operation may not be identical to the one used in the isospin argumentation. After all, the same physical system can have different values of the total angular momentum and isospin quantum numbers, the deuteron being the most familiar such example. Why can it not be that there are two different parities as well, one operating in conventional space, the other in isospin space? If that is the case it is no longer certain that the corresponding two parity eigenvalues are the same for every component of a given isospin multiplet. Moreover, even if the respective parities of the proton and neutron are equal, there is no more reason to assume that the corresponding value is positive than there is that it be negative, as already mentioned.

In summary, there are no less than three separate, experimentally un-provable, assumptions which must be made in order to arrive at the conclusion which has proven to have such dramatic consequences in the development of theoretical physics in the last 50 years, namely that the parity of the pion is negative. If the opposite conclusion is made, there is no violation of a parity selection rule required to explain why $K^\pm$ can decay into either two or three pions because both sets of products then have the same (positive) parity. Similarly there is no need to conclude that CP (or T) is not conserved on the basis of the neutral kaon's decay characteristics[211]. The prediction and observation of longitudinal polarization in beta decay has always been assumed[56] to be inconsistent with a positive value for the pion's parity, but as been shown in Sect. VIII.C,

recognition of the possibility that electrons and antineutrinos might enjoy continuous existence both before and after the onset of the corresponding nuclear transition invalidates any such conclusion. The question that must be asked under the circumstances is *whether one or the other of the logical uncertainties connected with the derivation of the pion's negative parity might actually be incorrect.* Is there perhaps another way to explain all these seemingly baffling observations that requires somewhat less dramatic theoretical developments?

The calculations of the various particle-antiparticle binaries and other systems discussed earlier lead to some definite conclusions about symmetry themselves. The Hamiltonian on which they are based commutes with T, C and P, as well as with the total angular momentum. The results of these calculations indicate that the neutron's wavefunction has $1/2^-$ symmetry, while each of those for the e^+e^-, p^+p^- and $\nu\bar{\nu}$ systems is 0^-, at least in their respective lowest-energy (translationless) states. The deuteron in this model possesses a $p^2e^-\bar{\nu}$ structure, whose ground state is computed to have 1^- symmetry {Sect. VII.E). The π^- parity is less certain based on what we know thus far from XBPS calculations, but the most likely chioce for its symmetry is 0^+, i.e. positive parity. This is because it has consistently been found that pairs of light fermions of opposite q/m_o values prefer 0^- states. As indicated in Table XVII, the negative pion appears to be composed of an electron, a neutrino and two antineutrinos and thus consists of two such pairs (Sect. IX.A). Moreover, considerations of particle balance in the $d\pi^- \rightarrow nn$ equation require that a $\nu\bar{\nu}$ product result as well, which again according to the calculations should have 0^- symmetry, i.e. negative parity.

The best inference from the above calculations is therefore: a) the parities of both d and n are negative, b) the parities of $\pi^\pm$ (and π^0) are positive, and c) the overall parity of the above reaction is negative. The last point follows because although nn is found to have positive parity, the equation of interest is only balanced with the addition of a single 0^- $\nu\bar{\nu}$ state. In this way the reaction is found to conserve parity, and thus to be an allowed process, in agreement with observation:

$$d(1^-) + \pi^- (0^+) \rightarrow n(1/2^-) + n(1/2^-) + \nu\bar{\nu}\ (0^-) \qquad\qquad \text{IX.1}$$

Hence no contradiction arises when the pions are assigned positive parity in this reaction. The calculations indicate that the parities of the proton and neutron are not the same, although it is clear that this relationship depends on the orbitals occupied by the individual proton components within a given nucleus. As mentioned above, there is no conflict between this finding and the

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assumption that these two particles are different components of the same isospin doublet[83] because the isospin analogue of the parity operator need not be identical to that acting in physical space. *With these assignments all* of *the arguments based on parity violation in the τ-Θ puzzle* or *the two- and three-pion decays of K^0 lose their validity.* If the intrinsic parity of a pion is positive, then nothing stands in the way of an odd or even number of such particles being produced from the same starting point. On this basis one is led to look for a different explanation for the longitudinal polarization phenomenon, one of which has already been formulated in terms of the XBPS model in the preceding chapter.

The details of the reaction given in eq. (IX.1) are also consistent with other characteristics of this process. First of all, it is clear that total angular momentum can be conserved, but only if J=1. The same conclusion was mentioned in the discussion of parity conservation given above, but the indication now is that the two-neutron state has ^{3}S symmetry, i.e. L=0, in contrast to what is usually assumed. With two proton $s_{1/2}$ spin orbitals involved, we have a concrete verification of the assertion that anti-symmetric spatial states can possess even L values. With the help of Table XVII we can describe the reaction in terms of a transfer of an electron and an antineutrino from the negative pion to the proton of the deuteron. The parallel spins of the proton constituents in the original nucleus are not affected by this transfer, so that a triplet two-neutron state must result. The $\nu\bar{\nu}$ fragment of the pion is then expected to be emitted in its 0^- massless state. In all this, both parity and total angular momentum must be conserved in the model because the Hamiltonian governing the transition commutes with both types of operators.

What about the reaction in which two neutrons and a photon are produced from $d\pi^-$? In this case one must only assume that the parity of the emitted photon is negative, i.e. the same as for the massless system from which it is formed. In a dipole-allowed radiative transition it is known that the perturbing field has 1^- symmetry, from which one must conclude that the non-massless, energetic photon resulting from excitation of the latter system has $0^- \otimes 1^- = 1^+$ symmetry. This does not mean at all that every photon with energy different from zero has 1+ symmetry, however, rather only that one resulting from a dipole-allowed process relative to a massless photon must have this value. Other states of e^+e^- have different values of J and P. Indeed, it is no longer clear that the evenness or oddness of J dictates a definite parity in the case of a photon. It is a relativistic particle, always moving with speed c when it possesses non-zero mass, and in that

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case the conventional idea of a wavefunction consisting of a product of an internal and a purely translational factor does not necessarily apply (see Sect. X.B).

The last of the three reactions suggested for $d\pi^-$ would yield $nn\pi^0$, but it is never observed. The above assignments indicate that parity is also conserved in this process, however. Does this prove that the present parity assignments are wrong? Not at all, because not every reaction allowed by symmetry must occur in nature. Other factors can be involved, most prominently the availability of conditions for formation of one or the other of the products. In the XBPS model an additional e^+e^- binary is required to balance the above equation because π^0 is assigned the composition $e^+e^-\nu\bar{\nu}$ (Table XVII). Since this massless binary system has negative parity, it cancels the deuteron's eigenvalue to give the same (positive) result on both sides of the equation. But is there any driving force to involve such an e^+e^- massless binary? In the model what must happen is that the electron and one antineutrino attach themselves to the deuteron's proton, inducing dissociation into two neutrons. There is a $\nu\bar{\nu}$ system left over from π^- as a result, but it apparently prefers to assume its massless state, as indicated in eq. (IX.1). The alternative of attaching the $\nu\bar{\nu}$ product to a neighboring e^+e^- massless binary to form a π^0 with 135 MeV rest mass is perhaps conceivable, but who can say it is not preferable that the $\nu\bar{\nu}$ species remain inert and instead have all the decay energy go to the departing neutrons? Or to give a part of the energy to a massless e^+e^- system so that it appears as a photon at the end of the reaction? In the future it may be possible to carry out calculations which compare these three reaction probabilities in a quantitative manner, but for now it should at least be clear that such an explanation for the presence or absence of these various reactions is a definite possibility. At the very least, it is in no way ruled out by anything one could possibly infer from symmetry arguments alone.

The composition vectors listed in Table XVII for K_S^0 and K_L^0 are suggested on the basis of the type and frequency of their respective decay modes. As before with the charged kaons, there is some justification for assuming that more than the four particles of a pion are contained in K_S^0 and K_L^0 because of their larger rest masses (3-4 times larger than $\pi^\pm$, $\pi^{0)}$. Because the rest masses of the neutral kaons are so similar to those of the charged $K^\pm$ species, it is natural to assume that the compositions of all these particles must be very similar, particularly with regard to the number and type of constituent electrons and neutrinos. The choice in Table XVII of a 13.13 composition for both K_S^0 and K_L^0 is tantamount to assuming that the two neutral kaons

correspond to very closely-lying states of the same system. The (assumed) energy difference between them (0.5 x 10^{-5} eV) is so small that it would be all but impossible to compute it in any *ab initio* model.

The most interesting feature of the decay of the neutral kaons is the fact that sets of both two and three pions are observed. In the present model, in which all the pions are suggested to have positive parity, one can' expect that even though both types of decay fragments are allowed by symmetry, the amount of phase space available in the two cases definitely favors the two-pion decays, as observed[203,204]. The K_S^0 system with a lifetime of 0.882 x 10^{-10} s exhibits only two-pion decays, while the K_L^0 species has a sixty-fold greater lifetime and produces $\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$ fragments in a total of 34.1% of its decays. At first it was thought on the basis of theoretical arguments involving CP conservation that K_L^0 would not exhibit any two-pion decays but in 1964 Christenson *et al.* and Aboshian *et al.* showed that this is not the case[68]. This work found that 0.2% of all charged modes are two-pion decays ($\pi^+\pi^-$). It is worth noting, however, that in 65.7% of all decays, the observed products are $\pi\mu\nu$ and $\pi e\nu$ which in turn are the known products of $\pi^+\pi^-$ decay, *i.e.* with one pion remaining intact and the other decomposing into a muon and a neutrino (or to an electron and neutrino after further decomposition of the muon). These types of decays do not normally enter into the parity discussions, however, for the simple reason that the electron, neutrino and muon are not assigned a definite parity.

The assignments in Table XVII also leave open the possibility that both K_S^0 and K_L^0 are their own antiparticles, just as is generally accepted for π^0. Originally theoreticians spoke of a single neutral kaon K^0, whose antiparticle is $\overline{K}^0$. The work of Gell-Mann and Pais[211] led to the designation of two separate particles K_1^0 and K_2^0 of different lifetimes. Subsequent experiments[68] ruled out the straightforward assignments of definite and opposite parities to these two species, however, contrary to what had been suggested in the above theoretical work. With 13.13 assignments for both K_S and K_L, it can be seen that their most common decay products can all be reached with the addition of a small number of massless e^+e^- and $\nu\overline{\nu}$ binaries as reactants, sometimes with the assumption that such systems go undetected as products as well. Accurate calculations seem to be the only way of obtaining more confidence in any of these assignments, but at least it is clear how they might have to be changed when more information becomes available, namely by suitable addition (or subtraction) of e^+e^- and $\nu\overline{\nu}$ binaries to the structures given in Table XVII.

The composition vector for the η resonance is more difficult to specify than for any of the lighter mesons. Its lifetime is relatively short (2.5 x 10^{-19} s, corresponding to a linewidth of 2.63 keV). The most common decay products are groups of two or three pions or several photons, with or without pion accompaniment. The two-photon decay occurs in 38.0% of all cases, compared to 30.0% for three $3\pi^0$ and 23.9% for $\pi^+\pi^-\pi^0$ decays. As usual all these products can be related by the gain or loss of an integral number of e^+e^- and $\nu\bar{\nu}$ binaries. The mass difference relative to the neutral kaons is relatively small (ca. 50 MeV), so that the composition of the η particle is likely to also be fairly similar, such as the 22.22 designation given in Table XVII. The only point which should be stressed in this assignment is that the hypothesized availability of massless binaries at the site of this and all other particle decays can be used to explain all of the observed products. There are a number of other heavier mesons which have not been listed in Table XVII, including the η', $\rho^\pm$, ρ^0, ω^0, φ^0, K^{*0} and $K^{*\pm}$ particles. These resonances play a key role in the quark model of elementary particles to be discussed in Sect. IX.J.

E. VIRTUAL PARTICLES IN THE XBPS MODEL

One of the most surprising aspects in the development of theoretical physics since the introduction of the Schrödinger[86] and Dirac[99] equations was the necessity of assuming that there are virtual photons in the neighborhood of atoms and molecules which need to be considered in the form of radiative corrections in computations for "isolated" systems[212-214]. In introducing these effects it is customary to go to some length to argue that real photons are not involved because the assumed entities violate energy and momentum conservation laws. It can be pointed out that such behavior is not inconsistent with the fundamental laws of physics, however, because Heisenberg's uncertainty principle[215] allows such minor deviations from them.

Because .of the indisputable accuracy produced in quantum electrodynamics calculations making use of radiative corrections, the assumption of virtual photons has become a fixture in theoretical physics. The idea received further impetus from Yukawa[88] in 1936 when he suggested that virtual particles with a mass of roughly 100 MeV /c^2 might perform a similar function in the transmission of nuclear forces. As already pointed out in Chapter IV, this revolutionary hypothesis was ultimately followed by the discovery of the pions in cosmic radiation. Attempts to formulate a theory of nuclear binding along the lines of quantum electrodynamics by

substituting pions for photons have generally been accepted to be quantitatively unreliable[90], but this fact has been ascribed to difficulties in treating such interactions at a suitably high computational level. In more recent times the W or intermediate boson was sought and found[216] as the analogue of the photon and pion in the weak interaction, lending further weight to this theoretical development.

It is well to mention that one of the main arguments in favor of the exchange of virtual particles is taken from general relativity theory[217], according to which it is assumed that no interactions at a distance exist. While forces such as gravity can be described quite well in terms of a formula involving the distance between two interacting bodies, it is argued that this circumstance does not prove that the actual mechanism from which the force arises can be explained solely on the basis of the two objects in question. Newton is often criticized for having assumed that the gravitational interaction is instantaneous, but in 1692 he wrote[218] that it was an "absurdity" to think that bodies act upon one another through a "vacuum, without the mediation of anything else." The long period of searching for an "aether" which transmits the electromagnetic force was in line with this view as well. The concept of virtual photons and virtual pions tries to get around such difficulties by assuming a) there is a vacuum, but b) particles can be created and annihilated anywhere at any time in it, and are thus always available for transmitting the observed forces. Once the creation-annihilation hypothesis is questioned, however, it is necessary to assume that massless binaries such as e^+e^-, $\nu\bar{\nu}$ and p^+p^- are always available in high concentration everywhere in the physical universe. Adoption of this alternative formulation thus suggests that the concept of virtual particles might no longer be required, or at least that the distinction between them and physically existent photons may be primarily of a heuristic nature.

To see how the computational techniques which are based on the assumption of virtual particles can be recast in a physical model in which neither a vacuum nor anything but real particles is accepted, it is helpful to consider the configuration interaction technique described in Sect. VI. C in a somewhat different light. Let us take the CI wavefunction for the helium ground state, for example. To a good approximation it consists of a single configuration in which the 1s orbital is doubly occupied. To obtain better agreement with experiment, however, it is necessary to mix in additional configurations, for example, the $2p^2$ species. The energy of the latter configuration is much higher than that of $1s^2$, and yet quantum mechanics allows the two

functions to form a linear combination which gives a *better* description of the helium ground state *than either of them alone.* One is simply employing a different language when this process is referred to as involving a "virtual" $2p^2$ state. In this way it is possible to incorporate into the present model the idea of virtual entities whose interactions violate conservation laws in a manner consistent with the provisions of the uncertainty principle. In order to go a step beyond this and bring in *virtual particles,* it is only necessary to have configurations containing spin orbitals corresponding to several particle types within the same system.

When a helium atom interacts with virtual photons according to the present model, it is necessary to deal with at least four particles, the two helium electrons and the e^+e^- complex which has been identified with the photon in Chapters II-III. The unperturbed system contains a massless 0^- photon state, while the radiative corrections correspond to a configuration of much higher energy in which the electron and positron are no longer bound together. Such effects are relatively small and it is thus reasonable to expect that their treatment in low-order perturbation theory is adequate to obtain the desired accuracy. The matrix element involved is provided by the methods of quantum electrodynamics[219], thereby circumventing the problem of working with explicit wavefunctions for photons which take account of the internal structure assumed for them in the present model. Even if exact solutions for the various e^+e^- states can be obtained from the XBPS model, it can still be anticipated that such an explicit CI treatment of the He atom plus photon would be much more difficult to carry through with the required accuracy than is the standard perturbational approach. A similar situation has already been met in Sect. VIII. A where the possibility of computing line-widths in terms of complex energy eigenvalues[189], in principle the preferred technique, was compared to the simpler approach involving the Fermi golden rule[190]. When very small imaginary parts of the energy eigenvalue are involved, better accuracy can invariably be obtained in the perturbational treatment because the terms omitted in such a procedure are negligibly small. One needs to carry out the computations with much more effort to approach the same level of accuracy when a strictly variational treatment is employed.

In this sense the possible advantages of assuming that real photons are involved in the quantum electrodynamics theory are only to be found on a conceptual level. The computational techniques of the existing theory are well-known to be capable of great accuracy and, except perhaps in rare instances not yet encountered, are in no need of further improvement. The situation is qualitatively different for nuclear interactions, however. The main reason for this

distinction appears to be less a matter of the relative strengths of photon and pion interactions, however, than the fact that the operators used to describe the two types of interactions in a quantum mechanical treatment are known far more accurately for the predominantly Coulomb forces governing atomic and molecular systems. One has an excellent starting point in the Dirac equation[99] or related two-component methods[102] from which to apply the necessary radiative corrections. The main effects are still to be found in the electrons and nuclei of the atoms and molecules of interest in quantum electrodynamics treatments, even though the most interesting results often emerge only after the photon interactions are included.

The main thesis of the XBPS model is that exponentially damped momentum-dependent operators of relatively short range are required to obtain a similarly useful starting point for the calculation of nuclear binding. By attributing a non-zero value to the charge-to-rest-mass ratio of the neutrino, it has been possible to describe the neutron and deuteron as collections of protons, electrons and antineutrinos interacting primarily through such short-range interactions, and thereby offsetting the large kinetic energies required for the confinement of light fermions within nuclear dimensions. By analogy to the quantum electrodynamics description of atomic systems, it can then be expected that the inclusion of pions into the theoretical treatment is required to obtain a truly comprehensive representation of the corresponding observed characteristics of nuclear interactions. The difference between the present model and earlier proposals to describe nuclear binding is that it foresees only a relatively minor role for pions, which might well be amenable to a perturbative treatment similarly as for photons in processes dominated by the electromagnetic interaction. The emphasis in the XBPS model is more on the nature of the quantum mechanical representation of the short-range interaction itself in terms of the exponentially damped Breit-Pauli terms such as spin-orbit, orbit-orbit and spin-spin coupling. This position no way contradicts the concept of certain particles being responsible for the transmission of certain forces, but it clearly places less emphasis upon the idea than is normally done in quantum field theory.

A more significant difference between the XBPS model and other suggested approaches is clearly that it denies the need for assuming *virtual* particles to describe any aspect of the corresponding experimental phenomena. Instead of a vacuum filled with nothing, it is assumed that free space as we perceive it is in reality permeated with large numbers of massless binary systems which are always available to participate in interactions with other (massive) particles.

These e^+e^-, $\nu\bar{\nu}$ and p^+p^- systems are not observable directly as long as they remain in their respective massless states, but they exist just as surely in this condition as do particles with non-zero relativistic masses. To describe the interaction of an atomic or nuclear system with such entities, it is simply necessary to expand the number of particles treated explicitly in the configuration interaction calculations, at least in principle. The resulting wavefunctions have contributions from configurations which contain excited states of such binary systems. As a result, their effects become observable in practical experiments, the most well-known, of which are the magnetic moment determinations for electrons and protons and the measurement of the Lamb shift[219]. A small admixture of such excited binary states in the wavefunction means that the measured properties of the overall system are directly affected by them.

Such a CI treatment, by construction, requires a set of configurations which share a common occupation of spin orbitals corresponding to definite numbers of the same type of particles, which is to say, a perfect elemental balance is a strict condition for treating a physical interaction in this manner. This does not exclude the introduction of external fields into the governing Hamiltonian, but in a broader sense it leaves open the possibility that any such additional effect ultimately has its origin in the existence of other particles in the neighborhood of the system in question. It is a "particles only" approach, often deemed to be *anathema* to the quantum field theory. There is no need to see a contradiction in these two views of physical theory, however. As usual it is the creation-annihilation hypothesis, in this case in the form of a vacuum model of free space, which only makes them seem mutually incompatible. If massless particle-antiparticle binaries exist, as the XBPS model's calculations indicate they do, then they must exist everywhere in great numbers at all times according to the laws of statistical mechanics. This possibility then allows a ready explanation of how action-at-a-distance effects are transmitted throughout space, and especially why it is necessary to assume fields external to the observed atomic and nuclear systems in order to obtain a suitably accurate description of their interactions.

F. INTERACTIONS OF PROTONS WITH PIONS IN THE XBPS MODEL

The most explicit evidence for the interaction of pions with nuclear matter is gleaned from elastic scattering of high-energy electrons off protons and neutrons[91,94,160]. Values for the proton form factors vary with momentum transfer in a manner corresponding to a charge distribution

which varies exponentially with the distance away from the proton's center of mass. This result can be satisfactorily interpreted in terms of a model in which the proton is surrounded by a cloud of pions. When an electron approaches a proton at sufficiently high energy it thus encounters not only the bare nucleon but also the less massive pions. As a result, it appears that the proton itself is not a point charge, unlike the electron and positron, for example. The calculations discussed in Chapter VII combined with the arguments of Sect. IX.A regarding the composition of pions in .the present model allow for a straightforward interpretation of the elastic scattering data, however, which puts matters in a different light and does not require the assumption of virtual particles in arriving at its conlusions.

As shown in Fig. 18, the XBPS model envisions a bare proton being continuously surrounded by numerous e^+e^- and $\nu\bar{\nu}$ massless binaries whose influence must be taken into account in explicit ca1culations in order to properly represent the physical situation. A minimum number of these binaries is required to explain the observed effects. The lowest-energy configuration is simply a product of the proton's spin orbital and the two-partic1e 0^- wavefunctions for one e^+e^- and two $\nu\bar{\nu}$ binaries in their respective massless states. The proton is able to approach the lighter particles relatively closely as a result of the form of the exponential damping factor in the XBPS Hamiltonian (see Table l and the discussion in Sect. VII.B). As a result, a higher-energy configuration is also needed to obtain a satisfactory description of the system. This corresponds to a neutron and a positive pion, which are formed by decomposing an e^+e^- binary to form a p^+e^- complex plus a positron. These species interact further with the $\nu\bar{\nu}$ binaries, with the antineutrino from one of them joining the proton-electron system to form a neutron, while the corresponding neutrino plus an additional $\nu\bar{\nu}$ pair combine with the positron to form π^+ (11.2 composition vector, see Table XVII). This (seven-particle) excited configuration mixes with the lower-energy proton-plus-massless-binary species (and other configurations of intermediate occupation) to form the final eigenfunction. According to the usual statistical interpretation of such wavefunctions, it follows that the system as a whole spends a certain fraction of its time in the excited neutron-plus-π^+ configuration, which circumstance leads to the deflection of high-energy electrons away from the proton itself. Since π^+ has a positive electric charge and a relatively small rest mass, it can be expected that the same configuration-mixing effect tends to raise the system's magnetic moment relative to what would be obtained if the e^+e^- and $\nu\bar{\nu}$ binaries always remained in their massless states.

The above effect is far greater than in the case of an electron interacting with photons, with the proton magnetic moment being 2.5 times larger than would be expected for a bare (Dirac) proton, compared to an analogous deviation of only 0.05% in the case of the electron. The point to be stressed in the present context, however, is that there is really no necessity to talk about virtual pions in describing this effect once allowance is made for the existence of massless e^+e^- and $\nu\bar{\nu}$ binaries in the neighborhood of the proton. Ultimately, it is the large mass of the proton which allows it to interact more strongly with such entities than do lighter particles such as electrons and neutrinos.
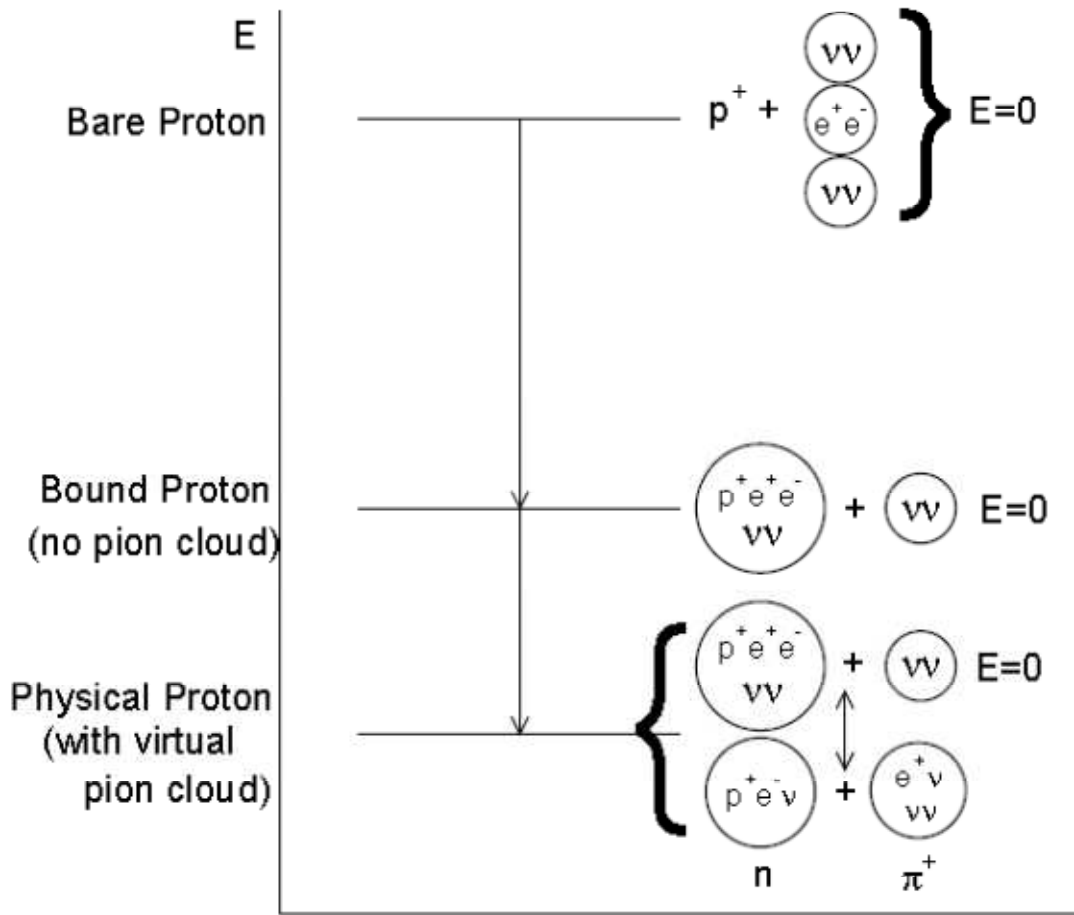
FIG. 18. Schematic energy diagram showing the relative stabilities of the bare proton (upper line). proton-photon-photrino complex (middle line) and physical proton with a virtual pion cloud, as foreseen in the XBPS model. Calculations indicate that the rest mass of the (physically unobserved) proton is several electronic mass units larger than the value of $1836 \, m_{oe}$ measured for the proton in its naturally occurring environment. The $n\pi^+$ virtual state is interpreted as a configuration interaction component of the actual (bound) proton wavefunction.

To give further consideration to this point, a series of calculations has been carried out for the tri-atomic $p^+e^+e^-$ system. In the 4s,2p basis employed for the $p^+e^-\bar{\nu}$ system in Sect. VII.B, a full CI energy of -186373.864 hartree results, which represents a proton binding energy of 148783 hartree to the e^+e^- ground state obtained at the same level of treatment. While this quantity is likely to become significantly smaller as the quality of the basis set is improved, it still indicates that there is a substantial interaction between the proton and such a particle-antiparticle binary. This result is clearly consistent with the arguments given in Sect. VII.B regarding the mechanism of proton binding to the $e^-\bar{\nu}$ complex, especially when one employs a q/m_o value of 1.0 a.u. for the antineutrino in the $p^+e^-\bar{\nu}$ calculations (Table X). Recognition of this point raises another question, however, namely whether such a large binding energy between the proton and the e^+e^- binary system is compatible with the theory of nuclear binding which has been discussed in Chapter VII. A calculation for the $p^2e^+e^-$ system for the same basis (an optimum scale factor of 0.10 is found in both cases, as compared to the value of 0.16 obtained above) yields a total energy of $–305503.616$ hartree, which corresponds to a binding energy for the second proton of 119221 hartree, 29562 hartree smaller than for the first. Since, according to the present model, there is always an abundance of massless e^+e^- species with which to interact, this result indicates there is a reluctance for the protons to group together around a single binary of this type. This is in sharp contrast to the experience with the $e^-\bar{\nu}$ complex when computed at the same level of treatment. There the first proton is not bound at all, existing in a $p^+e^-\bar{\nu}$ resonance state, whereas the second combines with it to form the strongly bound deuteron.

The results of the above calculations nonetheless suggest that the energy of the bare proton would actually be significantly greater if it were not always surrounded by e^+e^- (and $\nu\bar{\nu}$) systems, which according to the present model corresponds more closely to its physical state actually observed in experiments (Fig. 18). Increasing the rest mass of the proton would have almost no effect on the calculated total energies because they are referenced to the energy of each system's component particles separated to infinity. On this basis one can expect that the mass of the bare proton might be greater than that measured in the laboratory by as much as several electronic mass units. Since the polarization effects which cause this stabilization are charge-dependent, it can be expected that their influence on the neutron's rest mass *vis-a-vis* that in its isolated (bare) state is much smaller. Such a development would also be at least

qualitatively consistent with the premises of the isospin theory[83-85], in which the proton and neutron are assumed to be of equal mass in the absence of the electromagnetic interaction. Recognition of these effects would tend to indicate that the $|q/m_o|$ value which must be assumed for the neutrinos in order to compute a neutron ($p^+e^-\bar{\nu}$) rest mass in agreement with the measured value might need to be increased slightly if such environmental influences were properly taken into account in the theoretical treatment. It also can be noted that the above calculations are consistent with the well-known observation that the interactions of protons and other hadrons with photons are not correct1y described by quantum electrodynamics[220].

A more crucial question arises from this general discussion, however, namely do the electron scattering data for nucleons *imply that the proton is not* a *point charge?* The answer is not at all clear once it is realized that the extension in the charge distribution observed in the electron elastic scattering from protons may be tied up at least in large part with the fact that *the proton is not an isolated system under the governing experimental conditions.* It could be that the exponential nature of the charge distribution derives from the combination of *both* the proton and the e^+e^- and $\nu\bar{\nu}$ species to which it is tightly bound according to the present calculations (the pion cloud also results from this interaction in this view; see Fig. 18). To investigate this point further it is clearly necessary to consider the effects of probes in the scattering experiments which have significantly smaller de Broglie radii than the electrons employed in the above study[91,94,160].

When inelastic scattering processes are investigated with incident electron energies in the 10 GeV range[221], the nature of the cross sections is different than above and continuum states are observed whose charge distributions are no longer exponentially decreasing with the distance away from the proton's center. In essence the results indicate that point charge scatterers are now involved. There have been essentially two interpretations of this phenomenon, identifying the point particles with some internal structure of the proton[222-223] or with the bare nucleon itself[224]. In the present model it seems tempting to focus on the idea that as the energy of the electrons is taken up by the proton that they come into excited states which are no longer able to bind pions (or alternatively the e^+e^- and $\nu\bar{\nu}$ binary systems) even a small percentage of the time. A 10 GeV electron energy corresponds to a proton kinetic energy of 2-3.5 GeV in its own inertial system, which is comparable to what is computed for the p^+p^- binary in its massless state (Sect. VI.F). Under these conditions the interparticle distances are so small that even the advantage that the

proton normally enjoys by virtue of the form of the exponential damping factors in the XBPS Hamiltonian in the present model is unable to offset the enormous centrifugal (kinetic energy) effects impeding against binding. The proton is therefore no longer attracted by neighboring e^+e^- and $\nu\bar{\nu}$ species and behaves as an isolated system. The fact that the curvature in the measured cross sections for inelastic electron scattering steadily decreases with the energy of the continuum states is at least consistent with such an interpretation. We shall have occasion to return to this question later in this chapter when the subject of the quark model of elementary particles is taken up again (Sect. IX.J).

G. PROTON-ANTIPROTON INTERACTION

The reaction of a proton with an antiproton can be described in a perfectly analogous manner to that of an electron and positron in the XBPS model by virtue of the scaling properties of the Hamiltonian employed (Sect. V.D). The energy released is m_{op}/m_{oe} times greater in the p^+p^- interaction, and its mean interparticle distance is smaller by the same amount than in the e^+e^- case according to this treatment. It might be expected that when p^+ and p^- bind together that the result is a pair of photons with correspondingly greater energy than for e^+e^-, but the most commonly occurring process observed experimentally involves instead the production of a number of pions. It is therefore important to consider this process in somewhat more detail in order to find a possible explanation for this phenomenon.

Because the p^+p^- massless binary system is much less likely to interact with its surroundings by virtue of the relatively small value of its radius, it seems reasonable to assume that the energy given off as a result of its formation is borne by other particles in the neighborhood. Initially the proton and antiproton can be taken as being at rest in their center of mass, and it has been argued in Sect. III.C that the product of their interaction must also remain localized in the same region of space. In view of the zero rest mass of the $p+p^-$ binary, this situation cannot hold if it takes up any energy itself, in which case it must move with the speed of light out of the area. The same argument has been used to explain the quantized nature of photon interactions, particularly the well known fact that they tend to give up all rather than only part of their energy when absorbed by atomic or molecular systems. The enormous amount of energy released in the proton-antiproton interaction must be taken up by something, however,

and because of the conservation of momentum requirement, the most likely recipients in the present model are two photons, initially also possessing zero energy and therefore likely at rest in the neighborhood of the reaction.

In this respect the e^+e^- and p^+p^- particle-antiparticle reactions initially proceed in a very similar manner, but the amount of energy released in the latter process is sufficient to cause neighboring e^+e^- species to decompose into their elements, unlike the situation when a positron and electron react with one another. As shown in Fig. 19, a relatively uncomplicated scenario for the observed pion production can be imagined once two pairs of electrons and positrons are set free as a result of the proton-antiproton interaction. The conservation laws essentially require that the two e^+e^- species push off one another (see Fig. 4), so that initially one e^+ and e^- remain near the original center of mass, whereas the other two particles go off in opposite directions at high speed. The next step would then involve the attachment of $\nu\bar{\nu}$ binaries to each of the fragment systems: one to e^- to form a transient μ^- (Table XVII), one to e^+ to form μ^+, and finally one to the remaining e^+e^- pair to form a π^0 species. At such high energies the muons offer an attractive target to another $\nu\bar{\nu}$ binary, thereby producing the respective charged pions π^+ (ν) and and π^-($\bar{\nu}$). Alternatively, two $\nu\bar{\nu}$ species might be involved, with the unused ν and $\bar{\nu}$ particles taking up some of the released energy as well.
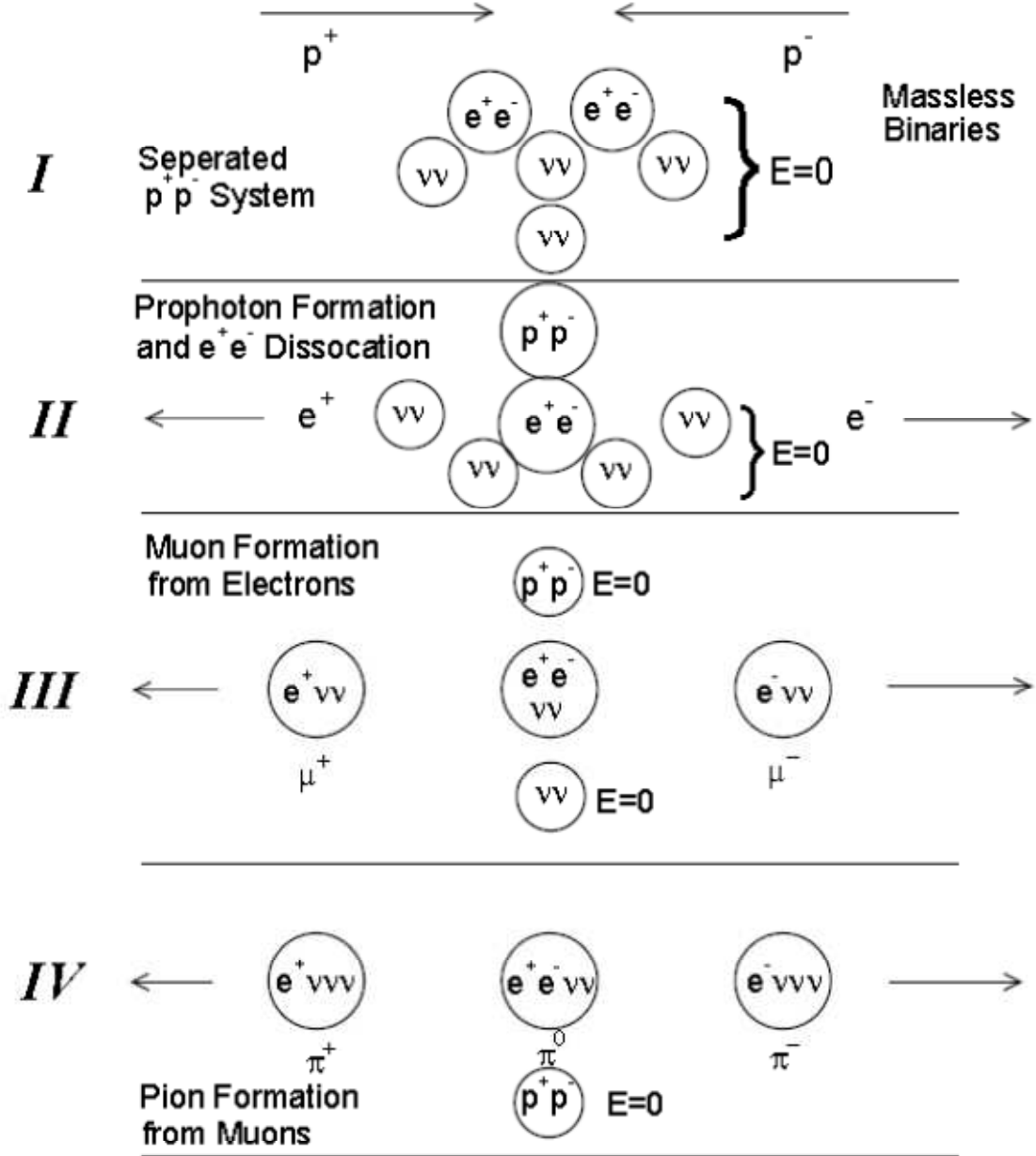
FIG. 19. Schematic diagram showing four stages of the proton-antiproton fusion process producing a system of three pions and the massless prophoton (p^+p^-) system. In the first stage the proton and antiproton are shown to collide with one another in the presence of massless e^+e^- and $\nu\bar{\nu}$ systems (I). Their fusion leads to the production of 1.88 GeV of energy which is taken up initially by two massless e^+e^- species, leading to their dissociation in the second stage (II). The latter particles further combine with $\nu\bar{\nu}$ species to form the $e+\nu\bar{\nu}$ (μ^+), $e^+e^-\nu\bar{\nu}$ (π^o) and $e^-\nu\bar{\nu}$ (μ^-) intermediate products in the third step (III), while in the final stage the muons react further to form the corresponding charged pion systems (IV). The prophoton product is assumed to remain in the translationless state upon its formation and thus escapes experimental detection.

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In accord with the notation used in Table XVII the reacting system involved is not 100.100, *i.e.* p^+p^-, but rather 123.123 (or 124.124), which yields as products a π+ (11.2), a π⁻ (2.11), a π^0 (11.11) and the 100.100 p^+p^- massless binary (plus an additional ν and v̄ in the second case). The mechanism described above leads to the production of only three pions, but it is obvious how alternative processes might occur which would produce still larger numbers of such mesons. Given the amount of energy produced by the p^+p^- interaction and the relatively small barriers to decomposition of the massless e^+e^- and ν v̄ binary systems, it is not surprising that a variety of such multiple-pion products is observed.

H. COMPOSITION OF HYPERONS IN THE XBPS MODEL

The next group of elementary particles, in order of increasing rest mass, are the various meta-stable baryons, or hyperons, as they are also frequently called. The suggested elemental composition of these particles is given in Table XVIII, along with a survey of their respective decay processes[203,204]. There is a well known baryon conservation law governing reactions of such systems, and this is obviously consistent with the main thesis of the present work, namely that all truly elementary (stable) particles are in continuous existence, regardless of the nature of the interactions to which they are subjected. These elements are the proton, electron and neutrino and their antiparticles, and since each of the hyperons decays into a collection of such particles of which one is always the proton (or antiproton), it is clear that each of them must contain such a heavy element if the creation-annihilation hypothesis is to be avoided. The simplest meta-stable baryon is the neutron in this view, with a 1.110 composition (Table XVIII), whereas the proton is one of the elements (100).

The next lightest hyperon is the Λ particle, which decays into the major sets of products pπ⁻ and nπ⁰. The latter differ by a single e^+e^- binary according to the assignments of Table XVII. The decay energy is of the order of the pion value, amounting to 177 MeV. The most likely composition (2.111) would thus seem to be one proton plus two antineutrinos, one electron and one neutrino, i.e. the components of $p^+ + \pi^-$. Alternatively, one might take it to be an excited state of the neutron itself which upon decay attracts a ν v̄ binary to its emitted e⁻ and v̄⁻ particles to form a negative pion. In view of the observation that a proton is surrounded by a pion cloud even when it is in its ground state, the former assignment is slightly preferred and given

explicitly in Table XVIII. With this choice only a single e^+e^- massless binary must be added to Λ to produce its $n\pi^0$ decay products. The other groups of fragments listed occur far less frequently and are easily correlated with the above structure for Λ when allowance is made for the usual addition or loss of particle-antiparticle binary systems. The un-symmetric nature of the Λ composition vector indicates that a distinct antiparticle exists with charge-conjugated composition, and this is observed as well.

TABLE XVIII. Classification of baryons by means of composition vectors. For notation see Table XVII.

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
proton	P	938.2592±0.0052	100	stable	-	-
neutron	n	939.5527±0.0052 (τ=918s)	1.110	$p^+e^-\nu$	1.0000	0
lambda	Λ	1115.59±0.05 (τ=2.521 x 10^{-10}s)	2.111	$p^+\pi^-$	0.642	0
				$n\pi^0$	0.358	10
				$p^+e^-\bar{\nu}$	8.13 x 10^{-4}	$\bar{1}$
				$p+\mu-\bar{\nu}$	1.57 x 10^{-4}	0
				$p^+\pi^-\gamma$	8.5 x 10^{-4}	10
sigma plus	Σ^+	1189.41±0.07 (τ =8.00 x 10^{-11}s)	11.111	$p^+\pi^0$	0.516	0
				$n\pi^+$	0.484	1
				$p^+\gamma$	1.24 x 10^{-3}	$\bar{1}$
				$n^+\pi^+\gamma$	1.31 x 10^{-4}	11
				$\Lambda e^+\nu$	2.02 x 10^{-5}	1
				$n\mu^+\nu$	< 2.4 x 10^{-5}	1
				$ne^+\nu$	< 1.0 x 10^{-5}	0
				$p^+e^+e^-$	< 7.0 x 10^{-6}	$\bar{1}$

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
neutral sigma	Σ^0	1192.48±0.l0 ($\tau < -1.0 \times 10^{-14}$s)	2.111	$\Lambda\gamma$	1.000	10
				Λe^+e^-	5.45×10^{-5}	10
sigma minus	Σ^-	1197.34±0.07 ($\tau = 1.484 \times 1\,0^{-10}$s)	2.120	$n\pi^-$	1.000	1
				$ne^-\bar{\nu}$	1.10×10^{-3}	0
				$n\mu^-\bar{\nu}$	4.5×10^{-4}	1
				$\Lambda e^-\bar{\nu}$	6.0×10^{-5}	1
				$n\pi^-\gamma$	1.0×10^{-4}	11
neutral xi	Ξ^0	1314.9±0.6 ($\tau = 2.98 \times 10^{-10}$s)	12.121	$\Lambda\pi^0$	1.000	1
				$p^+\pi^-$	$<0.9 \times 10^{-3}$	$\bar{1}\,0$
				$p^+e^-\bar{\nu}$	$< 1.3 \times 10^{-3}$	$\bar{1}\,\bar{1}$
				$\Sigma^+e^-\bar{\nu}$	$< 1.5 \times 10^{-3}$	0
				$\Sigma^-e^+\nu$	$< 1.5 \times 10^{-3}$	0
				$\Sigma^+\mu^-\bar{\nu}$	$< 1.5 \times 10^{-3}$	1
				$\Sigma^-\mu^+\nu$	$< 1.5 \times 10^{-3}$	1
				$p^+\mu^-\bar{\nu}$	$< 1.3 \times 10^{-3}$	$\bar{1}\,0$

TABLE XVIII continued

Particle	Symbol	Rest Mass (Lifetime)	Composition Vector	Decay Products	Fraction	BI
xi minus	Ξ^-	1321.29±0.14 ($\tau = 1.672 \times 10^{-10}$s)	3.121	$\Lambda\pi^-$	1.000	1
				$\Lambda e^-\bar{\nu}$	7.0×10^{-4}	0
				$\Sigma^0 e^-\bar{\nu}$	$<5.0 \times 10^{-4}$	0
				$\Lambda\mu^-\bar{\nu}$	$< 1.3 \times 10^{-3}$	1
				$\Sigma^0\mu^-\bar{\nu}$	$<-5.0 \times 10^{-3}$	1
				$n\pi^-$	$< 1.1 \times 10^{-3}$	0
				$ne^-\bar{\nu}$	<0.01	$\bar{1}$
omega minus	Ω^-	1672.5±0.5 ($\tau = 1.3 \times 10^{-10}$s)	13.131	$\Xi^0\pi^-$	-	1
				$\Xi^0\pi^-$	-	1
					-	2

The next three hyperons have nearly the same rest masses and are grouped together as Σ^+, Σ^0 and Σ^-, in order of increasing energy. Again two major decay modes are noted for Σ^+, namely $p\pi^o$ and $n\pi^+$. With the usual *caveat* an assignment of 11.111 for its composition is made in Table XVIII, which is identical to that of its $p\pi^o$ products. A single $\nu\bar{\nu}$ binary must then be added to obtain the $n\pi^+$ fragments, observed in 48.4% of Σ^+ decays. Otherwise there are six minor decay modes such as $p\gamma$ and $n\pi^+\gamma$, which are easily understandable in the usual way. The composition of Σ^+ implies that an antiparticle also exists, as observed.

The neutral sigma particle Σ^0 also has an antiparticle. Its decay products consist almost exclusively of a Λ species plus a photon or a separated e^+e^- pair. This suggests something akin to a conventional radiative emission is involved in the decay, so the Σ^0 structure is probably best assumed to be the same as that of the Λ particle. The assumption is that the 77 MeV higher rest mass of Σ^0 relative to Λ is simply a consequence of the mass-energy equivalence relation for two different states of the same system.

The heaviest of the sigma particles is Σ^-, again with its own antiparticle. It also has only a single major decay mode, producing $n\pi^-$. As such it is probably best to assume that the original system contains two electrons, thereby clearly distinguishing its composition from that of Σ^+. Since the other two Σ species are assigned a composition of five particles, it is reasonable to delete a $\nu\bar{\nu}$ pair from the $n\pi^-$ fragments in arriving at the Σ^- structure, so that a like number of constituents is assumed for all members of this hyperon family. A restructuring of the bonding between the Σ^- and $\nu\bar{\nu}$ species can then lead to $\Lambda e^-\bar{\nu}$ products (Table XVIII). As usual the possibility must be left open that each of these assignments may need to be altered by the addition or subtraction of e^+e^- and/or $\nu\bar{\nu}$ binary systems.

The next two hyperons in order of increasing rest mass comprise the Ξ family, Ξ^0 and Ξ^-. Both particles have essentially unique decay products, $\Lambda\pi^0$ and $\Lambda\pi^-$ respectively. Their difference in rest mass compared to the Σ family members is about 130 Me V, with Ξ^0 being more stable than Ξ^- by 6.4 MeV. This suggests a composition of several more particles than for the Σ species. Since Ξ^0 is more stable it seems reasonable to assume that it benefits more from the Coulomb interaction. Taking the number of constituent particles to be seven in each case leads to assignments of 12.121 for Ξ^0 and 3.121 for Ξ^-. Whether the combination of one proton, two

electrons and a positron (Ξ^0) reaps more benefit from the Coulomb interaction than just two electrons and a proton remains somewhat of an open question in this assignment, but at least the assumption that one can estimate such effects by algebraically adding the products of each pair of charges does speak in favor of it. As usual each of the minor decay products of both particles can be obtained with the help of integral numbers of particle-antiparticle binaries (Table XVIII) .

The last particle classified as a hyperon is the Ω^-, with a rest mass of 1672.5 Me V / c^2, some 340 MeV/c^2 greater than for the Ξ particles. It is also a fermion, but it is found to have a quartet state, in contrast to all the other hyperons. The compositions of two of its known decay products, $\Xi^0\pi^-$ and $\Xi^-\pi^0$, is the same according to Tables XVII and XVIII, namely 14.132. The other possibility (ΛK^-) has an additional $\nu\bar{\nu}$ pair. The assignment given in Table XVIII of 13.131 assumes that Ω^- has two more constituent elements than the Ξ family members and that $\nu\bar{\nu}$ massless binaries are always involved in its decays.

Before closing this section, however, it should be noted that there are a large number of other resonances found as a result of scattering pions from protons which are often referred to as excited nucleon states. Their decay products are generally of the same order. The main distinction between these two groups of systems lies experimentally in the magnitude of their lifetimes or linewidths. The N' and Δ resonances all have widths of ca. 100 MeV, corresponding to lifetimes which are shorter by a factor of 10^{12}-10^{13} compared to those of the hyperons. Especially in the present context, it seems pertinent to mention that a distinction on this basis is somewhat artificial. Designating something an elementary particle because it only decays with a lifetime of 10^{-10} s, while refraining from the same terminology for less stable systems yielding a very similar product spectrum, is in itself an indication that the whole classification system is somewhat inconsistent. The present view is that hyperons and the above broad resonances are all meta-stable combinations of protons and lighter stable fermions. They are distinguished from one another mainly on the basis of the number and type of such particles they contain, as well as the manner in which they are bound together by short-range interactions.

I. REACTIONS OF ELEMENTARY PARTICLES

Throughout this chapter we have been discussing reactions of particles generated in high-energy collisions, but the emphasis has been placed on one-particle decays. In this section we

will briefly sample other types of reactions occurring for such systems as a further illustration of the utility of the assignments given in Tables XVII and XVIII. We can start by considering the collision of two protons: $p^+ + p^+ \rightarrow d^+ + e^+$. As usual the equation as conventionally written is not balanced. On the left-hand side the composition vector is 200, i.e. two protons, while on the right it is 11.210. Balance is obtained by adding an e^+e^- and a $\nu\bar{\nu}$ binary to the initial system and by taking account of the fact that positron emission in such a process is inevitably accompanied by the release of a like number of antineutrinos. The result is 11.211 on both sides. Nothing is created or destroyed in this view. It only appears that just two protons are present at the start of the reaction. Instead there are always an infinite number of massless binaries available to participate in the reaction in a non-trivial way provided sufficient energy is available to cause their decomposition or attachment *en masse* to a neighboring system.

It only makes sense to include the binary systems explicitly in the reaction's equation when they have a different relationship to their surroundings before and after the process has occurred. The situation is thus wholly similar to that encountered when a chemical reaction takes place in a water solvent. We could write down 10^{23} water molecules on both sides of a typical equation, but this generally serves no useful purpose and clearly conceals the essential submicroscopic nature of the observed process. It occasionally happens, however, that a true picture of such a reaction only emerges after careful study, with the result that it is realized that one or more of the water molecules does play an essential role in the process, remaining for example as part of the product molecules generated by it. The massless binaries perform both the functions of reaction partner and catalyst in elementary particle reactions, depending on the specific case. One major difference between water and massless binaries, however, is that the latter cannot be observed in their state of lowest energy, which has been found in the present work to have 0^- symmetry. This characteristic makes a unique assignment of elemental composition to each participant in a given reaction effectively impossible. That need not be an important deficiency in the utility of the theoretical model, however, because the main interest is in the identity and probability of a certain reaction's occurrence. This aspect is not unduly affected by the miscounting of one or another of the binary systems, as long as the same error is made on both sides of the equation. These are the rules of the game which should be kept in mind in considering other reactions below.

Let us now consider a more complicated process in which a negative kaon collides with a high-energy proton. According to Table XVII the elemental composition of the initial particles is 13.122. The products of this reaction are K^0, K^+ and Ω^-, respectively 13.13, 22.13 and 13.131, which gives a total of 48.157. The difference between the two totals is 35.35, *i.e.* three e^+e^- and five $\nu\bar{\nu}$ species, which must be added to the reactants to restore elemental balance. These are distributed as follows: one e^+e^- and three $\nu\bar{\nu}$ s are needed to form K^0, an electron is added to p^+ + K^- and a neutrino is lost to give Ω^-, while the remaining particles go into the formation of the positive kaon (including the positron counterpart of the Ω^- electron and the neutrino lost from K^-). As usual the assignment is not unique. One would particularly like to know if the K^- elements are used exclusively in the formation of Ω^- or if they are distributed among two or more of the products. Only accurate calculations can presumably help to remove such uncertainties. The reaction is highly endothermic, which is consistent with the relatively large number of particle-antiparticle systems which become included in the process.

As another example let us consider the $p + \pi^-$ reaction which gives as products $n\pi^+\pi^-$. From the point of view of merely balancing the equation it is necessary to compare a proton with a neutron-plus-π^+ combination. The difference is again an integral number of binaries (12), which must be supplied on the reactant side of the corresponding equation. One can imagine the process starting with the decomposition of the e^+e^- binary. The electron combines with the proton and a species to form a neutron, while the e^+ and $\bar{\nu}$ counterparts result in the positive pion with the addition of the second $\nu\bar{\nu}$ species. In actuality it might well be that the initial π^- species is decomposed in the course of the reaction and simply reformed, perhaps with different constituent particles, at the conclusion of the process.

Another set of products observed in $p\pi^-$ collisions is ΛK^0. We begin with a 2.111 composition and end with $2.111 + 12.12 = 14.123$. The K^0 species is its own antiparticle and thus can be formed directly from only binary systems, provided the minimum of 496.7 MeV kinetic energy is provided at the center of mass in the original collision. The ν_e ^{37}Cl to e^- ^{37}Ar process can be similarly described by suppressing the common ^{36}Cl nucleus on both sides, which changes it to the typical beta-decay reaction $\nu_e n \rightarrow e^- p^+$. This equation is balanced by adding a massless $\nu\bar{\nu}$ species to the products which is assumed to go undetected. The νn reaction, which

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is crucial in the arguments supporting a second type of neutrino, as discussed in Sect. IX.B, is already balanced for μ^-p^+ formation. Only a suitable amount of kinetic energy is additionally required to make the latter process possible. In its absence a massless $\nu\bar{\nu}$ species can be formed, giving the appearance that only an electron and proton are emitted in the process. This is the only process which can occur unless the sum of the kinetic energy of the $\nu + n$ system in its center of mass and the decay energy of the neutron exceeds the 105 MeV rest energy difference between the muon and the electron.

As an example of a more complicated reaction, let us consider the result of a deuteron colliding with a K^+ particle. One of the more unusual possibilities is the formation[225] of the products $\bar{\Omega}^+\Lambda\Lambda p^+\pi^+\pi^-$. One starts with d^+ (1.210) and K^+ (22.13), or a total composition vector of 23.223. The Ω^- assignment in Table XVIII is 13.131, which implies 131.113 for $p^+\bar{\Omega}^+$. Adding a p^+p^- binary to the reactant side gives 123.323, which leaves a deficiency of two protons on the product side. These are covered by the Λ particles of 2.111 composition (Table XVIII). Adding these as products of the reaction gives a result of 135.335, which provides a potential elemental balance through the addition of one e^+e^- and two $\nu\bar{\nu}$ species as reactants. The $\pi^+\pi^-$ pair is unchanged by charge conjugation and thus can also be explained in terms of integral numbers of particle-antiparticle binaries becoming involved in this reaction.

In summary, the present analysis in terms of composition vectors of protons, electrons, neutrinos and their respective antiparticles at least provides a handy bookkeeping device for checking if a series of reaction products is feasible. In the model the occurrence of the given reaction depends on a number of factors, especially whether sufficient energy is available. The actual reaction probabilities can then be computed in principle, and in this way give a quantitative description of the various processes without the need of further assumptions. Traditionally one has tended to rationalize whether such reactions are allowed or forbidden by defining a series of quantities such as isospin[83-85], hypercharge[211,226] and muon quantum number[206] which have no counterpart in atomic and molecular calculations. In the XBPS model it can at least be imagined that these quantities need not be considered explicitly in calculations which are capable of providing a suitably quantitative description of such processes.

J. THE QUARK MODEL AND ITS RELATION TO THE XBPS TREATMENT

The prevailing theory of elementary particles is deeply bound up with concepts of symmetry, involving a number of quantities specifically introduced to aid in unraveling the tangled web that the experimental data for such systems has produced. As mentioned in Sect. IV.D, the basic assumption in many theories which have been put forward is that all matter is ultimately formed from a series of building blocks, but the models differ in their specific choices for these quantities. Between 1949 and 1956, Fermi[25], Yang[25] and Sakata[115] developed one of the first such variations on this theme. They suggested a set of elements consisting of the proton, neutron and lambda particles, thereby setting a precedent for allowing unstable particles to serve as elements. One of the most fundamental concepts employed in this approach was the provision that a boson can be constructed from an even number of such fermionic elements. Some of the new particles which could be expected from this model have never been found experimentally, however, which fact has generally brought it into disuse.

The suggestion of quarks as building blocks brought with it the new assumption, at least relative to the Fermi- Yang-Sakata model, that the building blocks need not be found among the list of observed particles. In 1964 when the latter model was first introduced by Gell-Mann[112] and Zweig[113], only three quarks and three antiquarks were needed to explain the structure of all known particles. In the meantime this number has been expanded to six each[114], and the resulting improved theory has won great respect among physicists for the detail with which it is able to describe observed relationships among this otherwise heterogeneous group of systems. Symmetry is the watchword in this theory and the more it has been refined and tested, the more it has come to be accepted as reality.

The present calculations are based on a model which does not require the existence of quarks or any other particles as elements except the proton, electron, neutrino and their antiparticles. It also does not make use of quantum numbers other than those known in the field of atomic physics at the time of the introduction of the Schrödinger[86] and Dirac[99] equations. On this basis it can be argued that the present model is relatively free of unproven assumptions, which is after all one of the most important criteria to be satisfied by any physical theory. Emphasis is placed to a large extent on the identification of a Hamiltonian which when employed in standard quantum mechanical procedures leads to a suitably accurate description of observed

phenomena, particularly for such quantities as the rest masses and lifetimes of elementary particles. As mentioned earlier, the fact that such a model might ultimately be improved to the point of delivering a high degree of quantitative reliablility does not necessarily clash with the precepts of the quark theory. It is conceivable that the elements assumed in the present model, particularly the proton, are composed of still less massive building blocks such as quarks or even more fundamental particles. Even if experiments continue to be unable to provide definitive, positive, direct evidence that quarks exist, such as isolating and identifying a particle with a charge of $\pm 1/3$ e or $\pm 2/3$ e, it can still be true that such entities nevertheless exist with essentially the same properties as are required for them in the theory. Similarly the fact that one or the other of its key predictions cannot be verified, such as for example that the proton decay with an extremely long but nonetheless finite lifetime[81,82], in no way constitutes a disproof of the model as a whole. In short, the possibility that two fundamentally different theories might explain the same set of observations inevitably leaves one with an uneasy feeling, but does not in itself amount to indisputable evidence that at least one of them is seriously flawed.

Yet one knows from experience with simpler problems in the field of mathematics that multiple solutions of different characteristics are more the rule than the exception. The goal of finding a set of elements or building blocks from which to synthesize all known material particles is similar to the familiar exercise in the theory of finite-dimensional linear spaces of determining a set of basis functions to span a given linear manifold. It is impossible to find a suitable choice of functions for this purpose whose number is less than the dimension of the corresponding space, but there is no difficulty in finding a solution *involving more than this minimal number*. It is easier to construct a polynomial fit to a given set of experimental results which satisfies a given least-squares criterion when the ratio between the number of terms in the expansion and that of available data points is relatively large. In general, one is inclined to favor solutions which rely on a minimal number of elements (basis functions or free parameters) to accomplish the desired purpose.

The quark model, in its most modern form holds that there are a total of twelve fermions of non-integral electric charge plus a number of leptons and their antiparticles which nature uses as building blocks. The XBPS model by contrast hypothesizes that only six elements are needed, the proton, electron, neutrino and their respective antiparticles. It also uses only two intrinsic properties of these particles in its formulation, namely their electric charge and rest mass (or the

ratio of these quantities in the case of the massless neutrinos). Other quantities such as angular momentum quantum numbers and parities arise naturally out of the solutions of differential equations. No direct use is made of quantities such as isospin, muon quantum number and hypercharge. It is proposed instead that quasi-degenerate groups of particles such as the pions arise naturally from the theory when one solves a Schrödinger equation in which the true elemental compositions of these systems appear explicitly. It is clear then that the present model at least attempts to represent the structure of elementary particles with a far less elaborate system of hypotheses than does the quark theory. It is equally clear that the success of the XBPS model ultimately lies in its ability to obtain reliable approximate solutions to the Schrödinger equations arising from it, without greatly increasing the number of free parameters and/or *ad hoc* assumptions needed to attain this goal.

If this goal can be achieved in a systematic manner, it will constitute strong evidence supporting the major assumptions which have been made in previous chapters, such as that of the ubiquitous presence of massless particle-antiparticle binary systems or the non-zero charge-to-rest-mass ratios of the neutrinos. In this sense there is a clear parallel between the present model and that of the quark theory. Both contain assumptions which of their very nature cannot be contradicted by experiment. In the latter case there is *no way to prove that quarks do not exist or that protons do not have a finite lifetime.* On the other hand, as already mentioned, these assumptions *might be verifiable,* at least in principle. It is not difficult to imagine experimental investigations which could produce such results, but if quarks really do not exist the wasted effort could be enormous. Similarly, the basic assumptions underlying the XBPS model appear to be impossible to refute by experimental means. How does one *prove that two particles pass out of existence,* for example, or how can one be *certain that* a *neutrino does not have* a *given charge-to-rest-mass ratio* when theoretical calculations indicate that conventional magnetic fields would be incapable of deflecting it regardless of what value it might have (Sects. VI.H-I)? The likelihood is thus that none of the above assumptions in either theoretical model will ever be definitively proven to be either true or false. Instead the theories must be judged on their respective abilities to make detailed predictions regarding other types of experiments, especially of those which will first become possible in the coming years.

Before concluding this discussion it is well to consider several further points, one of an experimental and two others of a theoretical nature. The deep inelastic scattering experiments[221]

mentioned in Sect. IX.F are often taken as an indication that the proton does have internal components which are point charges, just as the corresponding elastic form factors[91,94,160] are used to justify the position that the proton has a finite radius. If the assumptions of the XBPS model are correct, a quite different interpretation can be given to the same results. Accordingly, the proton, just as the electron (and neutrino), could be regarded as a point entity. The elastic scattering data[91,94,160] could simply be a consequence of the admixture of neutron-pion character (or binding with e^+e^- and $\nu\bar{\nu}$ binary systems; see Fig. 18) in the wavefunction of the proton in its normal environment, whereas the higher-energy data[221] for the corresponding continuum might be a reflection of the free proton's isolated state. In the XBPS model, the charge (probability) distributions of all these particles have finite dimensions, but the terms in the corresponding Hamiltonian are constructed entirely under the assumption that each of them is a point particle. From this point of view an experimental proof that the proton has a finite lifetime would be inconsistent with the present model because it would be incongruous to hold that a point-charge system could be meta-stable.

On the theoretical side it is interesting to compare the way isospin degeneracy is defined in the description of elementary particles as opposed to its original usage in the field of nuclear physics[83-85]. The classic example common to both fields is the nucleon doublet consisting of the proton and neutron, i.e. with I= ½. Each of these particles has its own antiparticle, which together form a *different* isospin doublet. The pi mesons on the other hand are said to form a single isospin triplet, consisting of the π^+, π^0 and π^- particles, even though the two charged members bear the same particle-antiparticle relationship as the proton and antiproton in the first example. Similarly the K^+ and K^- particles are paired in the same isospin doublet. According to this prescription the nucleons actually form an isospin quartet, but as already mentioned the standard theory assigns them to two different doublets instead. In so doing, however, one in effect asserts that there is an "accidental" degeneracy between the particles and antiparticles of the respective nucleon doublets. The question remains, however, why the particle-antiparticle relationship of the charged pions should be described as a true degeneracy in the theory while that of the proton and antiproton is given a less fundamental characterization.

The history of quantum mechanics has led to the belief that accidental degeneracies are either not real, as turned out to be the case in the $s_{1/2}$ - $p_{1/2}$ example for hydrogenic atoms[227], or that some "hidden" symmetry can be found in the Hamiltonian which leads to the realization that

a higher-order group is actually involved[228]. In the case of the proton-antiproton pair it can be claimed that the former attitude is probably justified because charge conjugation is not thought to be a true symmetry operation (see Sect. VIII.C), but this would imply that respective particles and antiparticles actually do not have the same total energy (rest mass). There is no evidence to support such a view, although one can always claim that this situation may one day change. Even if this eventuality should come about, however, one can wonder why there is no interaction capable of mixing the two "basis functions" anyway, similarly as happens in the famous example of ortho-para hydrogen conversion[229].

In their work on the neutral kaons (Sect. IX.D), which do comprise an isospin doublet according to the theory, Gell-Mann and Pais[211] did pursue the possibility of such hybrids of degenerate particles (K^0 and $\overline{K}^0$) being formed, a phenomenon referred to as hypercharge oscillations. Their conclusion was that a basis of hybrid kaons which would be separately symmetric and anti-symmetric with respect to the charge conjugation operation C should possess distinct decay lifetimes. The observation of three-pion decays several years later[230] gave strong support to this interpretation. As mentioned in Sect. IX.D, however, further experiments[68] indicated that the long-lived kaon (K_L^0; see Table XVII) exhibits both two- and three-pion decays, which led to the abandonment of the hypothesis of hybrid particles which are eigenfunctions of CP, but not the concept of hypercharge oscillations itself.

The question that seems pertinent in the present general discussion, however, is why such "linear-combination" particles *do not occur for multiplet partners possessing different electric charge,* such as for the three pions or the charged kaon pair. Such hybrid particles would necessarily have non-integral electric charges and consequently there is no evidence whatsoever that they exist. While it is always possible to come up with an argument based on one or more conservation laws to rationalize such a finding, it should at least be acknowledged that a far more straightforward explanation can be found by simply rejecting the idea in the first place that elementary particles can be treated as basis functions for irreducible representations of some mathematical group. Even in the case of the neutral kaons, it is still possible to interpret the experimental observations in terms of two particles (or two different states of the same particle) with well-defined lifetimes (K_S^0 and K_L^0). Their lack of electric charge simply makes it impossible to rule out that oscillations actually do not occur.

Especially when attention is focused on charged systems, it is seen that elementary particles do not generally behave like conventional angular momentum basis functions. For example, the spatial properties of the latter can be altered in a continuous manner simply by systematically changing the direction of a perturbing magnetic field. In this sense the analogy between angular momentum and isospin is not at all as closely drawn as generally assumed. *Where else in physical applications of group theory does* a *single basis for a degenerate irreducible representation play such an exclusive role as in this model for the structure* of *elementary particles?*

By contrast, the present model seeks to deal with the fact that two different particles of the same rest mass exhibit *distinct* properties, which are *fixed* characteristics of each system, by employing the following two basic assumptions: a) the true building blocks of nature are the proton, electron, neutrino and their respective antiparticles, each of which is an immutable element and b) the governing Hamiltonian commutes with the charge-conjugation operation, so that any given composite particle must have an antiparticle (in some cases identical with itself) with exactly the same rest mass. The possibility of encountering a single particle which is a *hybrid* of two or more other particles is therefore excluded in the present model, whereas it is clearly implied by any theory which treats these objects as basis functions for a particular group representation.

The other theoretical point mentioned above has more to do with the internal consistency of the XBPS model. Especially in view of the desirability of keeping the number of free parameters to a minimum in constructing theoretical models, it can be regarded as a positive aspect of the XBPS methodology that it appears capable of accomplishing the goal of obtaining maximum binding energies for particle-antiparticle pairs of exactly $2mc^2$ with the aid of only a single parameter, the exponential damping factor A. The scaling arguments of Chapters V-VI make clear, however, that this result is only valid for particles with electric charges of ±e or zero. In that sense the properties of the various quark particles (non-integral charge) are incompatible with the present assumptions of the XBPS model. The impasse can only be averted by allowing two additional damping constants, one for particles with ±1/3 e and one for those with ±2/3 e. Such an additional complication in the underlying theoretical framework might be acceptable on general grounds, but it would be a step backward and presumably should only be taken if more is

accomplished thereby than just ensuring that particular values of quark-antiquark binding energies are obtained.

In summary, the quark model and that presented in the present study differ in a number of important respects, although their ultimate goals are quite similar. The XBPS model employs notably fewer assumptions and seeks to compute rest masses of elementary particles in an *ab initio* fashion. The quark model derives its plausibility from its capacity to identify and rationalize an orderly grouping of meta-stable particles observed in high-energy experiments. In so doing it claims, for example, that a charged pi meson is a diatomic system composed of one type of quark and the antiparticle of another type. When the resultant particle decays a short time later, however, the fragments observed are ultimately electrons and neutrinos, i.e. leptons in the terminology of present-day physics, which do not appear to have a quark composition themselves. This model therefore relies firmly on the creation-annihilation hypothesis whose validity has been questioned in the present study, and which at the very least cannot be proven experimentally. The XBPS model by contrast asserts that the order perceived in the quark theory has its origin in the structure of a single Hamiltonian operator which governs the interactions of three well-established stable particles and their antiparticles. All other substances are thought to be compounds of such elements in various degrees of excitation whose composition can be inferred to at least within an integral number of p^+p^-, e^+e^- and $\nu\bar{\nu}$ binary systems on the basis of the identity of their various decay fragments.

X. THEORETICAL DISCUSSION: RELATIVISTIC TREATMENT OF TRANSLATIONAL MOTION

The main tool in the XBPS theoretical model discussed above is a Schrödinger equation whose Hamiltonian contains a relativistic free-particle form for the kinetic energy and potential terms which are short-range in nature. Specifically, the latter are exponentially-damped, momentum-dependent operators, which in the limit of velocities much smaller than that of light approach those used in the conventional Breit-Pauli approximation to the Dirac equation. Since this formalism is applied to describe interactions such as nuclear binding and elementary particle processes, it is evident that it must adhere to the strictures imposed by the theory of special relativity for phenomena involving high relative velocities. Theoretical aspects regarding this

point will be considered in the present chapter, with particular emphasis on the treatment of translational motion employed in the present model. This general topic will also be seen to have relevance in another important area of quantum mechanical theory, namely the particle-wave duality interpretation of physical processes.

A. RELATIONSHIP BETWEEN THE SCHRÖDINGER EQUATION AND MULTI-COMPONENT RELATIVISTIC FORMULATIONS

The Dirac equation[99] is the cornerstone of a four-component theory for one-electron atoms. Just as the Schrödinger equation,[86] it can be looked upon as an eigenvalue equation, i.e. $H_D \Psi = E\Psi$, for which the corresponding Hamiltonian consists of a 4 x 4 matrix of operators and the eigenvectors are corresponding four-dimensional vectors called spinors. For a time-independent H_D the same type of separation of time and spin-spatial variables can be obtained as for the Schrödinger equation. In this case E becomes a four-dimensional constant matrix and the spinors which serve as solutions to the corresponding time-independent Dirac equation are functions of spatial and spin coordinates only. Dirac was led to this equation by relativistic considerations[231], especially the desire to accurately describe atomic fine structure which was known to arise from *the motion of the electrons* and thus could not be satisfactorily represented by means of the original electrostatic Schrödinger equation. Since the goal of the present study has been the quantitative description of systems involving higher relative velocities than those of electrons in atomic systems, it is natural to ask whether the inherently relativistic nature of the interactions involved is compatible with a theoretical treatment in which the multi-component spinor wavefunctions analogous to those of the Dirac equation are not employed. To answer this question it is important to examine the nature of the Dirac four-component theory in some detail.

As already noted, the differential equation in this formulation employs fourth-order operator matrices. In the time-independent version the energy is represented by a constant matrix, whereas H_D itself contains non-vanishing elements of several types, including some at off-diagonal positions. As with all equations involving matrices, it is possible to apply a unitary transformation to each of the objects appearing in it to obtain an equivalent differential equation. The constant E matrix is unaffected thereby, but the spinor eigenfunctions as well as the Hamiltonian matrix will generally have a different appearance after such a transformation. An

interesting possibility suggests itself as a result, namely that a particular unitary transformation U might succeed in producing a diagonal form for the Dirac Hamiltonian H_D'. The transformed Dirac four-component equation thus takes on the following appearance:

$$\begin{pmatrix} H_1 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & H_3 & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix} \begin{pmatrix} \Psi_1' \\ \Psi_2' \\ \Psi_3' \\ \Psi_4' \end{pmatrix} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & 0 & E \end{pmatrix} \begin{pmatrix} \Psi_1' \\ \Psi_2' \\ \Psi_3' \\ \Psi_4' \end{pmatrix} \qquad \text{X.1}$$

where $U\Psi = \Psi'$ gives the relationship between the original and transformed spinor eigenfunctions.

Foldy and Wouthuysen[232] adopted such an approach and succeeded in diagonalizing the free-particle Dirac Hamiltonian with the help of

$$U = [(\alpha\,p)\beta + m_o + E_p]\,[2E_p\,(m_o + E_p)]^{-1/2} \qquad \text{X.2}$$

In this definition α_i and β are the four-dimensional Dirac matrices[231], and E_p and m_o are constant matrices for $(p^2 + m_o^2)^{1/2}$ and the electronic rest mass respectively (in a system of units in which $c = 1$). The corresponding diagonal elements in the free-particle H_D' are then $H_1 = H_2 = -H_3 = -H_4 = E_p$, which is the Einstein relativistic energy for a system of rest mass m_o and momentum $\mathbf{p}$. The same one-electron term also appears in the XBPS Hamiltonian (Table I). When the Coulombic potential $\varphi = -Z/r$ is introduced, the situation becomes more complicated, but Foldy and Wouthuysen suggested a series of contact transformations which in principle can achieve the desired diagonal form of H_D' in this case as well. By carrying out a finite number of such transformations it is possible to obtain an approximation to the Dirac equation which is closely related to that used in the Breit-Pauli formalism[233].

The main interest in the FW transformations in the present context is less in explicit mathematical details, but rather the theoretical implications of employing such formal diagonalization procedures in the first place. Once such a unitary transformation is applied, *the Dirac equation reduces to a series of four ordinary Schrödinger equations* involving H_1, H_2 H_3 and H_4, respectively, namely:

$$H_1\Psi_1' = E\Psi_1'$$
$$H_2\Psi_2' = E\Psi_2' \qquad\qquad\qquad\qquad\qquad \text{X.3}$$
$$H_3\Psi_3' = E\Psi_3'$$
$$H_4\Psi_4' = E\Psi_4'$$

The simplest way of proceeding is thus to solve each of these Schrödinger equations separately, obtaining four complete sets of eigenfunctions for each component. Under the circumstances it would no longer be necessary to employ spinor eigenfuntions with more than a single non-zero component. For example, if Ψ_1' is an eigenfunction of H_1 then the spinor (Ψ_1', 0, 0, 0) is an eigenfunction of H_D' as well, with the same energy eigenvalue as in the ordinary Schrödinger equation, $H_1\Psi_1' = E\Psi_1'$. If the individual eigenfunctions for each component Hamiltonian form an orthonormal set, as is always possible, such a choice of spinor wave-functions with only a single non-zero component would be orthonormal as well. A reverse transformation then allows one to generate spinor eigenfunctions for the original H_D operator from each of the above simpler spinor functions with a single non-zero component: $\psi = U\Psi$. There is thus clearly a one-to-one correspondence between the two sets of spinor eigenfunctions with identical eigenvalues.

As a result there are essentially (exactly in the case of free particles) four times as many spinor eigenfunctions of the Dirac equation as there are simple eigenfunctions of any one of the above Schrödinger equations involving H_1 to H_4, as has been pointed out earlier by Bethe and Salpeter[234]. The question that arises from this observation is whether the additional solutions of the Dirac equation are important physically. *In the case of the free-particle Hamiltonian, the answer is clearly no,* since the four diagonal Hamiltonian operators are identical except for sign. For each eigenfunction of H_1 there is an identical one for H_2, H_3 and H_4. The corresponding eigenvalues are equal for H_1 and H_2, and only of opposite sign for H_3 and H_4. It is common to think of the pairs of equal eigenvalues as corresponding to different spin directions of the free particle, but such a distinction is not essential. Instead one can have a complete set of solutions for both spin directions (assuming the system involved is a fermion) for each of these operators. This is the way the XBPS Hamiltonian is treated, for example, with all degenerate components being generated by the same operator. Under the circumstances it is a matter of semantics whether one refers to such a treatment as a one-component or a two-component method. At least

for the free particle, half of the solutions of the Dirac equation are simply redundant, serving no useful purpose from a physical point of view. Moreover, the negative-energy solutions don't really tell us anything new either. In summary, in the free-particle example, solution of a single Schrödinger equation tells us all we need to know.

When a potential is added to the Dirac Hamiltonian there is no longer such a simple relationship between the positive- and negative-energy solutions. This is because the potential has the same sign in all four diagonal terms of H_D whereas the kinetic energy terms for the first two components continue to differ in sign relative to the last two. Nonetheless, there is still a clear mathematical relationship between the two sets of spinor eigenfunctions which allows corresponding positive- and negative-energy solutions to be converted into another by simply changing the signs of E, p and the electronic charge e and interchanging the first two components with the last two[234,235] relative to the original solution. The new function thus corresponds to a system in which the electron of the original problem is replaced by its antiparticle. Dirac originally used this relationship to formulate his hole theory of the electron, which ultimately led to his prediction of the existence of the positron[1]. Furthermore, since the added potential only appears in the diagonal positions of H_D, it seems likely that the interesting simplifying feature of the transformed free-particle Dirac equation, namely $H_1 = H_2$ and $H_3 = H_4$ in the diagonal H_D' matrix, is also a characteristic of the corresponding hydrogenic version. In other words, all the physically meaningful positive-energy solutions of the Dirac equation with a central-field potential can also be obtained from a single Schrödinger equation.

The same type of analysis can be carried over to multi-component treatments of many-particle systems, such as the Bethe-Salpeter equation for helium-like atoms[236]. In this case one can generalize the Dirac approach to define corresponding differential equations whose solutions are spinors of dimension 4^N, where N is the number of particles. Such matrices can also be diagonalized, at least in principle, and the resulting H_i diagonal elements must have eigenfunction sets which can be used to produce the 4^N-component spinor solutions of the original system of coupled equations. If one continues to allow each of these Hamiltonians H_i to act on the entire spin space spanned by the particles, it again seems possible (and even likely) that all physically meaningful results for the corresponding positive energy states can be obtained by solving a single many-particle Schrödinger equation. Furthermore, as long as the

original multi-component set of equations is invariant to a Lorentz transformation[236.237], so must the corresponding diagonal (uncoupled) set as well.

In summary, because of the form of the Dirac equation and its generalizations, it appears to always be possible to obtain a completely equivalent version which decouples the various components. This being the case, there is no reason to rule out the possibility that a corresponding Schrödinger equation might exist whose solution contains all the physically meaningful information about a given system which can be obtained from quantum mechanical theory. Since an analogous multi-component treatment for nuclei and elementary particles is not known, it therefore seems reasonable to search for a concrete form of the Schrödinger equation which is capable of attaining the same objective.

Indeed, once such a form is known it is always possible to construct a multi-component version by writing down a diagonal matrix containing Hamiltonian operators corresponding to different combinations of particles and antiparticles and subjecting it to an arbitrary unitary transformation. The latter complication would serve no useful purpose, however. From this point of view the advantage of the original Dirac multi-component formulation is clearly that it provides a quite satisfactory description of the desired atomic interactions which is superior to any Schrödinger equation yet discovered. It brings with it a large number of redundant solutions, however, which either apply to a different physical system than that of primary interest, or are simply repetitions of other useful solutions. Under the circumstances, when dealing with interactions for which no appropriate multi-component formulation is known, such as in the present set of applications, there is no compelling reason to expect that a Schrödinger-type approach may not offer the most straightforward means of achieving a suitably accurate description of the phenomena of interest.

B. THE ROLE OF TRANSLATION IN THE MULTI-COMPONENT DIRAC THEORY

One of the mathematical characteristics of the multi-component Dirac equation[99] which sets it apart from its non-relativistic counterparts is that it can be brought into an explicitly covariant form[237]. The variables contained in it represent exclusively internal degrees of freedom, however. Implicit in this formulation is therefore the complete separation of internal motion from that of the center of mass. Such an arrangement for the theory seems intuitively obvious, but as pointed out in Sect. V.B., a mathematically sound procedure for effecting the

desired coordinate separation is only given explicitly for a non-relativistic treatment of the motion of several particles. In Sect. V.F it has been argued that the failure to obtain a clean separation of internal and center-of-mass coordinates in the XBPS Hamiltonian leads to some hitherto unexpected effects for the description of short-range interactions, particularly a relationship between the strength of the various Breit-Pauli terms (Table I) and the magnitude of the translational energy.

The question that therefore needs to be examined more carefully is whether the separation of internal and center-of-mass coordinates in the Dirac equation is consistent with experimental observations. In this context it is well to recall that the theory of special relativity[4] holds that the distance between two objects may be measured to be different in inertial systems moving with non-zero relative velocity to one another. In the c1assic example, both inertial systems move along the x axis and the distance x_1 - x_2 in one of them is perceived to be contracted in the other by a factor of $\gamma = (1 - v^2/c^2)^{-1/2}$, where v is the relative; velocity. It is not entirely obvious how this aspect of relativity theory should be carried over to quantum mechanics.

As an example, consider the computation of the mean distance between the electron and proton in the hydrogenic ground state. In quantum mechanics this quantity is given as the expectation value $<\Psi|(r_e-r_p)|\Psi>$ for the corresponding eigenfunction Ψ. The distance between these two particles is not fixed but the phenomenon of Lorentz contraction should still be relevant in this determination, that is, it seems reasonable to expect that a different result should be obtained if the center of mass of an H atom is at rest with respect to the observer or if it is moving relative to him at high speed. If the total wavefunction is always written as a product of two factors only one of which depends on the internal coordinates, however, as is the consequence of separating out the center-of-mass motion, it is clear that the expectation value in question cannot depend on the translational state of the system. The internal part of the wavefunction is completely independent of the motion of the center of mass. If, on the other hand, the two types of motion become increasingly intertwined as the translational energy becomes large compared to the system's rest energy m_oc^2, one can at least see a mechanism in which the Lorentz contraction/expansion effects might evolve in a natural way from the computations.

One might argue that it is not really necessary to obtain different results for internal properties of systems as a function of their translational state. Instead, one can compute the

property for the system at rest and use the relationships of special relativity theory[4], such as Lorentz contraction/expansion or time dilation, to obtain the corresponding results when the system is moving relative to the observer. The situation is similar to that encountered in the computation of the energy lost when positronium decays, namely $2\,m_{oe}c^2$ in accordance with the c1assical mass-energy equivalence relation. It seems fair to say that such interpretations may be overly simplistic, however.

In the case of property computations corresponding to states of high translational energy, this line of reasoning seems to suggest that the separation of internal and center-of-mass coordinates is an approximation whose .validity is lost at velocities close to the speed of light. An attempt to recast the XBPS Hamiltonian of Table I in terms of center-of-mass and relative coordinates leads to cross terms involving odd powers of both types of conjugate momenta, as already discussed in Sect. V.B. Although matrix elements for such terms cannot mix configurations of different translational momentum k, they can connect species of different angular momentum quantum number l.

Since the differentiations of the center-of-mass momentum operator always lead to factors of |k| in the corresponding matrix element expressions, it is clear that no such mixing can occur for zero translational momentum. By the same token, however, it seems reasonable to expect that the magnitude of such interactions increases steadily as the speed of the center of mass increases. This would mean in effect *that different internal configurations are used to describe states of high translational energy than are required for the corresponding system when it is at rest with respect to the observer.* Since the translational kinetic energy operator commutes with any Hamiltonian which is a function of the inter-particle distances only, none of this prevents the total eigenfunctions from having a well-defined translational energy. This effect is seen to arise naturally in the present formulation from the fact that the cross terms involving both types of coordinates have only vanishing matrix elements between functions of different k.

C. LORENTZ INVARIANCE CONDITION FOR QUANTUM MECHANICAL TREATMENTS: APPLICATION OF THE BOHR CORRESPONDENCE PRINCIPLE

The above discussion also raises a question about the way in which the Lorentz invariance condition of special relativity is applied in quantum mechanics. In the classical

theory[4], the discussion of relativistic effects is invariably tied up with the translation of various systems. The Lorentz invariance property of the Dirac equation[237] for a particle moving in a central field by contrast refers to a non-translating system, i.e. only internal coordinates are involved. In classical expositions of special relativity theory, however, the treatment of two interacting systems is generally given in terms of the original Cartesian coordinates of each particle. For example; in discussing the problem of two charged particles moving with the same velocity, Sard[238] proceeds as follows. In the inertial system of the charged particles themselves, the equation of motion is:

$$\mu_A \frac{d^2 y_A^t}{dt^{t\,2}} = \frac{e_A e_B}{(y_A^t - y_B^t)^2}$$

X.4

$$\text{and} \quad \mu_B \frac{d^2 y_B^t}{dt^{t\,2}} = \frac{-e_A e_B}{(y_A^t - y_B^t)^2}$$

where the two particles are denoted by the indices A and B, the direction y is perpendicular to their momenta, and y^t refers to a coordinate measured in the moving particles' own inertial system S^t. These equations express the fact that from the point of view of the particles themselves there are no magnetic forces (see the discussion in Sect. V.E).

   To transform to the laboratory inertial system S, it is necessary to apply the Lorentz transformation[4.132] to each of the quantities in the above equations, for example, $\mu_A = \mu_A^t \gamma$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ and v is the relative velocity of the particles. In this way one obtains a new equation of motion:

$$\mu_A \frac{d^2 y_A}{dt^2} = \frac{e_A e_B}{(y_A - y_B)^2 \gamma^2}$$

X.5

i.e. in terms of the laboratory quantities, including an additional factor of γ^2. Because the spread of the two particles in the y direction corresponds to a direction perpendicular to the velocity of the particles relative to the laboratory, it follows that this quantity is measured to be exactly the

same in both inertial systems ($y_A = y_A^t$), but in general other properties will have different values when viewed from the position of the charged particles as from that of the laboratory. The Lorentz transformation is supposed to predict these different observations in the two inertial systems.

In quantum mechanics the equation of motion is of a different form, however, namely one in which *the wavefunction of a system is sought rather than a fixed trajectory.* If the corresponding Hamiltonian is given in terms of the coordinates of each particle rather than assuming a separation into internal and center-of-mass coordinates, similarly as in the example from classical relativity theory discussed above, the following possibility suggests itself. The laws of nature must be the same in all inertial systems according to the principle of relativity[4,6,7]. *In the case* of a *quantum mechanical formulation this condition can be fulfilled if the Hamiltonian operator simply has the same form for all observers.* The coordinates on which this operator depends necessarily vary from one inertial system to another in describing a given system.

If two observers moving relative to one another start out with the same Schrödinger or Dirac equation $(H - E) \Psi = 0$, how then can they come to different conclusions about the properties of a system in their respective inertial systems? The answer lies in the fact that *there is more than one solution to the above quantum mechnical differential equation.* The computation of the properties of a given system thus requires *not only the solution of the corresponding equation, but also the correct identification of one of the resulting eigenfunctions* as that describing the state of the system actually observed. For example, the hydrogen atom ground state in the conventional language actually corresponds to an infinite number of different translational states. If two observers *disagree* on the magnitude of the translational energy of a given system, they will simply *choose different solutions* of *the same differential equation* to describe it. This possibility distinguishes quantum mechanics from its *classical* counterpart, for which *a definite answer is expected* for each set of initial conditions.

What these considerations show is that the principle of relativity can be incorporated into a quantum mechanical theory by simply requiring that the Hamiltonian H(q.p,t) of any system always be expressed in terms of the coordinates of a given observer. Applying the Lorentz transformation to H(q,p,t)-E(t) simply gives back the same expression in terms of the coordinates of a different inertial system, i.e. H(q',p',t') - E(t'). Under the circumstances *it is not really*

necessary that the two quantities be exactly equal, i.e. that $H(q,p,t)-E(t) = H(q',p',t')-E(t')$, in order to satisfy the principle of relativity. Rather it is sufficient that the *form* of H-E be the *same* for all observers, so that each of them can generate the *same* set of solutions to the corresponding quantum mechanical equations of motion from which the pertinent translational states corresponding to different perceptions of the same physical system can be properly selected. If the Hamiltonian is incorrect for any reason, there will inevitably be disagreement between prediction and observation on this basis, so this possibility still excludes the use of distinctly non-relativistic terms such as $p^2/2m$ in representing the kinetic energy. On the other hand, it suggests that failure to find a covariant form for H-E, i.e. one that satisfies the condition $H(q,p,t) - E(t) = H(q',p',t') - E(t')$ upon application of a Lorentz transformation, *does not constitute proof in itself that the corresponding quantum mechanical formulation is defective.*

Since the present discussion involves a comparison of the theory of quantum mechanics with its classical counterpart, it is relevant to consider the implications of the Bohr correspondence principle[119] in this regard. In accordance with this prescription, it must be expected that the quantum mechanical formulation reverts to the classical one in limiting situations in which the less general theory has established validity. One difficulty with applying this principle to a relativistic quantum mechanical theory is that two separate limiting processes are involved, as commonly achieved by setting Planck's constant h to zero and the speed of light in free space c to infinity. The minimal condition which must be fulfilled with regard to the theory of special relativity is that the classical limit of the quantum mechanical equations of motion can be cast in covariant form. For this purpose one typically employs the relativistic four-vectors[239] (**p**,iE/c and (**A**, iφ). The Breit-Pauli terms[102] are closely related to the classical magnetic interactions containing the vector potential A, as has been discussed in detail by Slater[240], so the ingredients for such a covariant limiting form are present when a Hamiltonian containing such interactions is employed. Because of the linearity of the Lorentz transformation, it is clearly essential that the r^{-3} dependence of the Breit-Pauli terms (see Table I) tend toward r^{-1} as a result of the limiting process. Since a constant velocity is always involved in such a transformation, however, it is not difficult to imagine how this occurs. Under these conditions the angular momentum **l** = **r x p** itself varies as the first power of r, and since the spin-independent Breit-Pauli terms are second-order in **1** (or **r • p** in the case of the Darwin term), the necessary changeover from r^{-3} to r^{-1} variation is ensured. The corresponding spin-dependent

terms can be plausibly ignored since they have no classical analogue and can thus be assumed to vanish in the pertinent limiting process.

The fact that the Breit-Pauli terms can be derived as an approximation to the Dirac equation[100-102], which itself is invariant to a Lorentz transformation, is perhaps the best indication that such a limiting relationship can be satisfied by a differential equation containing such momentum-dependent short-range interactions. The essential point remains, however, that it is not necessary that the relativistic quantum mechanical theory be Lorentz-invariant itself. As discussed above, the principle of relativity is automatically satisfied by employing the same functional form in all inertial systems for the (H-E) operator appearing in the pertinent Schrödinger equation. By treating the translational motion explicitly in such a formulation one can satisfy a more specific condition of the classical theory of special relativity, namely that observers in inertial systems moving with high velocity relative to one another *can measure different properties for the same physical system* (Sect. X.B). For this purpose it is only necessary that they choose eigenfunctions corresponding to *different translational energies* from among the complete set of solutions to a *Schrödinger equation which has the same form for each of them.*

D. THE ROLE OF SHORT-RANGE INTERACTIONS IN HIGH-ENERGY PROCESSES

Ever since the first scattering experiments on nuclei, it has been recognized that forces of shorter range than those of the Coulomb effect and gravitation are at work in nature. Yukawa[87] was the first to suggest that the interactions in question were exponential in character, but he assumed a potential which varies as r^{-1} as the internuclear distance approaches zero. In the XBPS model the short-range interactions vary as r^{-3} over a large range of interparticle distance, until the momentum-dependent damping factors begin to have a dominant influence on the theoretical description (Table I). Because the Lorentz transformation is linear in spatial and time coordinates, it is difficult to imagine an equation of motion involving such a potential which is invariant to such a change in variables. Nonetheless, by not assuming the usual separation in terms of internal and center-of-mass coordinates, it has been possible to employ such potentials in the XBPS model to obtain a realistic description of nuclear binding. In Sect. V. F it has been pointed out that the combination of short-range momentum-dependent potentials plus an interdependence of internal and translational motion allows a simple explanation for the fact that

particles and antiparticles can interact strongly with another. In the inertial system in which the center of mass is at rest, $\mathbf{p_1} = \mathbf{-p_2}$, from which it follows that particles of equal rest mass must move with the same speed relative to their midpoint. This circumstance allows maximum benefit to be taken of attractive short-range interactions which are momentum-dependent, whereas a much weaker effect is expected for two particles with greatly different rest masses.

It would appear then that not only is it consistent with relativity theory to employ such short-range interactions to describe high-energy processes, but even that it is actually essential that this be done in order to obtain satisfactory agreement with experimental observation. The arguments of the last section show how the results of the corresponding Schrödinger equation can be interpreted in a manner consistent with the principle of relativity, even though a covariant form for its Hamiltonian appears to be excluded. Furthermore, by not factoring out the center-of-mass motion, it is possible to explain at least in principle how the properties of physical systems can vary with their translational state, as the theory of special relativity tells us they must. None of this precludes the possibility of defining a Schrödinger or Dirac equation which is completely independent of center-of-mass coordinates. Formally, one must transform to the usual system of internal and center-of-mass coordinates and retain only the terms of the Hamiltonian which are independent of translational variables. Solution of the corresponding differential equation would then only produce information about states of zero translational energy. It is doubtful that such a simplified operator would be Lorentz-invariant, but its application would have an advantage relative to Hamiltonians of the XBPS type employed in the present work, in which the center-of-mass and internal types of motion are intertwined.

A related question to that discussed above is whether it is essential to employ a separate time coordinate for each particle in defining a proper equation of motion. Because of the assumed lack of simultaneity in different coordinate systems, it might be argued that such an arrangement is essential, but this view is subject to interpretation. In particular, if one assumes that all the particles of a given system are referenced to the position of an observer who is always located at the origin of the coordinate system in which the computations are carried out, it is permissible to employ a single temporal variable in describing the physical interactions. There is no need for more than a single clock with which to measure the time of events occurring in the same inertial system. This choice is also consistent with the assumption that each solution of the corresponding differential equation corresponds to a single total energy. As a result it is possible

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to describe the time-dependence of the XBPS eigenfunctions in the usual way as exp $(-iEt/\hbar)$, where E is the total energy of the corresponding state. Since the particles move independently of one another, it is necessary to employ a separate set of spatial and spin coordinates for each of them, but the former correspond to different length measurements in the same inertial system, thus requiring the use of only a single unit of distance (meter stick) to determine their values experimentally.

E. TRANSLATIONAL MOTION AND WAVE-PARTICLE DUALITY

The physical model on which most of the theoretical arguments of this study have been based relies heavily on the existence of subatomic particles from which more complex systems are formed. The questioning of the creation-and-annihilation hypothesis of matter is motivated in large part by experience with macroscopic systems, for example, which at least give the appearance of decomposing into ever smaller components when subjected to large forces, rather than dropping out of existence entirely. Nonetheless the theoretical framework chosen to provide a quantitative description of such particles is that of the Schrödinger or Dirac equations, in which *wavefunctions* are employed to compute the (average) properties of such systems. The concept of wave-particle duality should therefore be given careful consideration in the present discussion.

The wavefunction itself was interpreted by Born[241] as having only statistical significance, with $|\Psi|^2$ serving as a distribution function to be employed in computing average values. This identification is not in itself in conflict with the ancient philosopher's idea of an atom or particle. It still leaves open the possibility that particles always have a definite position in a given coordinate system, but emphasizes that observations of identical objects under the influence of the same system of forces over a long period of time are characterized by a definite statistical pattern. The motion of a planet in the gravitational field of a star can be charted over the course of centuries, for example, and from this information a distribution function can be defined which gives the percentage probability of finding its center of mass in a certain volume element independent of the actual time. Just because this distribution has a continuous form does not at all mean that one must ascribe delocalized characteristics to the planet.

Yet the concept of wave-particle duality is generally taken to mean that sub-atomic systems such as electrons and photons cannot be regarded in this way. Experiments can be

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carried out whose interpretation is argued to be impossible without giving up a strictly localized description of such systems[242]. Because of the fact that our view of physical reality is profoundly affected by the degree to which the particle model of subatomic systems succeeds in explaining experimental observations, it is important to give careful scrutiny to the wave-particle duality interpretation. In particular one should focus on the question: *does a single particle ever exist in more than one location at any one time?* It is probably safe to conclude that Newton would have vigorously opposed such a suggestion. He would have attempted to settle the matter exclusively on the basis of indisputable observations[15], being careful in his argumentation to avoid the slightest dependence on the assumptions of some theoretical model.

To begin such a discussion it is well to return to the treatment of translational motion in dynamical theory. Schrödinger was led to his operator substitution hypothesis[99] by studying the motion of a free particle, i.e. in pure translation. He employed de Broglie's ideas[19] regarding the equivalence of certain properties of particles and waves to define a prototype differential equation which could be generalized to deal with more complicated phenomena requiring the use of a potential. Einstein also used a free particle's motion to guide his development of the theory of special relativity[4]. These observations suggest that the essence of the wave-particle duality concept can also be found in the study of translational motion.

The free-particle wave function is $\exp[\pm i(k\,r - \omega t)]$, where $|k| = |p|/\hbar = 2\pi/\lambda$ and $\omega = E/\hbar = 2\pi\nu$. It should be clearly understood that such a wave function is a characteristic of a *single* particle in this picture[241.242]. A term such as "stream of like particles" in this connection merely implies that *each* such particle is described *by the same statistical distribution.* Experimentally one could determine the latter by subjecting *one* such particle to exactly the same experimental conditions a large number of times. The same distribution function is also relevant to the description of the motion of *many* such particles at one time, however, in which case the wave properties of the aggregate system can be observed directly, but this relationship can only hold if the particles in the aggregate system do not interact with one another. Otherwise each of their wave functions would necessarily be different than for their corresponding isolated state. There are limits to which it is feasible to consider a stream of electrons as free particles, entirely unaffected by each other, but this condition is fulfilled to a very good approximation in the classical electron diffraction experiment[26]. One can easily distinguish between an experiment carried out for a single particle and one for a collection of them, but as long as the above

condition of non-interaction is fulfilled, it is permissible to use the same statistical distribution function in both cases.

Since any free particle can be treated in the same way, it is also readily understandable why a statistical distribution based on the above wave function appears to be a *universal* property of pure translation. Even at relativistic speeds, the eigenfunctions of the total Hamiltonian for any isolated system, independent of its composition, are also eigenfunctions of the translational energy operator (see Sect. V.B) Thus such a relationship between the momentum and kinetic energy of the center of mass and the wavelength and frequency of the relevant statistical distribution would always be guaranteed.

With the above line of argumentation we could summarize the situation, at least tentatively, as follows: the wave properties of a given system only arise when *large numbers of such particles are observed under similar conditions over the entire duration of an experiment.* By contrast, the characteristic properties of a localized system exhibit themselves even if attention is restricted to the behavior of a single particle. The photoelectric effect[38] is a prime example of the latter type, and this experiment led Einstein to re-examine the long-held position that light is strictly a wave-like phenomenon[33]. The opposite side of the coin is the diffraction experiment, in which interference patterns are observed. It seems unavoidable to conclude that such spectral images do not result from a single photon (or other particle), in agreement with the above principle. Yet it is exactly experiments of the latter type which have led to the conclusion that a single particle can also exhibit wave characteristics.

This is the crucial contention which needs to be carefully examined. For if the wave properties commonly associated with particles only arise because of the collective motion of large numbers of them*, as suggested by the above statistical argument, there is really no need to speak of a wave-particle duality* to describe the relevant experimental findings. Instead, the de Broglie wavelength[19] and the Bohr frequency[5] can simply be characterized as statistical parameters needed to specify an appropriate distribution function for a large sample of indistinguishable particles, each of which possesses the same momentum and energy. Similarly one can also understand the Heisenberg uncertainty relation[215] as a purely statistical law, whereby the quantities Δp and Δq in this inequality are taken as rms deviations from mean values of complementary dynamic variables, as determined by carrying out *a series* of equivalent measurements on large *representative samples* of a given type of particle.

In short, if waves are nothing but streams of individual particles, then Newton was right when he argued in favor of his corpuscular theory of light[6]. Before taking a closer look at the key interference phenomena which are always cited in favor of the principle of wave-particle duality and against his position, however, it is well to examine another experiment which gave a decided impetus to the wave theory of light during Newton's lifetime.

F. LIGHT REFRACTION

The bending of light rays at an interface of two media of different density was one of the earliest phenomena to be characterized by a mathematical equation[243.244]. The sine law of refraction was discovered by Snell several thousand years after the other two laws governing this process had been discovered. Newton[6] attempted to explain such observations in terms of the particle model of light, while Huygens[33] argued that they could only be understood in terms of wavelike properties. Newton lost the argument, but it is interesting to see why. Since the angles of incidence and refraction were not changed in multiple passes of light rays through the same media, it could be safely assumed that the energy of the hypothesized particles of light remains constant. The fact that the light is bent downward upon entering a medium of higher density indicates that the potential V acting on the particles must be attractive, i.e. it decreases after crossing such an interface. Combining these two facts led unmistakably to the conclusion that the kinetic energy T of the light particles must be *greater* in the denser medium. Assuming that T was proportional to the square of the velocity, in accordance with the then accepted dynamical theory, thus led to the prediction that the speed of light must be greater in water than in air, which is incorrect[244]. Proceeding on the principle that an assumption which is contradicted by observation is false, it was thereupon concluded that this result refuted the particle theory of light once and for all.

Examination of the above argument shows that another error of a different kind was made, however, which ultimately invalidates the latter conclusion. In the first place the kinetic energy of the photon does not satisfy the non-relativistic relation employed therein. When the correct formula is used, one is still led to conclude that the *momentum* of the photon increases in going into the denser medium, but since the mass of such particles cannot safely be assumed to be constant in a relativistic treatment, *it no longer follows that the velocity of light must increase as*

well. The conclusion that light rays cannot simply be streams of photons because a purely mechanical treatment of the refraction phenomenon leads to a false prediction on this basis is therefore not justified. On the other hand, if it is assumed not only that light consist of particles but also that their collective motion conforms to a definite statistical distribution, a different result is obtained from the refraction analysis.

By interpreting the de Broglie[19] and Bohr frequency[5] relations in accordance with this view, one is led to the following conclusions: a) since the momentum of the photons increases upon entering the denser medium, the *wavelength* characterizing their collective motion must therefore *decrease* ($p = h/\lambda$), and b) since the energy of the individual particles is constant, the *frequency* associated with the statistical distribution must remain the *same* in both media ($E = h\nu$). Under the circumstances the speed of any given particle in the light ray can be computed as the product of the wavelength and frequency in each medium [improve this], on which basis one is led to correctly predict that the velocity of light is *smaller in the denser medium.* It actually varies in *inverse* proportion to the momentum, a possibility which could not be foreseen in the seventeenth century. Dicke and Wittke[244] have also used the de Broglie relation to derive the sine law of refraction explicitly, thereby showing that wave optics and Newton's classical mechanics are consistent in this respect as well. From the standpoint of Newton's dispute with Huygens, however, there is an important distinction between regarding light as a stream of particles moving in accord with a particular statistical distribution law instead of as an intrinsically wavelike quantity. In the first case, one can still speak of the individual units of light as particles which are always perfectly localized in space, whereas in the second, a delocalization is implied which is inconsistent with the concept of a particle.

To conclude this section, it can be noted that the above argument implies that the mass m_A of each photon increases as the *square* of the momentum upon entering the denser medium. Only in this way does *the* product of mass m_A and velocity v_A vary correctly as the first power of the photon's momentum. Moreover, the product $m_A v_A^2$ is seen to be equal to the (constant) energy of the individual photons, $E = mc^2$, where m is the relativistic mass of the photon in free space (in which case $v = c$).

G. PROBABILITY ADDITION LAW AND INTERFERENCE PHENOMENA

As remarked in Sect. X .E. one does not really come to the crux of the wave-particle duality question until interference phenomena are considered. The most frequently analyzed experiment is that performed by Young[43,245] in 1802, in which light from a single source is allowed to pass through two narrow slits separated from one another by some distance. The interference pattern which results is not the same as what is obtained by superimposing the results of two similar experiments in which only one of the slits is open for the same period of time. This result is generally interpreted as follows[246]. If the light emitted from the source were to consist of particles which are always localized, then each of them would have to pass through *either* one hole or the other. It is safe to say that Newton would have agreed with this statement. Since the holes are relatively far apart, it is impossible that particles going through one of them would be affected by those going through the other. Experimentally one can show that the interference pattern does not result because of collisions between individual photons, because the intensity of the light can be decreased to the point that it is impossible that more than one such particle be in the apparatus at any one time. Dirac summarized this situation[242] by saying that it is as if each photon "interferes with itself."

Since particles cannot interact with themselves but waves (or anything that is extended through space) can, the conclusion that light units (and everything else) somehow behave as both particles and waves depending on the set of governing conditions seems inescapable. For a stream of particles to give a different distribution on a screen depending on whether the two slits through which they pass are always open or whether they alternate in being open and closed in a complementary manner seems tantamount to ascribing some sort of intelligence to them. How can the photons at the source know which holes are going to be open and when? Yet the solution to this dilemma of attributing both wave and particle properties to matter is also not without its ambiguities.

Once the decision is made that a photon (or electron) must pass through both slits[247] in the Young experiment before reaching the screen, a simple variation presents itself. Send a single photon through the apparatus and replace the screen by a photographic plate[246]. The experiment mentioned above approaches this in principle, namely by letting a maximum of one photon through the apparatus at any one time. The fact that the usual intensity pattern still develops *after*

a *sufficiently long time* proves that the photons do not collide with one another, but as long as more than one partic1e is involved it is impossible to be certain that any one of them has passed through both slits on its way to the photographic plate. The lack of a suitably sensitive photographic plate renders such an experiment impractical[246], but its conceptualization at least illustrates that the principle of wave-particle duality implies a definite result of an experiment which has actually never been carried out. The possibility that a single photon makes more than one impact on the screen of the Young apparatus, or that it somehow recombines after passing through each of the slits so as to make only one indentation, is sufficiently improbable as to warrant skepticism in the absence of a means of positively verifying its occurrence. In recognition of the conceptual problems inherent in its logical structure, the wave-particle duality principle is typically described as a paradox[248].

It will be recalled. however, that the argument against an exclusively corpuscular (Newtonian) description for the photons in the Young experiment rests on the conclusion that at the time they leave the emitting source, there is no way they could be aware of the status of the two slits of the apparatus. If nothing occupies the space between the source and the slits, there is no reason to doubt this position, but in the course of examining the creation-annihilation hypothesis, a different picture has emerged. Instead it has been assumed that there is a high density of massless particle-antiparticle systems throughout all space. It also has been argued that such a model is consistent with the virtual photon formalism of quantum electrodynamics. If such entities need to be taken into account in the form of radiative corrections in order to obtain a highly accurate description of the fine structure of atomic systems, *it is not obvious that their influence can be entirely neglected in the present context either.*

Particularly if such massless species exist in the numbers required to explain emission processes without invoking the creation-annihilation hypothesis, it would seem to follow that there is a direct line of communication between the source and the two slits of the Young experimental apparatus, one that could cause the photons to distribute themselves differently on the screen depending on the nature of the environment a relatively large distance away. The fact that the single-hole intensity distributions do not satisfy an additivity law[247] to give the corresponding two-hole result is not in itself incompatible with the particle concept. Rather it reinforces the key assumption of the Born interpretation[241] that the wave function in quantum mechanical treatments needs to be multiplied with its complex conjugate to obtain a physically

meaningful distribution function for the system being treated. An accurate computation of this wave function *requires a complete specification of the physical system* at hand, which in the case of the Young experiment would mean not only the photons and the apparatus, but everything in between.

In this connection it is well to recall that the interference effects under discussion have never been observed when photons deriving from different sources are involved[246]. This implies that a very delicate balance is involved which leads to a mixing of the two parts of the wave function corresponding to the different slits of the Young apparatus only when a perfect symmetry exists. From this result it is clear that an extremely small perturbation is enough to destroy the constructive interference pattern. Closing one of the holes in the Young experiment might constitute such a small perturbation, the occurrence of which *if transmitted by a suitable medium to the source,* would be sufficient to "inform" the photons located there that the symmetry required for the production of interference phenomena is no longer present.

The advantage of the above interpretation is that it does not require[244] a modification of the fundamental definitions of particles and waves to make the experimental results understandable. Streams of particles of the same properties obey statistical laws embodied in the de Broglie[19] and Bohr frequency[5] relations. The individual particles are localized at all times, but *the quantum mechanical formalism is only able to provide* a *statistical distribution function* from which expectation values for their various properties can be computed. For this purpose an accurate description of the forces acting on the individual components of the system is necessary. In certain instances for which the results are very sensitive to the choice of experimental conditions, this requirement includes taking into account neighboring systems such as massless photons which are always present but whose influence can usually (but not always) be safely excluded.

H. THE EINSTEIN-PODOLSKY-ROSEN PARADOX

The last topic to be considered in the present section is the Einstein-Podolsky-Rosen (EPR) paradox[249]. Since it was first presented in 1935 there has been great interest in the question of what it implies about the role of quantum mechanical theory in describing general physical processes. The experiment on which this paradox is based is the same one with which the present study began, namely positronium decay. To properly appreciate the significance of the EPR paradox, it is important to first consider the essential features of the quantum mechanical theory

of measurement, known as the Copenhagen interpretation[250]. The guiding principle of this formalism is that one can only predict the result of a given experiment in terms of a probability distribution. The corresponding quantum mechanical wave function describing a given state of a system can always be expanded in terms of the eigenfunctions of a hermitean operator P corresponding to a property which is to be measured, i.e. $\Psi = \sum_i c_i \varphi_i$, where $P\varphi_i = p_i\varphi_i$. The probability that a given eigenvalue p_i will be obtained in the measurement is expected to be $|c_i|^2$ when Ψ is normalized to unity. The most controversial aspect of the Copenhagen interpretation is the question of whether such a probabilistic theory can be considered to give a complete description of physical reality (essentially the title of the original EPR publication), or whether a more detailed theory exists which is capable of giving more definite predictions of experimental results.

The positronium decay experiment represents a paradoxical situation *vis-a-vis* the Copenhagen interpretation because it shows that the value of a property of one of the two emitted photons (in the singlet decay) is always completely predictable based on knowledge of the outcome of the equivalent measurement carried out for the other[249]. From this interpretation it would seem to follow that if a property is measured for which the photon wave function is not an eigenfunction of the corresponding quantum mechanical operator, the result obtained can never be predicted with certainty, regardless of what other information is available. Since there are a variety of different measurements possible whose quantum mechanical operators cannot all be compatible (mutually commuting), the experimental evidence indicates that at least in the form given above this principle is violated in practice. While one can only speculate how two particles can always give complementary results when subjected to the same experiment, it seems certain that such a mechanism does exist on the basis of the above measurements[248,249].

In a broader sense the EPR paradox suggests that particles such as electrons, which are taken to be perfectly *indistinguishable* in a quantum mechanical treatment, actually enjoy a unique existence. For example, in a collection of a large number of hydrogen atoms, all in the $^2P_{1/2}$ state, there may be a mechanism by which an observer can know with certainty which of them will undergo radiative emission at a particular time. What is clear and indisputable is that quantum mechanical theory as we know it is incapable of providing such information. It is important to note in this context, however, that there really would be no way to *verify such theoretical predictions* for the outcome of the above experiment for the simple reason that it is

impossible to tell the individual particles apart. One cannot put a label on an individual hydrogen atom and then determine precisely when this particular system undergoes radiative emission. So in that sense it is impossible to devise a practical means of testing such a hypothetical extension of quantum mechanical theory.

The positronium decay experiment described above nonetheless succeeds in providing a type of photon labeling which satisfies the basic condition described in the last paragraph for hydrogen atoms. Accordingly, two otherwise indistinguishable objects are identified by virtue of their special relationship to one another. Under these circumstances it is still impossible to predict the outcome of every conceivable experiment on one of the two photons (i.e. that measured first), in accord with the Copenhagen interpretation. The knowledge of its relationship to the other product of a given positronium decay, however, is sufficient to allow for complete predictability of every complementary measurement subsequently carried out for the second photon.

There is no immediate justification for extrapolating this result to the much more sweeping proposition that the outcome of every experiment is completely predictable according to some unspecified theory, but this possibility is at least left open on such a basis. The required extension of quantum mechanical theory to achieve this goal might still involve the wave function for each of a given set of indistinguishable particles. This assertion follows from the fact that the definition of such functions is not uniquely provided in the existing theory. Since its predictions are always in terms of expectation values[241], it is clear that different functions can give identical results for all properties. It is always possible to introduce a finite discontinuity in such functions which leads to no change in any of its operator expectation values.

One of the more frequently mentioned examples of this nature involves the evaluation of the Darwin term for atoms when a *point-charge nuclear charge distribution* is assumed[251]. Because this operator is singular at the origin, the value of the wave-function there affects its expectation value, contrary to all other Hamiltonian terms (for example, those shown in Table I). If one excludes such physically unrealistic operators from the Hamiltonian, it is possible to vary the value of the wave function at the origin over an arbitrary range of finite values without affecting the associated total energy expectation value. It can be argued that no property which can be measured in practical experiments will depend on the value of the wave function at the

origin either, so in this sense there is no *unique* definition for any physically meaningful quantum mechanical wave function.

In summary, there are an infinite variety of functions whose overlap with one another is exactly unity. This being the case, one could use this ambiguity to store any conceivable amount of distinctive information regarding individual particles without affecting the equality of their quantum mechanical expectation values in any case. Under the circumstances it seems only prudent to at least hold open the possibility that the result of each experiment ever to be carried out upon it *is* uniquely specified. Presented with a set of such wave functions we would only verify that they lead to identical probability distributions, and consequently to the same expectation values for any conceivable observable. In other words, the very structure of quantum mechanical theory suggests a level of ambiguity which allows no definitive conclusion as to whether its probabilistic character is a signal of an intrinsic element of unpredictability of naturally occurring phenomena or not.

Since a possibly higher level of theory would not be subject to verification because of the impossibility of labeling otherwise indistinguishable objects, it might well be argued that there is no point in even looking for its concretization. From a purely scientific point of view this position is tenable. Nonetheless, the belief that the outcome of physical experiments is often just a matter of probability can lead to the generalization that almost everything is simply a matter of chance, and as a consequence, that causality is a proven impossibility in the natural course of events. The EPR paradox is an indication that such a judgment is at best premature. It *is* important to know if anything happens by chance or not, even if it is clear that in many situations it must remain effectively impossible to predict the (conceivably) inevitable outcome of a given experiment.

XI. CONCLUSION

In the present investigation attention has been centered primarily on .the pervasive assumption in modern-day physical theories that matter can be created or destroyed by means of a suitable addition or loss of energy. It has been emphasized that it is impossible to distinguish experimentally between a particle which is unobservable in its present state and one which has gone out of existence entirely. The concept of all material particles being composed of atoms or

elements which are impervious to the application of any force has played a crucial role in the development of the physical sciences over a period of several millenia. It has been argued in the present work that since the antithesis of this view, the creation-annihilation hypothesis, can never be proven by direct experimental observation, it is quite important to see if an alternative theory of physical transformations can be formulated which gives a plausible interpretation of all measured phenomena without giving up the principle of the indestructability of material elements.

Consideration of the decay of positronium and the subsequent production of photons, which has hitherto been assumed to involve the annihilation of an electron and positron, suggests a different explanation in terms of the formation of particleantiparticle binary systems with exactly zero rest mass. In order to give quantitative substance to this alternative hypothesis, attention is turned to the goal of finding a suitably concrete form for the system of interactions which would be capable of binding an electron and positron so strongly together that the energy lost in the process is exactly equal to the sum of their rest masses, $2\ m_{oe}c^2$ or 37557.73 hartree (1.02 MeV). Instead of simply deducing this result with the help of the Einstein mass-energy equivalence relation, it is proposed to consider positronium decay as a conventional radiative emission process in which the binding energy of the final state is to be computed with standard quantum mechanical methods once the nature of the associated interaction mechanism is identified.

In the course of studying other modern physics experiments on a qualitative basis it has been concluded that such a massless e^+e^- structure can plausibly be attributed to the photon itself, since it is known to have zero rest mass and to interact electromagnetically in a way that is at least consistent with a dipolar composition of this kind. Because of the well-known fact that photon emission processes occur at all locations in the universe, i.e. wherever a given excited state of a particle is found to undergo radiative decay, it follows that the proposed massless e^+e^- binary systems must exist everywhere in space with sufficiently high density in order to explain these phenomena without invoking the creation-annihilation hypothesis. Support for this assumption of ubiquitous photons with zero mass can be found in the black- body radiation experiment (Sect. II.D). The original quantum hypothesis of Planck holds that for every frequency of radiation ν absorbed by a blackbody at thermal equilibrium, there must be a higher population of photons with E = 0 than for E = hν or any other allowed energy value." Since there

are an unlimited number of such frequencies possible, it follows that the number of massless photons in the thermodynamical system is essentially boundless. It is then only a matter of theoretical interpretation whether occupation of such an $E = 0$ state is taken to correspond to a photon which has suddenly ceased to exist or one which simply defies experimental detection.

In this view the radiative emission process is not seen as involving the creation of a photon with $\Delta E = h\nu$, but rather as an exchange of energy between the original excited system and a photon in its neighborhood which initially possesses zero energy. The photon simply takes on an amount of energy which is lost by the other system in a transition to one of its lower-lying states. Furthermore, the requirements of conservation of energy and linear momentum are shown to be directly responsible for the quantization of such processes, since an arbitrary exchange of energy would require a smaller increase in the photon's momentum than that lost by the heavier system with which it interacts. Only by changing to a lower-lying internal state with nearly the same momentum as prior to the transition can the heavier system satisfy the $\Delta E = pc$ relation required by the photon's zero rest mass. The explanation of the Mössbauer effect is based on the same considerations, with the distinguishing feature that for the very large energies of nuclear emission processes, the two internal states of the heavier system involved in the transition are associated with momenta which differ far more greatly from another than those of different electronic states in an atomic transition.

One of the most critical aspects of the massless photon hypothesis is its questioning of the off-stated belief that zero energy and/or mass for any system necessarily implies its lack of existence (Sect. II.C). Instead it is pointed out that the Bohr frequency and de Broglie relations have zero frequency and infinite wavelength as limiting values when the energy and momentum of a photon approach vanishing magnitudes. Before this limit can be achieved by systematically reducing the energy of a single photon, frequency and wavelength values must be reached which by virtue of the above two relations already lie outside the range of experimental observation. Since photons whose (finite) de Broglie wavelength s are too large to measure are nevertheless assumed to exist, it is not unreasonable to expect that their zero-momentum counterparts may also be present, despite the impossibility of observing them directly. In this connection a flaw is pointed out in the argument which holds that a system with zero rest mass must move with the speed of light based on the dependence of the relativistic mass m on velocity. The latter relation

is satisfied for any velocity v smaller than c as long as m = 0, including v = 0. The ratio m_0/m is not uniquely defined under these circumstances and thus may exceed zero, implying that $\gamma^{-1} = (1 - v^2/c^2)^{1/2}$ also can be non-vanishing in this limit.

Consideration of other types of observed phenomena arising in the field of modern physics is found in no way to contradict the above hypothesis of ubiquitous massless photons. These processes include the photoelectric effect, Compton and Raman effects, Bremsstrahlung and electron production from photon collisions. At the same time, this line of argumentation calls into question standard interpretations of related elementary processes which involve the creation or annihilation of individual particles. The primary example considered in this connection is that of the beta decay of a neutron (Sect. IV.A). By following through on the proposal that no material particle is ever created or destroyed in physical transformations, one is led to conclude that a strict elemental balance must characterize any reactive equation, exactly as is assumed in ordinary chemical processes. In accordance with this view a neutron must be composed of a proton, an electron and an antineutrino *prior* to undergoing beta decay, because each of these particles is present at the completion of the process. An analogy is made between the neutron (and other unstable elementary particles) and excimer systems in the field of molecular physics, which are well known to correspond to meta-stable excited states whose ground state potential energy curves are completely repulsive. The energy given off in the decay of elementary particles is so large compared to that found in the operation of an excimer laser that a significant decrease in the sum of the rest masses of the products in such reactions is observed relative to the mass of the original meta-stable system. Similar arguments can be given for a whole series of elementary particles, such as muons, pions and other heavier species, all of which are known to decay with relatively short lifetimes to a collection of fragments of smaller aggregate rest mass.

In order to give quantitative substance to the above theoretical model, it is imperative that one clearly identify the nature of the interactions responsible for high-energy processes, especially the prototype example in which an electron and a positron combine to form a tightly bound binary complex with exactly zero rest mass. Emphasis is placed thereby on the fact that no corresponding state of a proton and an electron is known, i.e. the $1s_{1/2}$ state of the hydrogen atom is perfectly stable when left in an isolated condition, whereas that of positronium has only a short lifetime. Similarly, a massless p^+p^- binary must also be assumed to exist based on the analogous experimental results for the interaction of a proton and an antiproton. The term "phantom" will

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be used in the following discussion in referring to such massless particle-antiparticle binary systems in general, and the special name of "prophoton" has been given to the p^+p^- member of this family, to go along with the identification of its e^+e^- counterpart as the photon.

The high energies associated with the formation of these two phantons suggest that similar short-range interactions are involved as in the binding of nuclei, in which case an exponential form for the corresponding potential has been deduced from scattering experiments. Because of the participation of electrons, positrons and photons in the electromagnetic interaction, it is suggested that a good starting point in the search for such a potential is the relativistic Dirac equation or some approximation to it. It is noted that the Breit-Pauli reduction of this equation contains short-range terms varying inversely as the cube of the interparticle distance which are of the order of α^2 (10^{-4}-10^{-5} hartree) for typical electron-proton separations in an atomic system. The unbounded character of these terms in the limit of vanishingly small separations, particularly in relation to the corresponding relativistic kinetic energy, makes it clear, however, that the desired short-range bonding of elementary particles can only be satisfactorily described by interactions of this type if they are somehow modified to become considerably less attractive at extremely small interparticle distances.

An exponential damping of the Breit-Pauli interactions is thus suggested (Sect. V.C). This is in analogy to that proposed by Yukawa fifty years earlier for the description of nuclear binding, except that in that case it was applied to an r^{-1} potential. Similar quantities of r^{-3} type present in the $\mathbf{p} \bullet \mathbf{A}$ cross terms of the Maxwell-Lorentz Hamiltonian are kept in check by the $\mathbf{A}\bullet\mathbf{A}$ component of a perfect square, which in turn varies as r^{-4}. This suggests that the exponent of the damping function for the Breit-Pauli interactions should vary as r^{-1} or p, and that it should contain a factor of the charge-to-mass ratio of a given particle multiplied by a constant of order α^2. Since the motion of charged particles is involved in such interactions, it seems reasonable to choose a momentum-dependent exponential damping whose argument is always negative. Ultimately these arguments lead to the following explicit form for the damping function: exp (-A α^2 | $\mathbf{p}$ q/m$_0$|). At the same time, the unbounded correction to the Breit-Pauli kinetic energy, which varies as p^4, is introduced via the Einstein relativistic operator $(p^2 + m_0^2 c^4)^{1/2}$ - m$_o$ c^2 (with c = α^{-1} in atomic units). The resulting set of interactions is subsequently referred to as the exponentially damped Breit-Pauli Hamiltonian and is employed in a Schrödinger equation (XBPS) of the standard H Ψ = E Ψ form. The potentially crucial advantage of this Hamiltonian

is that it is bounded from below and thus can be treated using standard variational techniques, unlike the un- damped Breit-Pauli terms themselves. The explicit form of the XBPS Hamiltonian is given in Table I.

The above Schrödinger equation has an interesting scaling property of relevance to the positronium decay process. The total energy of any state of a particle-antiparticle system with a given charge and rest mass is exactly M times larger than that obtained by a coordinate scaling for an analogous binary system with the same charge but a rest mass which is smaller by the same factor M. This means that the same Hamiltonian which produces the $2\,m_{oe}c^2$ binding energy assumed for the e^+e^- phanton leads to a corresponding value for the p^+p^- prophoton system which is exactly $2m_{op}c^2$. The latter system's average interparticle separation is smaller by a factor of the proton-electron rest-mass ratio m_{op}/m_{oe} than that for the e^+e^- photon system. The form of the XBPS Hamiltonian is thus seen to guarantee the condition required by the Einstein mass-energy equivalence relation, namely that the maximum energy lost in a particle-antiparticle interaction is directly proportional to the rest masses of the isolated systems (Sect. V.D). This condition places a restriction on the form of the exponential damping factor in the XBPS Hamiltonian, specifically that the momentum p be divided by the particle's rest mass in the corresponding argument.

The use of the relativistic kinetic energy operator in the XBPS model raises a fundamental question of a different nature, however, namely how to deal with the translational motion of the combined system being treated (Sect. V.B). An analysis indicates that, contrary to the non-relativistic case, a transformation to center-of-mass coordinates for such a Hamiltonian does not lead to a total separation of the internal and translation motion. This fact is related to the requirement of the theory of special relativity that distances measured in an inertial system moving with constant velocity relative to the observer are found to be expanded. It has been argued that this phenomenon should be directly reflected in the corresponding expectation values of dynamical variables, specifically that such results must vary with the translational velocity of the system treated (Sect. X.B). Such findings clearly cannot result if it is simply assumed that the center-of-mass motion is completely separable, since properties involving only internal variables are thereby forced to be independent of the state of translation of the combined system. It was therefore decided to carry out calculations with the XBPS Hamiltonian in terms of the Cartesian coordinates of the constituent particles without introducing a center-of-mass

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transformation, so as not to prejudice the results of the treatment in this respect, despite the fact that this procedure brings with it certain computational difficulties otherwise avoided by such a change in coordinates.

Allowing the internal and center-of-mass motion to remain coupled in the theoretical treatment gives insight as to how the $e^+ e^-$ system can have a tightly bound ground state below the familiar 1 s state of positronium, unlike the case for the hydrogen atom of proton-electron composition. The essence of this argument is contained in the observation that the Σp_i condition for a binary system whose center of mass is at rest in the origin of the coordinate system implies that $\mathbf{p_l} = -\mathbf{p_2}$ or $\mathbf{v_1} = -(m_2/ m_l) \mathbf{v_2}.$ Since the mass of the proton is so much greater than that of the electron, this condition requires that their respective velocities be quite different from one another in the hydrogen atom. This circumstance is incompatible with the need to keep the particles close to one another while each is moving at high speed in order to take maximum advantage of the short-range momentum-dependent attractive terms in the XBPS Hamiltonian such as the damped spin-orbit coupling (Sect. V.F).

By contrast, the analogous condition for a phanton system leads to a perfectly correlated relative motion of the two particles, which lends considerable support to the proposition that such systems possess non-hydrogenic states of exceptionally large binding energies. As the translational energy increases beyond zero the correlated motion of the phantons is gradually disturbed, however. At first glance the smaller q/m_o value of the proton relative to the electron appears to be inconsistent with the requirement that $p^+ p^-$ have a much larger binding energy than the corresponding $e^+ e^-$ system, especially when one only considers the magnitude of their respective coupling constants in the short-range XBPS interactions. The same quantity also causes the argument of the exponential damping factors to be correspondingly smaller for the prophoton system, however, which in turn allows its two particles to approach each other far more closely than is preferable for the lighter electron-positron pair.

A full CI treatment of the Schrödinger equation discussed above for the $e^+ e^-$ system employing various primitive Cartesian Gaussian one-particle basis sets has indicated that the lowest-energy state for this system possesses 0^- symmetry and is symmetric with respect to the charge conjugation operation, which commutes with the corresponding Hamiltonian. The constant A in the exponential damping factors has been chosen so as to obtain the desired $e^+ e^-$ binding energy of $2\ m_{oe}c^2$ for an optimal choice of orbital exponents in the full CI treatment

(Sect. VI.D). The 0_s^- symmetry of the e^+e^- ground state is also found to possess a vanishing expectation value for the conventional (undamped) Darwin term, as well as for the corresponding δ-function term in the spin-spin interaction. This result shows in a particularly striking manner that the two constituent particles must effectively avoid one another completely in order to achieve maximum stability. The form of this wave function also allows for an efficient minimization of the center-of-mass translational energy. The value of the damping constant A is found to be 1.265 using the above criterion for a 3s,2p,2d basis. As a result, the range of the damping interaction conforms to the expectations discussed previously, becoming significant for interparticle distances smaller than or equal to $r = \alpha^2$. In accordance with the XBPS scaling property, the optimum prophoton wave function is characterized by Gaussian exponents which are larger by a factor of $(m_{op}/m_{oe})^2$ relative to those for e^+e^-, giving the desired $2m_{op}c^2$ binding energy relative to the separated proton and antiproton for the same choice of A as for the lighter phanton system.

One drawback of this approximate treatment is that the corresponding expectation value of the e^+e^- translational energy is rather large (0.88×10^6 a. u. in the 3s.2p,2d basis, for example). It is clear that a much more extensive treatment with a relatively large one-particle basis is necessary to reduce this quantity to vanishing magnitude. A re-optimization in terms of the difference between the expectation values of the total and translational energies respectively leads to an increased value for A of 1.727, and a somewhat more compact charge distribution for the electron-positron pair. Moreover, corresponding results for a variational treatment of this energy difference yield a somewhat higher value of 1.773, suggesting that in an exact solution of the XBPS, only a slightly larger value for this constant is required ($A \leq 2.0$).

Because the translational energy operator T commutes with the total Hamiltonian, it is clear that any exact solution for such a Schrödinger equation must also be an eigenfunction of both operators. Hence use of the same A value must produce the desired phanton binding energies for the $\langle T \rangle = 0$ solution in each of the three optimization procedures discussed above. Because the lowest-energy eigenfunction of H - T must not correspond to a minimal translation energy expectation value, it is preferable to employ only the full XBPS Hamiltonian in subsequent treatments involving other combinations of particles. This procedure is tantamount to regarding the computed variational energies as upper limits to that of the lowest translational state of a given system.

The description of the decay of the neutron in the XBPS model also requires an explicit representation of the interaction of a third particle and its antiparticle, namely the neutrino and antineutrino. It is necessary, consistent with the above considerations (Sect. VI.H), to assume that a corresponding phanton system of zero rest mass exists with a $\nu\,\bar\nu$ composition (photrino). The fact that the neutrino possesses no electric charge prevents the use of the above scaling property of the XBPS Hamiltonian in this instance, however, because it is only valid for particles having the same absolute magnitude of charge as the electron. Examination of the coupling constants in this Hamiltionian shows that the only way that it can lead to significant interactions for neutrinos is if their charge-to-rest-mass ratios are different from zero. This condition can only be met if the rest mass of the chargeless neutrino is itself exactly zero, something which is at least consistent with all experimental investigations as yet undertaken to measure this quantity. It is also clear that the use of the rest mass instead of the relativistic mass, $m = \gamma m_0$, in the coupling constants of this Hamiltonian is essential if this possibility is to have a significant impact on the description of the neutrino interactions. Accordingly, the sign of the corresponding Breit-Pauli interactions is determined on the basis of the product of the respective q/m_0 values of the interacting particles rather than for the product of their electric charges alone.

Subsequent calculations for the $\nu\,\bar\nu$ system have shown that the same value for the damping constant A which gives the desired binding energy of $2\,m_oc^2$ for the e^+e^- and p^+p^- phantons also works to a good approximation for $\nu\,\bar\nu$, i.e. with zero binding energy relative to the separated neutrino and antineutrino constituents. The agreement is particularly good if translational effects are considered explicitly in the optimization procedure. In addition, a different scaling property is found for the XBPS Hamiltonian which shows that the desired zero binding energy of the $\nu\,\bar\nu$ system holds independently of which $|q/m_o|$ value is assumed for the neutrinos. This result is consistent with the existence of more than one type (flavor> of neutrino in nature, but it also leaves the choice of this quantity's value open for other purposes, particularly to satisfy the requirement of computing the measured binding energy of the neutron relative to its decay products at rest.

The corresponding $\nu\,\bar\nu$ total energy curve can be thought of as containing a double minimum at E=0, one for the tightly bound phanton system and the other for the particles separated to infinity. Because of the lack of a (long-range) Coulomb interaction for neutrinos such a theoretical energy curve climbs immediately as the particles approach each other from

quite large distances, contrary to what occurs for the approach of a charged particle to its antiparticle. In the latter case the internal kinetic energy of the system varies more slowly with distance in this range than does the Coulomb attraction of the particles, an effect which is crucial to the success of the original Bohr model of the hydrogen atom. It can be argued that the failure of the neutrino to participate in similarly long-range attractive interactions is responsible for the extremely penetrating nature of its radiation, making it effectively incapable of overcoming centrifugal barriers separating it from other particles.

It is possible to understand the observed lack of conventional magnetic interactions for neutrinos on a similar basis (Sect. VI.I). From the point of view of a charged particle, all electromagnetic interactions are effectively purely Coulomb because magnetic contributions to the energy are always computed to be of vanishing magnitude in its own inertial system. Thus a non-zero q/m_o value is not necessarily inconsistent with the measurement of a vanishingly small magnetic moment for the neutrino. At the same time, it can explain why such particles undergo strong interactions at extremely small interparticle separations, as exemplified in the original Reines-Cowan antineutrino capture experiments. The fact that relatively high centrifugal barriers must be overcome before coming into a suitable range for which the damped Breit-Pauli terms can dominate is also consistent with the extremely small cross sections observed for such reactions. Moreover, the possibility of its participating in short-range interactions offers an explanation for the presence of an antineutrino in the meta-stable neutron, as demanded by the hypothesis of particle balance in all physical transformations. To obtain suitably strong binding with the electron, the other light component of the neutron in this view, it is necessary to assume that $\bar{\nu}$ has a positive q/m_o value, i.e. opposite to that of the electron (and also to that of its antiparticle).

The known q/m_o value of the proton is so small as to rule out its participation in XBPS interactions in the range of interparticle separations typical for nuclear binding for which this quantity appears as a coupling constant. This circumstance practically eliminates any chance of such Breit-Pauli terms as the spin-spin, orbit-orbit and spin-other-orbit interactions being directly involved in the binding of protons, but it leaves open a distinct possibility that the spin-same-orbit and Darwin terms with electrons and antineutrinos can affect them in a significant manner. One knows from observations of atomic spectra, for example, that the electron undergoes a spin-same-orbit inter action with the nucleus which does not depend on the mass of the heavier

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particle, but rather on the square of the mass of the electron itself. Preliminary calculations with the XBPS model demonstrate that such a proton-electron interaction can be enormously attractive in the desired range of interparticle separation. The opposite effect is found for the proton-antineutrino spin-same-orbit interaction, but the calculations demonstrate that the most stable state of the $p^+e^-\bar{\nu}$ system adjusts its charge distribution to obtain a significant net binding between the proton and the $e^-\bar{\nu}$ complex. In addition to this polarization effect, another dominant factor responsible for proton binding is its attractive Darwin term interaction with the antineutrino, which easily outweighs the effect of the corresponding proton-electron repulsive contribution. By decreasing the q/m_o value of $\bar{\nu}$ below that of the positron, it is found that the total energy of the most stable state of the $p^+e^-\bar{\nu}$ system can be adjusted to become equal to that known experimentally for the neutron, i.e. 28781.31 a.u. higher than that of its separated component particles. This occurs for a q/m_o value of +0.63 a.u. for the antineutrino in the 3s,2p,2d basis mentioned above, i.e. about two-thirds that of the electron (Sect. VII.B. A value of unity for this quantity gives a much lower energy for the $p^+e^-\bar{\nu}$ system, showing it to be bound by 62528.34 a.u. relative to its separated constituents (employing the same basis), a change of 91309 a.u. or 2.48 MeV, so it is clear that these results are quite sensitive to the numerical value chosen for this charge-to-rest-mass ratio.

The last comparison offers a clear perception of the mechanism of proton binding in the present model. The $e^-\bar{\nu}$ complex can be looked upon as a clone of the prototype e^+e^- massless binary, especially when q/m_0 for $\bar{\nu}$ is assumed to be the same as for the positron. Because of the nature of the coupling constants in the spin-same-orbit and Darwin terms in the XBPS Hamiltonian, the proton can be strongly attracted to $e^-\bar{\nu}$, especially if the charge-to-rest -mass ratio of $\bar{\nu}$ is relatively large. Reducing the value of this quantity below unity necessarily causes the stability of the $e^-\bar{\nu}$ complex to be lessened, but at the same time it increases the magnitude of the net proton binding energy to this system. The symmetry of the resulting $p^+e^-\bar{\nu}$ metastable state is consistent with this interpretation. The proton finds itself predominantly in an $s_{1/2}$ orbital, while the $e^-\bar{\nu}$ species prefers the 0^- character exhibited by both e^+e^- and $\nu\bar{\nu}$ in their respective lowest states, giving an overall $1/2^-$ symmetry to the resulting $p^+e^-\bar{\nu}$ wave function. There is an indication that the computed proton binding energy is overestimated in the computational treatment employed, since improvements in the AO basis tend to make it more difficult for the proton to polarize the $e^-\bar{\nu}$ charge distribution. On this basis it can be expected that the q/m_o value

for the antineutrino needed to give the experimental neutron binding energy in the exact XBPS treatment is somewhat larger than found in the best calculations carried out in the present study. Moreover, it seems likely that as the one-particle basis becomes more flexible that the description of the tight-binding phanton states as well as that of the $e^-\bar{\nu}$ complex will allow more effective use of the short-range Breit-Pauli attractive interactions, thereby drawing the two component fermions somewhat closer together in an improved level of treatment. This conclusion is consistent with the results of the different e^+e^- optimizations discussed first in which the influence of translational energy is explicitly taken into account.

The qualitative picture which emerges from these considerations is thus that the proton experiences a net attraction to the $e^-\bar{\nu}$ complex primarily because the antineutrino's charge-to-rest-mass ratio is somewhat smaller in absolute value than that of the electron. This condition makes the $e^-\bar{\nu}$ system significantly less stable than e^+e^- in its lowest state, but it also makes the former binary more attractive to protons than its phanton counterpart. This attraction is insufficient to produce a bound $p^+e^-\bar{\nu}$ system, however, i.e. to bind a single proton, and this circumstance is assumed to be responsible for the known instability of the neutron. According to the present model, decay of the neutron occurs via a spin-flip mechanism (Sect. VIII.B), whereby the predominantly singlet nature of the 0^- $e^-\bar{\nu}$ complex is suddenly destroyed. Instead the spins of the proton and electron begin to alternate, causing the antineutrino to be ejected, always with the same spin as that of the original neutron. This course of events is consistent with the measured spin relationships of the emitted particles in the decay of polarized neutrons. The presence of a slight triplet admixture in the $e^-\bar{\nu}$ complex is thus seen as a contributing factor to the finite lifetime of the neutron, and to have an important influence on the nature of weak-interaction decays in general. At the same time the computed strength of the electron's attraction to the proton and antineutrino helps to explain why the resulting tri-atomic system only has a magnetic moment on the order of a nuclear Bohr magneton (Sect. VII.C). An analoguous situation can be found in the study of the Compton effect, for which it is found that collisions of photons with inner-shell electrons can only be successively described by taking the mass of the scattered system to be that of the entire atom to which it is bound.

The above model for the structure of the neutron offers a basis for obtaining a quantitative description of the nuclear binding process. A second proton can be bound to the $e^-\bar{\nu}$ complex by the same mechanism as the first, making the deuteron a stable system of $p^2e^-\bar{\nu}$ composition. The

lowest state is found to be a closed-shell singlet, however, in disagreement with experiment. The situation is corrected, however, by including a δ-function potential which only affects the protons of the system (Sect. VII.E). It is completely similar to the corresponding spin-spin interaction in the XBPS Hamiltonian, except that the q/m_o quantities are set equal to unity rather than to their normal proton values. This term greatly favors the triplet state of the deuteron, making it the ground state of the system (1^-). This effect arises primarily from the fact that the corresponding interaction for the closed-shell 0^- state is quite repulsive, whereas the triplet has virtually a zero expectation value because of the nearly perfect cancellation of the Coulomb and exchange integrals for such a δ-function potential. The binding energy of the 1^- $p^2e^-\bar{\nu}$ state in the best treatment as yet carried out is somewhat larger than the experimentally known value for the deuteron, but there is at least evidence that the essential nature of the Hamiltonian which governs the nuclear binding process is properly described by the above set of interactions.

It should be emphasized that this result is obtained with the aid of the same two parameters, the damping constant A and the q/m_o value for $\bar{\nu}$, which are deduced from the previous computations in the XBPS model for the three phanton systems and the $p^+e^-\bar{\nu}$ neutron structure. The essential feature of this development is the hypothesis that *the electron and antineutrino form a central potential in their preferred 0^- state which is capable of strongly attracting a given number of protons.* The proton-proton δ-function spin-dependent potential is closely related to the Breit-Pauli interactions, as mentioned above, but is clearly distinguished from its electromagnetic counterpart by virtue of the magnitude of its coupling constant. An equivalent antiproton-antiproton interaction must also be assumed, but a corresponding proton-antiproton version does not appear to exist since it would destroy the otherwise perfect symmetry which exists between the e^+e^- and p^+p^- systems.

The above description of the deuteron binding process can then be extended at least qualitatively to the structure of heavier nuclei, of which ^{3}He and the hypothetical ^{4}Li system have been explicitly treated in this work (Sect. VII.H). A key feature of this model is the identification of a neutron as a proton which is relatively tightly bound to an $e^-\bar{\nu}$ unit inside the nucleus. There is thus a clear relationship between this proposal and the main topic considered in the present study, namely the questioning of 'the creation-annihilation hypothesis, particularly in the form first suggested by Fermi according to which the electron and antineutrino are assumed to not be physically present inside the bound nucleus. The exponential form of the damped Breit-Pauli

interactions allows for two regions of similar potential for protons relative to the tightly bound but meta-stable $e^- \bar{\nu}$ system, which in turn allows for the existence of a pair of (mutually orthogonal) proton orbitals of similar energies. Hence the conventional interpretation of nuclei as combinations of protons and neutrons is consistent with the present model. The overall spin of the nucleus is determined exclusively by the proton constituents in this view, i. e. the ordinary proton and that of the neutronic $p^+ e^- \bar{\nu}$ system. The latter conclusion rests primarily on the finding of a singlet structure for the $e^- \bar{\nu}$ complex, which characteristic explains in a quite simple way why the electron and antineutrino do not appear to affect the magnitude of the nuclear spin despite the fact that they each possess $s = 1/2$ themselves.

An important consequence of the above identification is that it gives a perfectly straightforward explanation for the observed adherence of both types of nucleons to the (generalized) Pauli principle (Sect. VII.H). In this view no separate postulate is required to explain the relevant experimental data, since all that is ever involved is the exchange of indistinguishable protons. While such an interpretation is easily reconciled with the general procedures employed in isospin theory, it does call into question the latter's underlying precept, as introduced in 1932 by Heisenberg, namely that the proton and neutron are simply two different states of the same system which are characterized by the same spatial and spin wave functions. Specifically, it holds open the possibility that the governing Hamiltonian is completely independent of the hypothetical isospin coordinates. This position clearly does not diminish the practical utility of this theory, as applied for over a half-century in several important fields of physics, but it does raise questions as to whether the isospin quantities should be regarded as true constants of motion in the description of physical interactions.

Once the above interpretation has been made, it is possible to understand the Aufbau principle for nuclei in a straightforward manner in terms of protons and neutrons, consistent with the shell model of nuclear structure (Sect. VII.G). The increased binding energies of successive protons is thereby associated with an overall contraction of the nuclear system which occurs because of the increase in the number of proton-electron and proton-antineutrino attractive interactions resulting therefrom. The well-known volume effect observed for nuclei appears to be closely tied up with the fact that the electrons and antineutrinos must avoid each other to a high degree (in order to have a small Darwin term) in order to maximize their Breit-Pauli attraction for one another. In this view the proportional increase in volume with atomic number is directly

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related to the corresponding increase in the number of $e^-\bar{\nu}$ bosonic complexes needed to form the required number of neutrons to bind the nucleons together.

In addition to the above explanation for the origin of the strong and weak interactions of nuclear systems, the XBPS model allows for a re-interpretation of virtual processes in quantum electrodynamics and related theories (Sect. IX.E). Accordingly, when one speaks of virtual photons affecting the properties of a given system, one can describe the interactions in terms of the original atom or molecule combined with one or more e^+e^- and $\nu\bar{\nu}$ phantoms. In the lowest-order representation only the massless photon is involved, but an excited configuration in which the electron and positron are separated makes a contribution to the overall wave function in higher order. An analogy is made to CI treatments of electronic structure, in which it is routine to mix in configurations of relatively high energy, which one can refer to as virtual states, in order to obtain a more accurate description of the physical system.

The ubiquitous presence of real photons in the universe, as assumed in the XBPS model, thus provides a straightforward explanation for the fact that perfect agreement with experimental data for an apparently isolated system cannot be obtained without making some provision for interactions external to it, i.e. virtual processes. In this view the results of quantum electrodynamics can be obtained, at least in principle, by including real e^+e^- binaries explicitly in the theoretical treatment, in which case variational methods would be applicable. Since the internal structure of the photon has little to do with the electromagnetic interactions responsible for such effects, it is reasonable to exclude such details in the theoretical treatment employed. Instead, low-order perturbation theory based on matrix elements between the $E = 0$ and $E \neq 0$ e^+e^- states which are derived on the basis of certain assumptions about the nature of the photon field can be employed with high accuracy, as prescribed in the standard quantum-electrodynamics approach.

The same type of argumentation applies to virtual pions in nuclear interactions, even though a first-order perturbation theory treatment is unrealistic in this instance. To make the analogy more concrete, it is necessary to ascribe a definite elemental composition to the pions (Sect. IX.A). Since it is known that π^+ decays into μ^+ and ν, and μ^+ itself into e^+, ν and $\bar{\nu}$, the particle balance hypothesis suggests that the charged pions have a tetra-atomic composition, i.e. they consist of a single electron and three neutrinos. The (virtual) pion cloud in nuclear structure theory can thus come about as the result of an interaction of a proton with three phantoms,

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namely one photon and two photrinos (Sect. IX.F). An excited configuration in which the above particles are redistributed to form a neutron ($p^+e^-\bar{\nu}$) and a positive pion ($e^+\nu\bar{\nu}\nu$) thus makes a significant contribution to the overall wave function of the system in a CI expansion of which the leading term is the proton-phantoms complex.

The form of the exponential damping factor in the XBPS Hamiltonian offers a clear mechanism by which such configuration mixing can occur, since it allows the proton to interact strongly with the lighter particles through the spin-same-orbit and Darwin terms, for which only the large q/m_o values of the electron and antineutrino serve as coupling constants. Because its orbital is effectively un-damped in such interactions, the proton is relatively free to maximize this effect through contraction of its charge distribution, being hampered in this respect by only the accompanying kinetic energy enhancement. The present calculations indicate that the binding energy of the bare proton to a single e^+e^- species is on the order of 1.0 MeV or more. On this basis it can be argued that the actual mass of the bare proton is larger by several electronic mass units than that measured for the proton in its natural environment. Further calculations indicate that the binding energy of a second proton to such an e^+e^- unit is significantly less than the first, which helps to explain why nuclei form themselves around electron-antineutrino pairs instead of such particle-antiparticle binary systems.

The finding that the bare proton is strongly bound to the e^+e^- (and $\nu\bar{\nu}$ but not p^+p^-) phantoms in the XBPS calculations suggests a simple explanation for the existence of the "pion cloud", as indicated by elastic electron scattering experiments in the GeV range, which show a definite extension of the proton's charge distribution. Consistent with this interpretation is the fact that higher-energy inelastic scattering resonance continua of protons in the 2-4 GeV range exhibit point-charge characteristics. According to the de Broglie relation the interparticle distances involved in the elastic scattering processes correspond to a range in which the proton can interact strongly with neighboring photons and photrinos, whereas those associated with the inelastic scattering processes are too small to produce similar bonding effects, with the result that the properties of the bare (Dirac) proton are observed under these conditions. Ultimately, it is simply the large mass of the proton and its consequently small q/m_o ratio which in this view is responsible for its perception as other than a point-charge particle in low-energy scattering experiments.

The proton-antiproton (annihilation) interaction can be understood on this basis as well (Sect. IX.G). Just as in positronium decay, it is assumed in the present model that (at least) two (massless) photons take up the energy given off in this process. Because the magnitude of the energy expended is so much greater in this case, however, the e^+ and e^- components of the interacting photons are set free. One positron and one electron form the basis for charged pion formation combining with a photrino and a ν and $\bar{\nu}$ respectively to form π^+ and π^-. Another (e^+, e^-) pair can also combine with a photrino to give a neutral pion. In this interpretation the proton and antiproton do not lose their existence in the process, but instead form a massless prophoton at rest in the original center-of-mass coordinate system. The fact that the $E = 0$ form is apparently always reached in the process is consistent with the observation noted above, namely that only in this state can a system of zero rest mass travel with less than the speed of light. Intuitively, such a result seems essential in order to have the required transition from the initially stationary p^+ and p^- systems to occur with any finite probability. The same argument also explains why all the energy of a photon is given up in the photoelectric effect and other absorption processes, even though the energy and momentum conservation laws do not in themselves require this result (see Sect. III.C). At the same time, the greater activity of pions as compared to muons in the prophoton formation process itself is attributed to the unsaturated structure of the latter, which can be thought of as heavy electrons with $\nu\bar{\nu}$ adjuncts. At scattering energies below 100 MeV, this distinction between muons and pions is no longer critical, which explains why the muons are stable with respect to pion formation under such conditions, as is observed in the study of cosmic rays.

In addition to providing a theory of elementary particles which is governed by a strict elemental balance of protons, electrons, neutrinos and their antiparticles, the XBPS model also gives insight into other processes which have aroused the interest of physicists over the past several decades. The longitudinal polarization of electrons and neutrinos in beta decay is noted to follow a simple rule (Sect. VIII.C), whereby the direction of each of their momenta is invariably found to be (predominantly) parallel to that of its magnetic moment. In the case of neutrinos this conclusion is based on the signs of their respective charge-to-rest-mass ratios needed to obtain the observed binding energies of the neutron and its antiparticle. Heavier systems such as the proton and muon do not follow the above rule, but this behavior is easily understood on the basis of the conservation of energy and linear momentum laws. The latter would always have to be

violated in such cases in order to allow conformance with the above correlation found between the respective directions of the magnetic moments and momenta of lighter particles.

With these observations in mind, it can be noted that the concept of a neutron or muon as a "molecule" composed of certain elements, rather than as a single fundamental particle itself, opens up a new possibility for explaining the observed polarization effects in the decay of these systems. Each of the constituent particles in such a decaying system can be expected to be subject to its own distinct field (due to damped Breit-Pauli interactions in the present model), and this circumstance could lead to correlations between the momenta and spins of such species similar to those observed. If the decay particles are simply created at the time of the decomposition of a single particle, no comparable assumption is warranted in the absence of external fields, which is the basis of the argument against parity conservation in such experiments. Thus the creation-and-annihilation hypothesis for material particles can be seen as a key underlying assumption in the long-accepted interpretation of the longitudinal polarization phenomenon accompanying these decay processes.

Once the creation-and-annihilation hypothesis is seen to be unessential in explaining other classes of observations in modern physics, however, the possibility arises that it also might not be essential in this area either. This observation in turn suggests *that parity might well be conserved in all physical processes after all.* Instead the above correlation found between momentum and magnetic moment directions is seen to be consistent with the expected effects of the short-range interactions of the Breit-Pauli Hamiltonian, which would tend to force the spins of each of the decay particles into alignment with the respective partial fields acting on them. If the field gradients are always positive at the location of the particles immediately after the decay process begins, the latter would not only rotate so that their magnetic moments (or their equivalent in the case of the short-range neutrino interactions) become parallel to their respective fields, they would also be accelerated in that direction as well (Stern-Gerlach effect). Such an *inertial* effect for the incipient fields arising at the time of beta decay would thus explain the longitudinal polarization phenomenon without requiring that the Hamiltonian employed not commute with the parity operation. In addition, it can be noted that most (if not all) experimental attempts to provide evidence for parity violations in other contexts are inconclusive because they employ external perturbations which themselves do not commute with this operation. Typical examples are use of an electric field to induce a dipole moment in an atom or of a radiation field

to demonstrate optical rotation. Since each of these fields is *ungerade,* it is impossible to know if the effect observed stems from some inherent property of the free-space Hamiltonian or from the effects of the applied perturbation itself.

Recognition of the above uncertainties in the parity non-conservation argument suggests (Sect. IX.B) that one should re-examine the theoretical arguments which led Yang and Lee to propose the idea in the first place. Their solution to the τ-Θ puzzle involves a clear assumption about the parity of the pion, namely that it is negative. One usually attempts to justify this choice on the basis of experiments in which low-energy negative pions are scattered from deuterons, but it is well-known that no *definitive* conclusion based on experiment alone can be reached regarding the parity of any of the participants in this reaction. Instead, one ultimately uses the fact that a choice of negative parity for the pion would allow an assignment of positive parity to both the proton and neutron, which at best is only made plausible by an argument based on isospin theory. Nor can it be argued that *only* by assuming a negative pion parity can any of the experiments be satisfactorily interpreted which are thought to provide evidence for parity violations in nature. In fact, the situation is quite the opposite, namely if the parity of the pion is positive, there are no difficulties whatsoever in understanding why a given system might decay into both an even and odd number of them, as is observed for both charged and neutral kaons (and other systems). With this background in mind, it is interesting to note that the calculations of the XBPS model indicate that the parity of each pion is positive, while it is that of the neutron in the relevant $d\pi^-$ scattering experiments which is negative. This finding ultimately rests on the fact that the phanton ground state is computed to have 0^- symmetry in the present model, and the same result is obtained for the analogous $e^-\bar{\nu}$ complex which is thought to be a constituent of the neutron. Finally, it is important to note that if parity really is conserved in all natural processes, as these considerations strongly suggest, then there is also no reason to doubt that both the charge-conjugation and the time-reversal operations are perfectly conservative, since previous claims to the contrary have been based on the same types of experimental evidence as discussed above.

Another puzzling characteristic of the behavior of neutrinos may also be connected with the existence of massless phanton systems (Sect. VIII.D), namely the unexpectedly low cross sections observed for the reactions of solar neutrinos arriving at the earth's surface. Under laboratory conditions the reaction producing the neutrinos is assumed to involve a massless $\nu\bar{\nu}$

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system according to the present model. The antineutrino is used to form a neutron, while the neutrino is released with high energy. Because of the 10^6-10^7 °K temperature in the sun's core, a different distribution of photrino translational states is expected there than on the earth, in close analogy to what one knows for the blackbody radiation spectra of photons under the same two sets of conditions. If a significant number of dissociated $\nu\bar{\nu}$ species are present in equilibrium with their bound counterparts in the solar environment, it seems at least conceivable that a rather large fraction of the key fusion processes known to occur there would be induced by free antineutrinos. In such reactions no neutrinos would be emitted thereby destroying the simple relationship which has been used in predicting the outcome of the solar neutrino measurements made at the earth's surface. The key unknown quantity in this proposal is clearly the height of the energy barrier for photrino dissociation, but the expected higher reactivity of free antineutrinos *vis-a-vis* bound $\nu\bar{\nu}$ species could also be a significant factor in reducing the fraction of neutrino-producing fusion processes occurring on the sun as compared to what is observed at ordinary temperatures. This example provides a clear illustration of how the phanton hypothesis differs from the creation-annihilation concept. The present model assumes a well-defined structure for the $\nu\bar{\nu}$ system normally required to cause the fusion reactions of interest to be balanced, but this leaves open the possibility that such a composite particle may behave differently under one set of experimental conditions than another. By contrast, to remain consistent with the creation-annihilation hypothesis, it must be assumed that the environment can have no effect of this nature, because in this view the neutrino and antineutrino are thought to be non-existent prior to the reaction's occurrence.

The present study has also led to a reassessment of certain theoretical aspects connected with quantum mechanical theory in general, one of which has been alluded to in the preceding paragraph. To begin with, it is argued that the coupling between particles and antiparticles implicit in the multi-component Dirac formalism is probably not essential (Sect. X.A). Instead, it can be assumed from linear algebra that a unitary transformation exists which leads to a diagonal form for the corresponding Hamiltonian representation without affecting the constant energy matrix for a time-independent process. In the transformed representation the desired results can always be obtained *through the solution of ordinary (one-component) Schrödinger equations.* The example of the free-particle Dirac equation demonstrates that only one such diagonal term in

the transformed Hamiltonian may actually be needed to obtain all physically meaningful information regarding the original system of interest. Especially since the XBPS Hamiltonian has a domain including all possible spin orientations of the system being treated, there is no reason in principle that it should not be capable of a complete description of its states.

One can consider such a Hamiltonian as a representation of all physical interactions from the point of view of an observer who is at rest at the origin of the coordinate system employed. This interpretation necessitates the introduction of a separate spatial coordinate vector for each particle under consideration *but only a single time variable.* Such a Hamiltonian depends only on the coordinates and conjugate momenta in the observer's inertial system and otherwise contains only relativistic scalars such as the charges and rest masses of the particles being considered, while the energy operator E is that originally introduced by Schrödinger. Solution of the corresponding differential equation does not lead .to a single result, however, but rather an infinite number of eigenvalues and eigenfunctions, representing all possible degrees of translational motion as well as the complete set of internal states of a given system. Observers in inertial systems moving relative to one another simply associate a different energy and charge distribution with the same physical event.

In this view, classical relativistic relationships such as time and mass dilation and Lorentz-Fitzgerald contraction must be used to predict *which energy and wavefunction each observer will decide agrees best with his own perception of the same experiment.* This leaves open the possibility that the principle of relativity can be completely satisfied (see Sect. X.C) in such a theoretical formulation by employing *the same form for the (H-E) operator in each inertial system, without requiring that it be Lorentz-invariant itself.* By employing the relativistic free-particle kinetic energy operator for each particle in the XBPS Hamiltonian, one guarantees Lorentz invariance in the limit of large interparticle separations, while the use of the Breit-Pauli interactions at least potentially accounts for the Lorentz invariance of classical electromagnetic interactions as well. Whether any Hamiltonian can be found which contains suitably damped short-range interactions of the type required to obtain a consistent representation of the strong and weak interactions while still having H-E remain invariant under an arbitrary Lorentz transformation is exceedingly doubtful, however, primarily because of the fact that only the first derivative of the time coordinate appears in the energy operator itself.

The above argument indicates, however, that this circumstance may not preclude a satisfactory representation of high-energy phenomena as long as the principle of relativity is fulfilled in the proposed manner. Ultimately, one can justify this view with the aid of the Bohr correspondence principle. In other words, the quantum mechanical Hamiltonian only needs to satisfy the Lorentz invariance condition in the classical limit in which Planck's constant effectively vanishes. Neither the strong nor the weak interaction can be properly formulated within the framework of classical physics, however, and thus the inability to describe them in a covariant form in no way contradicts the observation that all phenomena in the classical limit are uniquely represented through the application of suitable Lorentz transformations. Once one leaves the classical domain, however, it is only clear that *the principle* of *relativity itself* must remain in force, but as the above argument shows, the quantum mechanical formulation in terms of the Schrödinger or related differential equations allows this condition to be satisfied without requiring Lorentz invariance of the associated H-E operator.

The fact that the XBPS Hamiltonian cannot be separated into two parts which depend exclusively on internal and center-of-mass coordinates, respectively, means that its eigenfunctions cannot be factorized in the usual way. Instead, a coupling of the two types of motion is effected through the mixing of free-particle eigenfunctions of the same |k| but different angular momentum quantum number 1. Hence k is retained as a good quantum number, as must be the case in view of the commutation of the corresponding operator with the total Hamiltonian, but the internal portion of each wavefunction may change with the degree of translational motion. In this way the two eigenfunctions selected by observers in two different inertial systems moving relative to one another can correspond to different translational states of the same internal state *without having identical properties,* something which is clearly required by the time dilation and Lorentz contraction phenomena of the theory of special relativity (Sect. X.B). The Breit-Pauli interactions are partially responsible for the coupling of the internal and center-of-mass motion and the extent of such effects is seen to increase in a straightforward manner with the magnitude of the translational energy. The covariant formulation of the Dirac equation implies that a separation of the center-of-mass motion is always possible, but this has the consequence that internal properties such as the electron-proton separation in hydrogenic systems are independent of the magnitude of their translational energy, which is inconsistent with the Lorentz contraction phenomenon. Ultimately, it can be argued that the insistence on such a covariant form for the Dirac theory not only is responsible for the high accuracy of its predictions in the field of atomic and molecular physics, but also for

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its well-known inability to account for higher-energy interactions such as occur in the study of nuclei and elementary particles.

By assuming that photons and other phantons are present in large quantities everywhere in the universe, it is possible to return to the early view of fields in physical theory as arising exclusively from the interactions of particles. A picture of physical reality emerges which is similar to that espoused by Newton three centuries ago, as well as to the atomic theory of Democritus of much earlier origin. The conclusion that particle dynamics could not explain such properties of light as refraction at an interface of two media of different density can be shown to have rested on an improperly non-relativistic description of the motion of photons (Sect. X.F). Employing the de Broglie and Bohr frequency relations leads to a different conclusion, however, namely that the wavelength of light must decrease upon entering the medium of higher density while the frequency remains the same. Consequently, it must be expected that the velocity of light decreases upon entering the denser medium, as actually observed, even though its momentum increases at the same time.

The conclusion that photons have an intrinsic angular momentum is also shown to depend on the assumption that they are created at the time of emission accompanying atomic transitions. The present calculations indicate that the lowest massless state of the photon is a singlet and that the experimental finding of a non-zero spin for energetic photons simply verifies the fact that in electric-dipole transitions one unit of angular momentum is transferred from the atomic system to the photon with which it interacts.

More generally, it is pointed out that the *wavelike* properties of photons and other material objects can only be observed when a *stream of such particles* is allowed to interact with the measuring device, even though no more than one such particle may be in the apparatus at any one time. In this view there is simply a connection between the temporal and spatial distributions of a collection of particles of the same type having the same translational energy and momentum, as given by the de Broglie and Bohr frequency relations. It is this systematic character of the motion of particles under the same conditions which produces the experimental results generally interpreted as being consistent with wavelike behavior. The Born interpretation of the absolute square of the wave function as a probability distribution is compatible with such a view, especially when the effects of the system's translation are taken explicitly into account. The main question which has raised doubts about such a purely statistical interpretation has revolved around the finding that probability itself is not additive in the theory, but rather only the wave functions which are used to compute the pertinent

distribution function. In the Young interference experiment with coherent radiation, it has been consistently argued that photons at the source have no means of "knowing" whether one or both slits of the apparatus are open. The key observation is that the intensity distribution observed for the case when both slits are open is not simply obtained by adding the corresponding distribution which results when one of the slits is open and the other closed to that resulting for the complementary arrangement. It has often been argued that this result is only understandable *if a given particle can pass through both openings* on the way to the screen, i.e. that it possess delocalized character generally only associated with wavelike substances.

By rejecting the assumption that space can be free of material particles and replacing it instead with the concept of ubiquitous photons everywhere in the universe, a mechanism presents itself, however, which does allow particles at the source of the Young apparatus to be aware of the condition at the slits allowing passage to the screen beyond (Sect. X.G). As a consequence, the need to redefine the original meanings of particle and wave in order to explain the Young interference phenomenon is eliminated. Thus even the paradox of wave-particle duality is seen to be intimately tied up with the hypothesis that material elements can be created and destroyed in physical transformations. By rejecting this position and holding instead that indestructible elements do exist in nature, it is found that there really is no proof that massless particles cannot exist in sufficient density throughout the universe and thus produce the results of the Young interference experiment which are observed.

Finally, a mechanism is suggested whereby the definition of the quantum mechanical wave function can be extended to enable it to contain more information about a given system than could be deduced on the basis of expectation values of dynamical variables. The possible relevance of this subject to the understanding of the Einstein-Podolsky-Rosen paradox has been discussed (Sect. X.H), particularly from the point of view of whether the results of experiments involving individual particles might not in fact be generally predictable. In cases where no physical means exists to distinguish one particle from another, however, it is clear that even such additional theoretical information would not be sufficient to allow an assignment of different outcomes of experiments to specific members of the ensemble.

In summary, the present model asserts that a consistent theory of elementary particle interactions can be built upon the premise that all matter ultimately is composed of electrons, protons, neutrinos and their antiparticles. Such a model is characterized by very simple conservation laws, the most basic of which requires that the same elements are present before, during and after every physical process. Each of the above elements is assumed to be perfectly stable and to possess a point charge distribution. There is no unambiguous experimental finding

which stands in contradiction to this view. A model based on the existence of only point particles as the constituents of all matter is the ultimate concretization of the theory of the Greek philosopher Democritus which holds that the only physical realities are "Atoms and Voids", and most importantly in the present context, that the former are absolutely indestructible. A point particle has no volume and thus can never be divided into still more elemental parts. To the elemental balance principle is added the energy, linear and angular momentum conservation laws. Parity is assumed to commute with the corresponding Hamiltonian, similarly as the charge conjugation and time reversal operations.

On this basis, it is suggested that there may be no need to assume that particles exist which are not composed of the above elements. The present model also does not make use of a number of theoretical quantities invented in the quantum age, such as isospin, hypercharge and the muon quantum number. Such a development in no way precludes the utility and relevance of these properties. Rather, it questions whether they are *essential* to the goal of understanding all processes involving elementary particles. In a similar vein, while the quark model in its various forms has been quite successful in predicting the existence of hitherto unknown elementary particles, as well as elucidating the symmetry relationships which exist among such systems as a whole, it can still be argued that such explanations may not be *unique*. Furthermore, there is a fundamental relationship connected with treating particles as basis functions for an irreducible representation of a group which does seem to be lacking on the basis of experimental observations. The classical applications of group theory in quantum mechanics, such as the description of angular momentum, allow for equivalent irreducible representations for which different basis sets are employed (for example, real and complex 1^2 eigenfunctions). One knows, for example, that the choice of such angular momentum basis functions can be systematically varied by changing the direction of an applied magnetic field for atomic Zeeman effect experiments. To demonstrate the analogous relationship in elementary particle physics, it is necessary to find different *linear combinations* of degenerate particles, as it were, for a particular irreducible representation. When the original basis functions are of opposite charge, as is the case for pions and kaons, for example, this would mean isolating a particle with non-integral electric charge, something which has never been accomplished in any experiment to date. While it is easy for a theoretician to construct arguments which rationalize the failure to discover such hybrid charged particles in nature, it also can be argued that the real reason for this development is that the true elements which nature has provided us are completely immutable, regardless of any external force which might be applied to them.

In the absence of the positive identification of particles with fractional electric charges, one can at least ask the question as to whether a theory which only requires the existence of

known stable particles as the sole building blocks of nature may not ultimately be sufficient to explain all measureable phenomena. The present model thus envisions the universe as consisting exclusively of point particles joined together by forces which, with the exception of gravity, are not greatly different than those envisioned in the Dirac equation. In particular, the same Hamiltonian is used to represent the electromagnetic and weak interactions, and additional proton-proton interactions have been investigated, suggested to a large extent by the success of the nuclear shell model, which give promise of ultimately incorporating a quantitative description of the strong interaction in the present model as well. The fact that the creation and annihilation of material particles can never be unambiguously verified by purely experimental means should serve as a strong impulse toward the further development of such an alternative theoretical model, one that avoids making the above assumption and instead requires a strict elemental balance in all naturally occurring processes.

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ACKNOWLEDGMENT