Proof of Internal Contradictions in the Lorentz Transformation: The End of Space-time Mixing

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Abstract

A simple example is considered which shows that the Lorentz transformation (LT) is physically invalid because it leads to the conclusion that clock rates depend on the speeds of distant objects. This lack of internal consistency in relativity theory is traced to an undeclared assumption Einstein made regarding a normalization factor appearing in his original derivation of the LT. An alternative Lorentz transformation (GPS-LT) is obtained by replacing this false assumption with another that demands that a strict proportionality exists between the rates of clocks in different inertial systems, exactly as is assumed in the methodology of the Global Positioning System (GPS). The GPS-LT is consistent with all known experimental observations as well as with Einstein's relativistic velocity transformation (RVT). The success of the GPS-LT in removing the inherent contradictions of the LT demonstrates that Einstein's famous position that space and time are inextricably mixed is fundamentally in error. The relativity principle (RP) when applied to the Ives-Stilwell transverse Doppler experiment and the muon decay studies is also shown to prove that Fitzgerald-Lorentz length contraction (FLC) does not occur in nature

and that the dimensions of accelerated objects actually increase by the same fraction in all directions as the rates of clocks are slowed, i.e. isotropic length expansion accompanies time dilation in a given rest frame.   The GPS-LT is also consistent with Newtonian absolute remote simultaneity and does away with Einstein's symmetry principle whereby two clocks in relative motion can supposedly both be running slower than each other at the same time. The accompanying theory restores the principle of objectivity of measurement that was universally believed until the dawn of the 20th century.

Introduction

The Lorentz transformation (LT) is the cornerstone of Einstein's Special Theory of Relativity (STR)[1]. It satisfies the condition of Lorentz invariance, and leads to a number of predictions such as time dilation and FitzGerald-Lorentz length contraction (FLC). The LT satisfies Einstein's two postulates of relativity: the relativity principle (RP) first introduced by Galileo in 1632, and the light-speed postulate (LSP), which holds that the speed of light ($c=2.99792458 \times 10^8$ ms^{-1}) is independent of both the state of motion of the light source and the observer. It has been subjected to many experimental tests over the past century and there is widespread, although not universal, agreement among physicists that it has been confirmed in all cases.

A physical theory must do more than agree with experiment, however. It must also be consistent with established logical principles, and it should be single-valued, i.e. it must provide a unique answer to any question that falls within its range of application. In the following it will be shown that the LT does not satisfy either of these theoretical criteria and in fact also is inconsistent with well-known experimental tests that were carried out as early as 1938 [2,3].

2. Violation of Einstein Causality for Clock-rate Ratios

The LT consists of four relationships between the spatial and time coordinates measured by observers in two different inertial systems S and S' for the same event. Einstein [1] assumed that the origins of the corresponding two coordinate systems coincide for a starting time $t=t'=0$, i.e. $x=x'=0$, $y=y'=0$ and $z=z'=0$. The LT equation relating the two time variables is given as follows (x is the location of the event as measured by a stationary observer in S; S' is moving away from S along the x axis with speed v):

$$t' = \gamma (t - vxc^{-2}) = \gamma \eta^{-1} t, \tag{1}$$

with $\eta = (1-vxc^{-2}t^{-1})^{-1}$ and $\gamma = (1-v^2c^{-2})^{-0.5}$.

It should be noted that the variables t', x and t in eq. (1) are actually intervals relative to their respective origins: $t'=0$, $x=0$ and $t=0$. The corresponding intervals between *two* events are obtained by subtraction to be $\Delta x = x_2 - x_1$ etc. Since speeds are always defined as the ratio of

spatial and time intervals, it is important to have an alternative version of eq. (1) for these quantities. If v is assumed to be constant, this alternative relation for spatial and time intervals is seen to be:

$$\Delta t' = \gamma \left(\Delta t - v \Delta x c^{-2} \right) = \gamma \, \eta^{-1} \Delta t, \tag{2}$$

with $\eta = (1 - vc^{-2} \Delta x / \Delta t)^{-1}$.

Both the above equations have played a very important role in theoretical physics. Poincaré [4] was the first to point out that they indicated that the long-held concept of absolute remote simultaneity of events [5] might be incorrect because there can be a null time difference in one inertial system ($\Delta t = 0$) without the other vanishing as well. According to eq. (2), this situation can occur whenever both $v \neq 0$ and $\Delta x \neq 0$. He emphasized that no experimental data existed at that time which would rule out this possibility from occurring in nature. It was also pointed out that the LT indicates that the Newtonian concept of complete separation of space and time may also not be correct since the timing result ($\Delta t'$) of one observer may depend on the spatial coordinate (Δx) of the same event measured by another observer who is stationary in a different rest frame, and not just on the latter's timing result (Δt). Space-time mixing is an important assumption in modern-day cosmology, as for example in string theory [6].

Consider the following simple application of eq. (2), however. The two observers follow the course of a far-distant object traveling at constant velocity **u**. The stationary observer in S finds that the object moves a distance Δx during an elapsed time Δt, i.e. $\Delta x = u_x \Delta t$ (how far the object moves along the y and z axes is not important for the present discussion). He therefore uses eq. (2) of the LT to compute the corresponding elapsed time $\Delta t'$ measured for the same event. Dividing this result by Δt leads to the following relation:

$$\Delta t' / \Delta t = \gamma \left(1 - vc^{-2} \Delta x / \Delta t \right) = \gamma \left(1 - v \, u_x \, c^{-2} \right) = \gamma \, \eta^{-1}. \tag{3}$$

Note that $\Delta t' / \Delta t$ is the ratio of the elapsed times for this event measured in S and S', respectively. *It is also the ratio of the clock rates in the same two inertial systems,* however, since the elapsed time is by definition proportional to the corresponding clock rate. As a result, eq. (3) allows us to conclude that the ratio of clock rates depends not only on the relative speed v of S and S' but also on u_x, the x-component of the object's velocity measured by the stationary observer in S.

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This result is completely unacceptable from a physical standpoint, however. It means that a change in velocity of the object, which is potentially light-years distant, affects one or both of the rates of proper clocks in S and S'. Moreover, there is nothing standing in the way of applying eq. (3) to a series of objects moving at a wide variety of velocities.

It might be thought that the above problem with the LT is that it involves three bodies, the two observers in S and S' and the distant object under observation from both. One can just as well use event calculus in connection with eq. (1) to come to the same conclusion. First, the two observers consider an event at location x_1,t in S, from which they conclude that the ratio of their respective clock rates is t'/t. Then they simply turn their attention to a different event that occurs at exactly the same time but at a different location x_2 in S. The conclusion from eq. (1) is that the value of t' must have changed because the value of x is now different while that for t is the same as before. As a result, the t'/t clock-rate ratio must have changed as well, despite the fact that the only perceivable "cause" for this effect is that the location of the event under mutual consideration by the stationary observers in S and S' is not the same in the two cases.

One can also conclude that such a change in the clock-rate ratio would stand in contradiction to Newton's First Law (law of inertia). Since the two clocks are both moving in pure translation (with no unbalanced forces), neither of their rates can be expected to change and therefore the corresponding ratio must also remain constant.

In short, the above example proves that eqs. (1-3) are not valid physically, which also means that the LT itself is not acceptable as a component of a theory of relativity. Instead, what we have is a clear violation of Newtonian causality [7]. Therefore, any conclusions that have previously been made on the basis of the LT need to be carefully reconsidered.

3. The Global Positioning System-Lorentz transformation (GPS-LT)

In order to understand how Einstein arrived at a physically invalid space-time transformation, it is important to critically examine the derivation of the LT he gave in his original work [1]. Lorentz noted as early as 1899 [8,9] that there was an undefined degree of freedom in the most general space-time transformation (GLT) that leaves Maxwell's equations

invariant. One can express this relationship by inserting a *normalization* factor ε in each of the four equations below:

$$\Delta t' = \gamma\,\varepsilon\,(\Delta t - v\Delta x c^{-2}) = \gamma\,\varepsilon\,\eta^{-1}\,\Delta t \qquad\qquad (4a)$$

$$\Delta x' = \gamma\,\varepsilon\,(\Delta x - v\Delta t) \qquad\qquad (4b)$$

$$\Delta y' = \varepsilon\,\Delta y \qquad\qquad (4c)$$

$$\Delta z' = \varepsilon\,\Delta z, \qquad\qquad (4d)$$

with $\eta = (1 - vc^{-2}\Delta x/\Delta t)^{-1} = (1 - vu_x c^{-2})^{-1}$. Exactly the same equations were derived by Einstein [1] based on his two postulates of relativity, except that he used a slightly different notation than Lorentz (he referred to the normalization factor as φ instead of ε). He eliminated the uncertainty posed by the degree of freedom in the GLT by asserting (see p. 900 of Ref. 1) that "φ is a temporarily unknown function of v." He then showed on the basis of symmetry considerations that φ=1 is the only allowed value for the normalization function under these circumstances, thereby producing the LT upon substitution in eqs. (4a-d); this includes the offending relation in eq. (2). However, it is important to understand that a clear assumption is involved in the above conclusion. *It amounts to a third postulate of relativity theory.*

The fact that Einstein did not declare it as an additional postulate is at the very least an interesting fact of history, but this would be an insignificant development if the assumption were actually true. The analysis of the previous section indicates instead that the normalization constant (Lorentz's ε or Einstein's φ) must be chosen so as to satisfy the condition, $\Delta t'=\Delta t/Q$, where Q is the ratio of proper clock rates in the two inertial systems S and S'. It depends only on characteristics of these two rest frames and is completely unaffected by the motion of distant objects, specifically not on u_x in the example of the previous section. The corresponding value of ε is obtained easily by equating the value of Δt' in eq. (4a) to the value in the above proportionality condition:

$$\Delta t' = \gamma\,\varepsilon\,(\Delta t - v\Delta x c^{-2}) = \gamma\,\varepsilon\,\eta^{-1}\Delta t = \Delta t/Q, \qquad\qquad (5)$$

One therefore concludes that the physically allowable value is:

$$\varepsilon = \eta/\gamma Q. \qquad\qquad (6)$$

Substitution of this value in the GLT of eqs. (4a-d) leads to the desired alternative

Lorentz transformation (GPS-LT):

$$\Delta t' = \Delta t/Q \tag{7a}$$

$$\Delta x' = \eta\, (\Delta x - v\Delta t) \tag{7b}$$

$$\Delta y' = \eta\Delta y/\gamma Q \tag{7c}$$

$$\Delta z' = \eta\Delta z/\gamma Q. \tag{7d}$$

The GPS-LT satisfies both of Einstein's postulates of relativity while at the same time insuring that there is no contradiction involving the ratio of elapsed times $\Delta t'/\Delta t$. By contrast, the LT, which is given below, demands that this ratio be a function of $\Delta x/\Delta t$, in clear violation of Newtonian causality:

$$\Delta t' = \gamma\ (\Delta t - v\Delta x c^{-2}) = \gamma\, \eta^{-1}\, \Delta t \tag{8a}$$

$$\Delta x' = \gamma\ (\Delta x - v\Delta t) \tag{8b}$$

$$\Delta y' = \Delta y \tag{8c}$$

$$\Delta z' = \Delta z, \tag{8d}$$

It is important to recognize that the relativistic velocity transformation (RVT) can be obtained from each of the above three space-time transformations by simply dividing each of the various spatial equations for $\Delta x'$, $\Delta y'$ and $\Delta z'$ by the corresponding relation for $\Delta t'$. The result in each case is given below, with $u_x' = \Delta x'/\Delta t'$, $u_x = \Delta x/\Delta t$, etc.:

$$u_x' = (1 - vu_x c^{-2})^{-1}(u_x - v) = \eta\, (u_x - v) \tag{9a}$$

$$u_y' = \gamma^{-1}\, (1 - vu_x c^{-2})^{-1}\, u_y = \eta\, \gamma^{-1}\, u_y \tag{9b}$$

$$u_z' = \gamma^{-1}\, (1 - vu_x c^{-2})^{-1}\, u_z = \eta\, \gamma^{-1}\, u_z. \tag{9c}$$

The normalization factor ε in the GLT of eqs. (4a-d) is simply cancelled out in each of the divisions and therefore does not appear at all in the RVT [note also that η appears in all three equations by virtue of eq. (4a) of the GLT].

A number of the most important results of relativity theory actually result directly from the RVT, and thus do not rely in any way on Einstein's assumption about the normalization factor. These include the aberration of starlight at the zenith [10] and the Fresnel light-drag experiment [11], both of which were quite important in Einstein's thought process [12]. The RVT also guarantees compliance with the light-speed postulate. It is used directly in the derivation of the

Thomas precession of a spinning electron [13,14] and thus the LT is not essential in this case either. Moreover, the proof that Maxwell's equations are invariant to the GLT in eqs.(4a-d) demonstrates that the value chosen for ε/φ is inconsequential for this purpose as well. Indeed, it was this fact that caused Lorentz to introduce the normalization factor ε in the general transformation in the first place [9].

The GPS-LT can be obtained somewhat more directly by combining eq. (5) and/or eq. (7a) with the RVT of eqs. (9a-c), i.e. by multiplying the various velocity components with the corresponding times in S and S', respectively. The clear distinction between the GPS-LT and the LT is that there is *no space-time mixing* in the former set of equations. The arguments in Einstein's version of relativity for *remote non-simultaneity of events* as a necessary condition for satisfying the LSP are therefore *negated* by the GPS-LT [15]. There is also no possibility of forcing a violation of Newtonian causality through time reversal [16] since the constant Q in eq. (5) is necessarily positive. It is seen that by multiplying each of the four LT equations with the same factor η/γ on the right-hand side, one obtains the corresponding four equations of the GPS-LT.

While it is clear that the GPS-LT satisfies the light-speed postulate because of its direct relationship to the RVT, it still remains to show that the choice for the normalization factor in eq. (6) also satisfies the other of Einstein's relativity postulates, the RP [1]. This question is closely tied up with the condition of Lorentz invariance that is a key feature of the LT. Squaring and adding the four relations of the GLT in eqs. (4a-d) leads to the following result:

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = \varepsilon^2 (x^2 + y^2 + z^2 - c^2 t^2). \qquad (10)$$

The value for the normalization factor of $\varepsilon = 1$ assumed by Einstein [1] to obtain the LT leads to the aesthetically pleasing and transparently symmetric form that is so familiar to theoretical physicists. Most importantly, Einstein's version of eq. (10) satisfies the RP since it looks exactly the same from the vantage point of both observers. It is less obvious how any other choice of ε can satisfy the latter requirement, and this has been used to justify adopting the value of unity in deriving the LT. Specifically, the question arises as to whether the choice of $\varepsilon = \eta \, (\gamma \, Q)^{-1}$ in eq. (6) that leads to the GPS-LT is also consistent with the RP.

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To consider this possibility it is helpful to first write down the corresponding result obtained from the inverse of eqs. (4a-d), which can be obtained most simply by algebraic manipulation of eq. (10):

$$x^2 + y^2 + z^2 - c^2t^2 = \varepsilon^{-2}(x'^2 + y'^2 + z'^2 - c^2t'^2). \tag{11}$$

To satisfy the RP, the latter equation must be consistent with an alternative form of eq. (10) that is obtained by switching the roles of the two inertial systems and the respective observers in these rest frames. This is accomplished by exchanging all primed and unprimed subscripts and changing the sign of their relative speed from v to –v, with the result:

$$x^2 + y^2 + z^2 - c^2t^2 = \varepsilon'^2(x'^2 + y'^2 + z'^2 - c^2t'^2). \tag{12}$$

Satisfaction of the RP therefore demands that a) the normalization factor ε' be defined in a completely analogous manner as ε and b) that the following relation between ε and ε' be satisfied:

$$\varepsilon^2 \varepsilon'^2 = 1 \tag{13}$$

It is obvious that Einstein's value of $\varepsilon = \varepsilon' = 1$ satisfies both of the above requirements. Indeed, a value of $\varepsilon = \varepsilon' = -1$ also is not excluded by these conditions.

The value of ε in eq. (6) that is used to derive the GPS-LT of eqs. (7a-d) leads to the following relation when substituted in eq. (13):

$$\eta^2(\gamma Q)^{-2} \eta'^2(\gamma Q')^{-2} = 1, \tag{14}$$

whereby η' must be obtained from $\eta = (1 - vc^{-2}\Delta x/\Delta t)^{-1} = (1 - vu_xc^{-2})^{-1}$ in the standard way, i.e. consistent with condition b) above, by exchanging corresponding primed and unprimed values and changing v to –v: hence, $\eta' = (1 + vu_x'c^{-2})^{-1} = (1 + vc^{-2}\Delta x/\Delta t)^{-1}$. The value of γ remains the same because it is a function of v^2, and the value of Q' = Q^{-1} is fixed by forming the inverse of eq. (7a), i.e. $\Delta t = Q'^{-1} \Delta t' = Q \Delta t'$. *Thus, Q and Q' bear a reciprocal relationship to one another, as one expects from an objective theory of measurement.*

Substitution in eq. (14) thereby simplifies the condition of relativistic invariance to:

$$\eta^2\eta'^2\gamma^{-4} = 1. \tag{15}$$

From the definitions of η and η', it follows that [17]

$$\eta\eta' = \gamma^2 \tag{16}$$

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This result is obtained by using eq. (9a) of the RVT to define u_x' in η' in terms of u_x. It is obviously compatible with eq. (15), as required by the RP.

The above discussion demonstrates *that space-time mixing is not essential to satisfy the RP.* The direct proportionality assumed in eq. (7a) between the respective clock rates in S and S' is quite consistent with experimental findings, including the GPS methodology, but it also seemingly conflicts with the conventional view that all inertial systems are equivalent and therefore indistinguishable [18]. Galileo's original arguments when he introduced the RP in 1632 shed considerable light on this issue. He used the example of passengers locked in the hold of a ship who were trying to determine whether they were still located at the dock or were moving on a perfectly calm sea [19]. His main point was that it would be impossible for them to make this determination on the basis of their purely *in situ* observations. More interesting in the present context, however, is that this argument does not exclude the possibility that objects on the ship, including the passengers themselves, did not undergo changes in their physical measurements as a result of the ship's motion. Rather, the assertion is that *all such changes must be perfectly uniform*, and that this is the fundamental reason why no distinction can be observed without carrying out measurements outside the ship's hold. That interpretation is also consistent with Einstein's original work [1] in which he concluded that acceleration of a clock leads to a decrease in its rate. After the acceleration phase is concluded and a new state of motion is reached, it seems reasonable to assume that the clock's rate continues to be slower than in its original state. The RP simply states that the rates of all clocks are altered in the same proportion when they make the transition between the same two inertial systems. Similarly as with the First Law of Thermodynamics, it does not matter which intermediate states were reached in the process as long as the initial and final states are identical [20].

4. Asymmetric Time Dilation

One of the basic goals of relativity theory is to establish the relationship between the

measured values of a given quantity obtained by two observers in relative motion to each other. The LT was used to derive two key effects involving measurements of space and time variables: time dilation and FitzGerald-Lorentz length contraction (FLC). Both are characterized by a symmetry principle in STR whereby two observers in relative motion each find that the other's clock is running slower than his or the other's measuring rod is contracted relative to his. These results conform to a *subjective* view of the measurement process, i.e. it is purely a matter of perspective which clock is running slower or which meter stick is shorter.

The first experimental test of time dilation was carried out by Ives and Stilwell in 1938 with their study of the transverse Doppler effect [2]. The results confirmed Einstein's prediction [1] that the frequency of light ν_r observed in the laboratory would always be less than the value of the emitted frequency ν_e from a moving source [21]:

$$\nu_r = \gamma^{-1}\nu_e .\tag{17}$$

Note that, in agreement with Einstein's symmetry principle, the above equation implies that the measurement process is *subjective*. In this experiment, the light source was accelerated in the laboratory where the receiver is at rest. According to eq. (17), a decrease in frequency would also be observed if the tables were turned and light emitted from the laboratory were observed in the rest frame of the original moving source. In other words, each observer would find that it was the other's clock that had been slowed by time dilation as a result of their relative motion. This result was believed to be the inevitable consequence of the RP. Since the Ives-Stilwell study was only a "one-way" experiment, however, it was incapable of verifying this aspect of Einstein's prediction.

This situation was remedied with the high-speed rotor experiments carried out by Hay et al. in 1960 using the Mössbauer technique [22]. In this case it was the absorber/detector rather than the light (x-ray) source that was subject to acceleration since it was mounted on the rim of the rotor. The empirical findings for the shift in frequency $\Delta\nu/\nu$ are summarized by the formula:

$$\Delta\nu/\nu = (R_a^2 - R_s^2)\ \omega^2/2c^2,\tag{18}$$

where R_a and R_s are the respective distances of the absorber and x-ray source from the rotor axis (ω is the circular frequency of the rotor). It shows that a shift to higher frequency (blue shift) is

observed when R_a is greater than R_s, as in the present case. The corresponding result expected from eq. (17) would be:

$$\Delta \nu / \nu = \gamma^{-1}(|R_a - R_s| \, \omega) - 1 \approx -(R_a - R_s)^2 \, \omega^2 / 2c^2, \qquad\qquad (19)$$

i.e. a red shift should be observed in all cases in accordance with the symmetric interpretation of time dilation. However, the results shown in eq. (18) indicate on the contrary that the effect is *anti-symmetric*, in clear contradiction to both eq. (17) and the LT. Hay et al. [22] nonetheless declared that their results were consistent with Einstein's theory [1] without mentioning the difficulty with the prediction of the LT. They also noted that eq. (9) can be derived from Einstein's equivalence principle [23], which equates centrifugal force and the effects of gravity. Subsequent experiments by Kündig [24] and Champeney et al. [25] also found that their results were summarized by eq. (18). Kündig stated explicitly that the results confirmed the position that it is *the accelerated clock that is slowed by time dilation*, thereby asserting that the measurement process is *objective* in this experiment, contrary to the prediction of eqs. (17) and (19).

A more detailed discussion of the transverse Doppler experiments and their relation to the LT may be found in a companion publication [26]. The main conclusion in the context of the search for an internally consistent version of the Lorentz transformation is that *the amount of the time dilation increases with the speed v_{i0} of the x-ray source relative to the axis.* Specifically, it is proportional to $\gamma(v_{i0})$ [27-28]. A completely analogous result was obtained in the Hafele-Keating experiments with atomic clocks located on circumnavigating airplanes [29-30], which show clearly that it is the speed v_{i0} relative to the earth's center of mass that ultimately determines their rates. In that case the elapsed time τ_i on a given clock satisfies the relation:

$$\tau_1 \gamma(v_{10}) = \tau_2 \gamma(v_{20}). \qquad\qquad (20)$$

The Hafele-Keating experiments also provide the basis for the methodology of the Global Positioning System (GPS). It is assumed that the rates of satellite clocks satisfy eq. (20) as well as a comparable relation for the gravitational red-shift [31]. In particular, it is found that the satellite clocks run slower than their counterparts on the ground when gravitational effects are excluded. Thus, *the symmetry principle predicted by the LT is contradicted by the everyday*

operations of GPS. Exactly the same formula [26] applies to the rotor experiments [22, 24-25], in which case the axis of the rotor serves as reference for the speeds of the absorber and x-ray source that are to be inserted in the γ factors. Expansion of eq. (20) with $v_{i0} = R_i\omega$ and $\tau_i = v_i^{-1}$ leads directly to the empirical formula given in eq. (18).

Thus, eq. (20) can be called the *Universal Time-Dilation Law* (UTDL). It is a simple matter to convert this equation into the form of the GLT for time dilation given in eq. (4a), i.e. eq. (7a) of the GPS-LT. In this equation Q is the ratio of clock rates in S and S' as determined by the UTDL. Accordingly, Q>1 if the clock at rest in S' runs more slowly, and by virtue of the fundamental objectivity of the revised theory, Q<1 if it runs faster than that in S. In the typical case where the clock at rest in S' has been accelerated relative to S before returning to a state of uniform translation, $Q=\gamma(v)$. Eq. (20) is more general since it also accounts for the situation when both clocks being compared are moving relative to the original rest frame S_0. It is clearly necessary in applying eq. (20) to first identify the above rest frame; it has been referred to as the objective rest system (ORS) in earlier work [32]. The relative speed v of S and S' is not directly involved in the UTDL, thereby eliminating the subjective character of the measurement process otherwise inherent in Einstein's LT.

5. Isotropic Length Expansion

The GPS-LT and LT also differ sharply with regard to length variations. In the following example the two observers (O and O') are initially at rest in inertial system S. They each measure the diameter of a sphere and agree that it has a value of D m. O' then places the sphere on his rocket ship and moves away from O. After some time he assumes a constant relative velocity v in the common x-x' direction so that he is now at rest in inertial system S'. He then repeats the length measurements on the sphere and finds in accordance with the RP that its diameter still has a value of D m in all directions. According to the FLC, O finds that the sphere has contracted along the x direction, but that its dimensions along all perpendicular directions have remained the same. Thus,

$$\Delta y = \Delta y' = D. \tag{21}$$

There is another way to carry out these measurements, however, namely to take

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advantage of Einstein's LSP [1]. Indeed, the modern-day definition of the meter [33] as the distance traveled by a light pulse in c^{-1} s ($c=2.99792458\times10^8$ ms^{-1}) requires that the diameter be measured using clocks that are at rest in S and S', respectively. The theory assumes that the two clock rates are not the same because of time dilation on the rocket ship and therefore that the measured elapsed times for the light to traverse the sphere satisfy the GPS-LT relation of eq. (7a) with $Q = \gamma$:

$$\Delta t' = \gamma^{-1} \Delta t. \tag{22}$$

Accordingly, the above distance values have the following relation:

$$\Delta y' = c\Delta t' = c(\gamma^{-1}\Delta t) = \gamma^{-1}c\,\Delta t = \gamma^{-1}\Delta y = D. \tag{23}$$

The conclusion is that the two observers must *disagree* on their measured values for the diameter of the sphere and *by increasingly larger amounts* depending on how close their relative speed v approaches c, i.e. $\Delta y = \gamma D$ from eq. (23). This is in clear contradiction to what was determined in eq. (21) on the basis of the FLC.

It needs to be emphasized that all of the above values come directly from application of Einstein's theory [1]. There is never a question about how the corresponding measurements to obtain the various quantities mentioned in eqs. (21-23) are actually carried out in practice. For example, it might be thought that the contradiction can be removed by simply arguing that the various results are not obtained at the same time. The problem with that approach is that S and S' move with constant relative velocity and thus there is no reason to expect that any of the measured values will change with time. The above example has been referred to in earlier work [34] as the "clock riddle" to distinguish it from the far better known "clock paradox" used to illustrate the essential role of acceleration in time dilation [35].

Comparison of the two theoretical methods for length measurements in the direction parallel to **v** also uncovers another discrepancy. According to the FLC , the length of the sphere should contract on the rocket ship (S'):

$$\Delta x' = \gamma\Delta x = D. \tag{24}$$

Since the rates of clocks are independent of orientation, one expects a perfectly analogous prediction to eq. (23) in this case, namely:

$$\Delta x' = c\Delta t' = c(\gamma^{-1}\Delta t) = \gamma^{-1}c\ \Delta t = \gamma^{-1}\Delta x = D. \tag{25}$$

Instead of observing a contraction in the parallel direction, O actually finds that the sphere's diameter has *increased* by the same fraction as above ($\Delta x = \gamma D$). The conclusion is that *isotropic length expansion* accompanies time dilation in S', not the type of anisotropic length contraction expected from application of the FLC. As with eq. (3), the clear indication from this discussion is that the LT, from which the FLC is derived, is not a valid physical transformation.

The above discussion has been purely theoretical. What does experiment have to say about whether the lengths of objects expand or contract? Examination of previous claims of length-contraction observations [36] shows that they involve distributions of a large ensemble of particles such as electrons. As such, they ignore the effects of de Broglie wave-particle duality [37] which is known to produce a decrease in the wavelength of the distribution in inverse proportion to the momentum of the particles ($p = h\lambda^{-1}$). It should be noted that STR length contraction (FLC) has a substantially different dependence on the speed of particles than does the de Broglie duality [38]. For example, doubling v in the latter case leads to a reduction in the de Broglie wavelength of the particles by 50%, where if the STR length contraction is invoked, a

much smaller decrease is expected, namely by a maximum factor of $\dfrac{\gamma(2v)}{\gamma(v)} \approx 1 + 1.5v^2c^{-2}$.

A better place to begin is the Ives-Stilwell study of the transverse Doppler effect [2]. A light source with a standard wavelength λ_0 is accelerated and the wavelength λ of the radiation is measured in the laboratory. Two values are obtained for opposite directions of the light source. Averaging of these two values therefore eliminates the first-order Doppler effect caused by the motion of the light source to and from the observer, respectively. It is found that the average wavelength is *larger* than the standard value. Einstein's LSP is then assumed, from which is

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concluded that the average frequency ν measured in the laboratory is inversely proportional to the average wavelength and therefore that $\nu < \nu_0$. For example, if the speed of the light source is the same as above, this means that $\nu = \nu_0/2$. This value of the frequency is then taken to be experimental proof that clocks in the rest frame of the light source run slower than their identical counterparts in the rest frame of the laboratory, in quantitative agreement with STR. Yet, the experiment actually measures wavelengths directly and finds that they are *larger* in the laboratory than in the rest frame of the light source: $\lambda = 2\lambda_0$. The analogous conclusion that the experiment demonstrates that *lengths expand instead of contract* is never made in textbooks discussing this experiment.

Sometimes, the argument is made that the observed result can be ignored because length contraction only refers to "material objects." This conclusion overlooks the effect of Einstein's first postulate of relativity, however, the RP. It states that the observer co-moving with the light source will measure the standard wavelength value for the light source, i.e. $\lambda' = \lambda_0$, even though his colleague in the laboratory measures a larger value for the same radiation. The only rational conclusion from the RP is that the diffraction grating (or comparable measuring device) in the rest frame of the light source has increased by the same fraction as the wavelength, so no change is noticeable. The observer himself must also have experienced the same amount of length expansion in all directions since otherwise he would be able to distinguish between the two rest frames, in direct contradiction of the RP.

The situation is made clearer by considering the results of another experiment. Rossi et al. [3] showed that the *range of decay* of meta-stable particles such as muons *increases* when they are accelerated in the upper atmosphere. Because of the RP, the corresponding range must be smaller for observers co-moving with the particles. Although the original authors did not

mention it, their results have been hailed as a confirmation of length contraction in various textbooks [39,40]. The truth is that this experiment tells us just the opposite. The reason the observer moving with the muons measures smaller distances is precisely because the length of his meter stick has *increased* as a result of the acceleration. The numerical value of a measurement is inversely proportional to the unit in which it is expressed. When the meta-stable particles are produced in collisions, the rates of all clocks in their rest frame slow down and the lengths of all objects increase in the same proportion so that measured speeds of other objects are unaffected by these changes; the units of both time and distance change by the same fraction. The Rossi et al. experiment is therefore just another confirmation of isotropic length expansion accompanying time dilation, not anisotropic length contraction as the LT unquestionably predicts.

6. Conclusion

Investigation of the time equation of the Lorentz transformation (LT) shows that it requires the ratio of clock rates (t/t') in two different inertial systems to be a function of the location of the event in question. The LT therefore violates the causality principle and is consequently invalid. This analysis also shows that the only way for a space-time transformation to avoid a violation of the causality principle is to have the above ratio of clock rates be constant as long as the two rest frames continue to travel at constant velocity. This condition is shown to be consistent with Einstein's two postulates of relativity. It requires that the normalization factor in the General Lorentz transformation (GLT) introduced by Lorentz in 1898 have a value of $\varepsilon = (1-v^2c^{-2})^{0.5}(1-vxc^{-2}t^{-1})^{-1}Q^{-1}=\eta(\gamma Q)^{-1}$, where Q satisfies the clock-rate proportionality relation $t'=t/Q$. This value replaces that assumed by Einstein [1] in his original derivation of the LT,

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namely $\varepsilon = 1$.

The resulting alternative Lorentz transformation (GPS-LT) is given in eqs. (8a-d). It is consistent with the relativistic velocity transformation (RVT) introduced by Einstein in the same work. This is an important observation since some of the most important results of relativity theory such as the aberration of starlight at the zenith and the Fresnel light-drag experiment are actually obtained directly from the RVT and therefore do not depend in any way on the LT itself. On the other hand, the ALT is not consistent with a number of controversial predictions of the LT such as FitzGerald-Lorentz length contraction (FLC) and the symmetry principle which holds that two clocks can both be running slower than each other at the same time. It is found instead that time dilation occurs asymmetrically, as for example is assumed in the methodology of the Global Positioning System (GPS). Surprisingly, the GPS-LT also indicates that isotropic length expansion accompanies time dilation in a given rest frame. The latter prediction finds confirmation in the results of the Ives-Stilwell transverse Doppler study and the Rossi et al. measurements of the average length of decay of accelerated muons. It is also obviously consistent with the requirement of the light-speed postulate that the wavelength of light always be proportional to the corresponding period.

Ultimately, the main result of the present study is that a consistent version of relativity theory can be formulated on the basis of the assumption of a strict proportionality between clock rates in different inertial systems. When clock rates slow, both the unit of time and the unit of distance increase by the same fraction so that the corresponding unit of velocity is not changed. This relationship is clearly consistent with the LSP. The clock-rate proportionality factors satisfy a Universal Time-Dilation Law [see eq. (20)]. A key difference between the GPS-LT/RVT version of relativity and that employing the LT is that the speeds of clocks must be measured

relative to a definite rest frame (objective rest system ORS) in order to compute the necessary time-dilation factors from the UTDL. This conclusion is consistent with the Hafele-Keating measurements of the rates of atomic clocks carried onboard circumnavigating airplanes, for example, and also with the results of the transverse Doppler studies employing high-speed rotors.

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