The Giant Twin's Travels

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Abstract

There are two versions of time dilation in Einstein's original paper on special relativity theory (SRT). It is pointed out that they differ in a crucial respect, namely whether the phenomenon is symmetric or asymmetric. The Clock Paradox arises from belief in the latter version, which asserts that a clock runs slower when it is accelerated than an identical clock that remains at rest in its initial rest frame. The symmetric theory of time dilation is based on the Lorentz invariance condition of the Lorentz transformation (LT). Experiments have invariably shown that it is always possible to know which of two clocks is running slower, and therefore have confirmed Einstein's asymmetric version of time dilation. The question is therefore considered whether the lengths of accelerated objects are properly described by the LT, despite the fact that it fails to predict the observed asymmetry in clock rates. The present work calls attention to the fact that Einstein's derivation of the LT is based on an *undeclared assumption* in addition to his two postulates of relativity. It is shown that assuming instead that clock rates are strictly proportional to one another leads to a different version of the Lorentz transformation that still satisfies the two postulates of relativity and the velocity transformation of SRT while successfully describing the asymmetric character of time dilation.

Keywords: time dilation, twin paradox, postulates of special relativity, Lorentz transformation (LT), velocity transformation (VT), alternative Lorentz transformation (ALT)

I. Introduction

One of the most frequently discussed aspects of the special relativity theory (SRT) is the twin paradox. The interest in this topic goes back to a comment that Einstein made in his original paper [1] about clocks in motion. He concluded that a clock at rest on the Equator should have a slower rate that an identical clock at rest at one of the Earth's Poles. This led to speculation [2] that two twins would differ in age if one of them goes on a long journey into outer space at very high speed while the other remains at rest at his home base, specifically that the former would come back significantly younger than his stay-at-home brother. Subsequent experiments carried out on circumnavigating airplanes [3-4] proved definitively that at least Einstein's supposition about clock rate variations was correct. Whether that proves that the traveling twin would come back younger is clearly still a matter of great uncertainty, but the success of the experimentation with clocks raises another question that has thus far been largely ignored. What happens to the size of the twin as he makes his way through space at such high speed? The other bastion of SRT is the Lorentz contraction effect [1]. It states that he will decrease in size along the direction of motion but will otherwise stay the same, causing him to be distorted in much the same way as an image in a fun-house mirror. Time and space are famously interconnected in relativity theory, so there is no reason for confining this discussion to the question of aging. As will be shown below, a clear picture emerges from both experiment and theory as to what happens to the dimensions of the twin as a result of his high-speed travels.

II. Changes in the Moving Twin's Circulation

One of the most basic characteristics about time dilation is that is *uniform* for all clocks at rest in a given inertial system. This condition is required by the relativity principle (RP) since any violation of it would allow one to distinguish between one such rest frame and another. Furthermore, it also holds for all other types of natural timing devices, including the periods of biological processes occurring in plants and animals.

Consider, for example, the circulation of blood through the human body. If the accelerated twin A travels with constant speed $v= 0.866$ c (c=2.9979×10^{8} ms^{-1}) relative to his initial location, his clocks, i.e. those that are at rest in his inertial system, will run exactly half as fast as those that are stationary in his brother B's rest frame. The RP therefore stipulates that A's blood will circulate with *twice* the period it would have under the same conditions if he was not moving relative to B.

There is another measure of circulation, however, namely the *speed* at which blood moves through the body. Relativity theory is also quite clear on this point as well: the speed w between two fixed points is the *same* for all observers regardless of their state of motion. This principle has been applied in the discussion of muon decay [5], for example, as well as in examples given in textbooks [6,7] to illustrate the effects of high-speed travel. It also can be said to be a consequence of the RP, since its violation would mean that w/c ratios would necessarily vary and therefore allow one to distinguish between different inertial systems.

The above comparisons raise an obvious question. How can twin B measure the period of A's blood circulation to be twice the normal value and still find that the speed with which the blood travels through A's veins and arteries is exactly the same as before his twin brother was rocketed into space? Moreover, how can one explain that the ratio of A's circulation period to the speed of the blood is independent of his orientation? Since speed is by definition the ratio of the distance D traveled to the elapsed time T required to complete the journey, there is only one acceptable answer for the above questions: A's blood must travel twice as far during the circulation period under the conditions on the rocket ship, so that the D/T ratio remains unchanged. This conclusion in turn implies that twin A is now twice as tall and twice as wide as before he parted from twin B.

In summary, when one applies Einstein's two postulates of relativity directly to the problem of predicting changes in the dimensions of the accelerated twin A, a qualitatively different result is obtained than one expects from SRT and its prediction of Lorentz contraction. Instead of his height decreasing by half along his direction of relative motion to twin B, it doubles instead. In addition, A's breadth does not remain unchanged but also doubles as a result of his changed state of motion. This is clearly uniform length expansion rather than the asymmetric length contraction claimed by SRT.

III. Empirical Evidence for Relativistic Length Expansion

The theoretical arguments of the previous section need to be considered in light of available experimental evidence. It is obvious that an observer cannot actually measure the length of an object moving away from him at high speed, so all that one can do is to carry out experiments that allow as straightforward a deduction as possible of such effects. One way of accomplishing this objective has been largely ignored, even though the experiment in question is quite well known, namely the change in wavelength of radiation emitted from a moving source. A transverse Doppler effect was predicted in Einstein's original paper [1] and it was first verified experimentally 33 years later by Ives and Stilwell [8]. The result is

that the observer in the laboratory finds that the wavelength of the radiation has increased. At the same time, the corresponding frequency has decreased, and this is universally accepted [9] as an example of time dilation in the rest frame A of the moving source. It is interesting to note, however, that the same authors decline comment on the significance of the wavelength increase. According to the RP, someone in rest frame A must measure the same frequency and wavelength as observed in the laboratory L when the source is at rest there. The only way to explain this is to assume that the diffraction grating at rest in A must also have increased in dimension by the same fraction as the wavelength. Moreover, the increase in wavelength must be the same independent of the direction of the light source's motion relative to L. This can be seen by eliminating the non-relativistic angle-dependent term in the relativistic Doppler formula [1].

These considerations show unequivocally that there is a uniform increase in the dimensions of all objects at rest in A, which in turn is perfectly consistent with the theoretical conclusion regarding changes in the height and width of the accelerated twin in the example discussed in Sect. II. If Lorentz contraction had occurred instead, one would expect that the observer in L would find a decrease in the wavelength of the radiation along the direction of motion after eliminating the non-relativistic angular dependence in the Doppler formula, and no change at all in the corresponding perpendicular directions.

Lorentz contraction is also claimed to be the cause for the observed increase in the range of accelerated muons in the atmosphere [5], but once again it is found upon closer examination that this conclusion is faulty. There is complete agreement that the lifetime of the accelerated muons in their rest frame A is longer than for muons at rest on the Earth's surface L, and this clearly explains why the range of the atmospheric muons is longer. It is also true because of the RP that an observer moving with the atmospheric muons does not notice any change from the normal lifetime and range before disintegration. There is also agreement that such an observer must find that the distance between the original position of the muons in the atmosphere and the Earth's surface is smaller by the same factor than the corresponding distance measured by the observer on the Earth's surface. Note that the clear assumption is that the unit of speed is the same in both the A and L rest frames, causing the two observers to agree on the speed of the atmospheric muons as they move in space.

Where the standard argument in the literature [5-7] goes off the track is by concluding that the above experience proves that there is length contraction in the rest frame A of the atmospheric muons. To show that this is not the case, it is first necessary to see that the reason the observer in A consistently measures smaller elapsed times is because his *unit of*

time is greater than for the observer in L, i.e. his clock runs slower than that at rest on the Earth's surface. By the same token, the reason that the observer in A consistently measures shorter distances is that *his unit of length is greater* by exactly the same factor, i.e. his meter stick is *longer* than that used in L. In other words, once again the conclusion is that the lengths of objects increase upon being accelerated.

Another way to put this is to recall that the numerical value of any measured quantity is inversely proportional to the unit in which it is expressed. All lengths of objects in the rest frame A have *increased* by the same fraction in all directions relative to their values when they are at rest in L. Consequently, the observer in A consistently measures smaller values for the same distance as the observer at rest on the Earth's surface.

This is the case even though each observer will continue to believe that it is his meter stick that is perfectly standard. The latter state of affairs occurs because all changes in the dimensions of stationary objects in any given inertial system are *uniform*. The only way that an observer can notice that either his own dimensions or those of stationary objects in his rest frame have changed by virtue of their mutual acceleration is to measure the lengths of other objects that are *not* stationary. The same situation holds for the rates of clocks and also for all other physical properties one can possibly imagine. Otherwise, the RP would be easily demonstrated to be invalid on the basis of *in situ* measurements.

In summary, as before with the transverse Doppler effect [8], length expansion in all directions accompanies time dilation in an accelerated rest frame, not length contraction along the direction of motion.

Other arguments that have been brought to bolster the Lorentz contraction supposition center on the characteristics of high-speed electrons and nuclei. One observes an increase in particle density [10], for example, as well as enhanced ionization characteristics when the particles are accelerated. The claim that these experimental results can only be explained by Lorentz contraction overlooks the fact that the de Broglie wavelength λ of the distribution of the particles [11] is inversely proportional to their momentum p. Accordingly, one expects the product of p and λ to equal Planck's constant h. It is at least simple in principle to distinguish between the two effects. In the latter case the key quantity is the momentum p of the particles, and thus the observed wavelength will depend strongly on their inertial mass m. By contrast, the amount of relativistic length contraction only depends on the speed of the particles. This distinction makes it impossible to claim that a decrease in de Broglie wavelength is the same as relativistic length contraction. Since individual electrons are closer together than they were prior to being accelerated, it follows that the frequency of the

5

accompanying synchrotron radiation will be increased, so one does not need length contraction to explain the observed results in this case either [10]. An observer moving with the free electrons will find that the wavelength of their distribution is even shorter because his unit of distance is larger than for his counterpart at rest in the laboratory. He will also find that the synchrotron radiation has a higher frequency from his vantage point.

IV. Degree of Freedom in the Lorentz Transformation

The preceding discussion shows that SRT actually comes to different conclusions depending on how it is applied. As discussed in the previous section, it is easy to find examples in the literature where each of the approaches in question has been applied with apparent success. However, the fact that they lead to lead to opposite conclusions for length variations in the context of the Twin Paradox has thus far escaped attention. In one case it is clear that the accelerated twin should undergo length contraction along his line of motion and no change at all in the perpendicular directions. In the other, it is equally clear that his dimensions must increase by the same fraction in all directions. It is obvious that both conclusions cannot be correct, so it is imperative that one find a flaw in the argumentation leading to (at least) one of them.

To begin this exercise, it is important to see that both approaches make use of Einstein's two postulates of relativity [1]. The difference lies in the fact that the length contraction variant makes direct use of the Lorentz transformation (LT) to come to its conclusions, whereas the other ignores it entirely. This circumstance makes it highly advisable to go back to the original derivation of the LT to see if an explanation for the seemingly confusing state of affairs might be found there. It needs to be clearly understood that the above two postulates are not sufficient in themselves to arrive at a unique version of the desired space-time transformation. This point was made clear in Lorentz's early papers [12] when he wrote down the most general version of the transformation that leaves Maxwell's equations invariant. Einstein was aware of this problem and his solution (see p. 900 of ref. 1) was to assume that an undefined function φ in Lorentz's general transformation can only depend on the relative speed v of the two inertial systems whose coordinates appear in these equations. He did not declare this as an assumption, however, and consequently made no attempt to justify it. He subsequently used a symmetry argument to show that under these circumstances φ must have a constant value of unity, which conclusion then leads directly to the LT.

What is generally overlooked in the above derivation is that it is not essential that $\varphi = 1$ in order to satisfy the two postulates of relativity. Put in another way that is more directly relevant to the present discussion, concepts such as Lorentz invariance and Lorentz contraction are merely forced onto relativity theory as a result of the $\varphi = 1$ condition and are not the inevitable consequence of Einstein's light-speed postulate and the RP. The only way to be certain that Einstein's choice is the correct one is through experimental verification. For example, it needs to be demonstrated that the symmetry implied in the Lorentz invariance condition actually occurs in nature. The fact is that the Hafele-Keating measurements [4] and earlier experiments studying the transverse Doppler effect for a light source and detector mounted on a high-speed rotor [13] prove that it is always possible to determine which of two clocks is running slower, namely the one that undergoes the greater acceleration relative to a definite reference system. This result is generally attributed to Einstein's Equivalence Principle [14] and it is also clearly consistent with Einstein's example of clocks located at different latitudes on the Earth's surface [1], as discussed in the Introduction.

Nonetheless, it needs to be clearly understood that the above results are definitely inconsistent with the symmetry implied by the LT [15], and this fact opens up a clear possibility for understanding why the two approaches come to different conclusions about the way the accelerated twin's dimensions vary even though they both make use of Einstein's relativity postulates: Einstein's assumption to eliminate the degree of freedom in the general version of the Lorentz transformation is incorrect. It needs to be replaced by another assumption that recognizes the fundamental asymmetry that results when objects are accelerated. In previous work [16-17] it has been shown that a straightforward way to accomplish this objective is to assume that clock rates in different rest frames generally differ by a definite proportionality factor. This assumption leads to an alternative Lorentz transformation (ALT [18-19]) that is consistent with both the fundamental asymmetry observed in time and space measurements made by different observers as well as Einstein's two postulates. The ALT leads to the same relativistic velocity transformation (VT) as in Einstein's original work [1] and is therefore consistent with all the confirmed predictions of his theory that are based on this aspect of SRT. However, it does not support either Lorentz length contraction or the symmetry principle demanded by the Lorentz invariance condition, and thus, unlike the LT, is consistent with all the empirical evidence discussed in Sect. III that supports the conclusion of isotropic length expansion accompanying time dilation.

V. Conclusion

History has shown that Einstein's conclusion that the rates of clocks decrease upon acceleration is correct. However, since time and space are interrelated through his light-speed postulate, this knowledge can also be used to make an additional conclusion about the way the dimensions of objects change when they are accelerated. For example, we can predict how the height of a twin changes as he speeds away on a rocket ship by passing light over his body and measuring the corresponding elapsed time for this to occur. Because of the RP, the accelerated twin will not notice any difference on his onboard clock, but his stay-at-home brother will find that his value for the elapsed time keeps getting larger as the rocket ship's speed increases. This is the same phenomenon as is well known for the decay lifetimes of accelerated muons. Since the speed of light is exactly the same for both, this means that the height of the accelerated twin must have *increased* as a result of the change in his state of motion. Moreover, the ratio of the twins' elapsed times must be independent of spatial orientation no matter how fast the rocket ship moves away.

The above argumentation is perfectly straightforward and is based exclusively on Einstein's two postulates of relativity [1], yet the conclusion is opposite to what one normally expects from SRT. Instead of anisotropic length contraction, one finds that isotropic length expansion accompanies the slowing down of clocks. This conclusion is consistent with the fact that the wavelength of light always increases in transverse Doppler experiments at the same time that the frequency of the accompanying radiation decreases. Lorentz contraction is derived from the Lorentz transformation of SRT, which in turn is also based on Einstein's two postulates, so the disparity in the two sets of conclusions certainly warrants a clear explanation. It is not difficult to find it. In deriving the LT in his original work [1], Einstein made an additional assumption that was not declared as such. He claimed that a function that appears in the most general form of a space-time transformation that leaves Maxwell's equations invariant can only depend on the relative speed v of the two inertial systems being compared. The same or equivalent assumption is used in all other derivations of the LT as well as in other well-known formulations of relativity theory [20-21], but it lacks experimental justification.

The key point that needs to be recognized in the context of the Twin Paradox and in the relevant experimental data is that the asymmetry in the rates of clocks in different inertial systems is not consistent with Lorentz invariance. This in turn means that the LT does not correctly describe the phenomena in question. Nonetheless, the RP and Einstein's light-speed

postulate can still be satisfied without making the above assumption, while at the same time guaranteeing the observed asymmetry in clock rates. This can be done by assuming instead that clocks in different inertial systems run at different rates that are in a fixed ratio: $t = Qt'$. Making this assumption leads to an alternative Lorentz transformation (ALT) that no longer forces the undesirable symmetry characteristics of the LT and also clearly eliminates the space-time mixing feature of SRT that is also inconsistent with the available measurements. The ALT leads to the same velocity transformation as Einstein derived in his original work and therefore explains the aberration of starlight at the zenith and the Fresnel light-drag experiment just as well as when the LT is used.

In summary, the experiments that show that the rates of clocks are slowed by acceleration prove that the dimensions of objects increase by the same fraction in all directions. The moving twin retains the same proportions as he had prior to leaving his original rest frame, but he grows in stature as a result. He doesn't notice these changes himself because of the RP, but his brother is able to make this determination for him along with finding that his brother's biological processes have slowed on the rocket ship.

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