Newton's Law of Inertia and Clock-rate Proportionality: Einstein's Historic Mistake

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Abstract

A straightforward extension of Newton's First Law leads to the conclusion that the properties of objects in pure translation should remain constant for an infinite period of time. The Principle of Causality demands this because of the absence of unbalanced forces which might otherwise affect their values. Consequently, one expects that the rates of two proper clocks in different inertial systems *should remain strictly proportional to one another*. It is shown that this conclusion is inconsistent with the Lorentz transformation (LT) of Einstein's Special Theory of Relativity (STR), which holds that two clocks in relative uniform motion must each be running slower than the other (Einstein's Symmetry Principle). Instead, time dilation is expected to be asymmetric, i.e. it is always possible in principle to know which of two clocks is running slower (Universal Time-Dilation Law). Transverse Doppler measurements that have been carried out with high-speed rotors, as well as the study of the rates of circumnavigating atomic clocks carried out by Hafele and Keating in 1971, are in quantitative agreement with clock-rate proportionality in different rest frames. The same relationship is assumed in the operation of the Global Positioning System (GPS) between elapsed times for a given event that are measured on satellites and corresponding values obtained with atomic clocks located on the earth's surface. It is shown that there is an alternative version of the Lorentz transformation, designated the GPS-LT, which incorporates the proportionality of clock rates and elapsed times. It nonetheless satisfies both of Einstein's postulates of relativity and is also consistent with the relativistic velocity transformation (RVT) derived in his original work. The GPS-LT differs from the original LT in that it eschews space-time mixing and is consistent with the absolute simultaneity of remote events enunciated by Newton and his contemporaries. It is also characterized by a different relationship between space and time coordinates than the well-known LT condition of Lorentz invariance. An amended version of the Relativity Principle (RP) is proposed on this basis which assumes that the *units in which the laws of physics are expressed* differ from one inertial system to another. The GPS-LT is therefore consistent with the *uniform scaling* of corresponding physical properties in different rest frames. Finally, it predicts *that isotropic length expansion accompanies time dilation* in a given rest frame, not the familiar type of anisotropic length contraction posited by STR. Experimental evidence supporting this conclusion is obtained from the Ives-Stilwell transverse Doppler experiment and studies of the range of decay of meta-stable particles.

Keywords: Einstein's Symmetry Principle (ESP), asymmetric time dilation, clock-rate proportionality, Universal Time-Dilation Law (UTDL), Lorentz transformation (LT), relativistic velocity transformation (RVT), alternative Global Positioning System-Lorentz transformation

(GPS-LT), absolute remote simultaneity, isotropic length expansion, uniform scaling of physical properties, amended version of the Relativity Principle (RP)

I. Introduction

The First Law of Kinematics (Law of Inertia) states that objects move with constant velocity in the absence of unbalanced forces. The present work examines relationships between the properties of objects which are at rest in different inertial systems, i.e. in states of uniform translation. This analysis points out a heretofore unnoticed connection between the above law of Newtonian physics and Einstein′s Special Theory of Relativity (STR) [1].

II. Clock-rate Proportionality

Consider Clock A moving at constant velocity. Since there are no unbalanced forces, it is clear that its rate will remain constant for an infinite period of time under these conditions. Similarly, Clock B, identical to Clock A but at rest in a different inertial system, will also have a constant rate, but not necessarily the same as for Clock A. Indeed, one of the key predictions of Einstein′s original paper on relativity [1] is *time dilation*, the possibility that the rates of clocks may differ from one inertial system to another. However, it follows from the above considerations that their *ratio must be constant* since neither of the two individual rates changes with time. This relationship will be referred to in the following as *clock-rate proportionality*.

Suppose then that Clock A runs Q times faster than Clock B, i.e. $Q > 1$ *is time-independent.* When both clocks are used to measure the elapsed time of the same event, the value obtained with Clock A will be Q times greater than that found using Clock B. For example, Clock A at rest on the earth's surface would find that the time required for the earth to revolve once about its axis is $\Delta T_A = 86400$ s (24 hrs). If $Q = 2$, the corresponding time ΔT_B measured on Clock B would be only half this amount, namely 43200 s. Experimental data obtained with circumnavigating airplanes [2,3] are consistent with this analysis. The results show that the rates of clocks flying in different directions around the globe, and also of one at the airport of departure and return, are indeed strictly proportional to one another.

By contrast, relationships of elapsed times Δt, $\Delta t'$ measured by respective observers in two different inertial systems S and S' are predicted from STR [1] by using the following equation from the Lorentz transformation (LT):

$$\Delta t' = \gamma \left(\Delta t - v \Delta x c^{-2} \right) = \gamma\, \eta^{-1} \Delta t \,. \tag{1}$$

The equation is given in terms of intervals of space Δx, Δy and Δz and time Δt and their primed counterparts, i.e. $\Delta x = x_2 - x_1$, $\Delta x' = x_2' - x_1'$ etc. for two events [c is the speed of light, v is the relative speed of the participating inertial systems S and S' moving along a common x,x′ coordinate axis and $\gamma = \left(1 - v^2 c^{-2}\right)^{-0.5}$. In addition, the quantity η is defined in eq. (1) as

$$\left(1 - v c^{-2} \frac{\Delta x}{\Delta t}\right)^{-1}.$$

As a typical example, consider the case of an object moving along the x axis with speed $u = \Delta x / \Delta t$. A stationary observer in S measures the value of the distance traveled to be Δx in the corresponding elapsed time Δt. Eq.(1) is then used to determine the corresponding elapsed time measured by the stationary observer in S'. The ratio of these elapsed times is:

$$\frac{\Delta t'}{\Delta t} = \gamma \left(1 - vc^{-2}u_x\right) = \gamma \, \eta^{-1} \quad . \tag{2}$$

The above ratio is seen to depend on the object′s velocity component u_x in the x-direction. This result is certainly not intuitive since the same two clocks are involved in each set of measurements. The usual explanation for this in STR is to say that a given event occurs at a different time for the stationary observer in S than for his counterpart at rest in S' (remote non-simultaneity of events). Clock-rate proportionality in S and S', which one expects from Newton′s First Law for inertial systems, rejects this possibility, however. It says that a stationary proper clock A in S *can be adjusted so that its rate is always equal to that of an identical proper clock A' at rest in S'*. As a consequence, elapsed times for the same two events read from the adjusted clock that is stationary in S must be identical to those read from the stationary proper clock A' in S'. In this way, it is seen that the ratio of elapsed times in eq. (2) is equal to that of the respective clock rates. Since the latter ratio must be a constant, it follows that *eq. (2) does not conform to physical reality and therefore that the LT itself may be invalid.*

The adjustment procedure described above is used successively (pre-correction technique [4]) in the timing procedures of the Global Positioning System (GPS) and thus enjoys overwhelming confirmation from experiment. That in itself is not surprising because failure to obtain this level of agreement would constitute proof that the rates of stationary clocks in inertial systems can change with time in spite of the absence of external forces. It is well to note at this juncture that experiments at CERN [5,6] have shown conclusively that the *level of acceleration of clocks is immaterial in determining their rates*, so even the presence of small forces acting on the GPS satellites does not alter the above conclusion.

It is therefore necessary to replace the elapsed-time equation of the LT for two inertial systems with one that is consistent with Newton's Law of Inertia. This is accomplished with the relation given below, namely:

$$\Delta t' = Q^{-1}\Delta t \,, \tag{3}$$

where Q is a constant depending only on the relationship between the two inertial systems. Before discussing how Q can be calculated on a general basis, it is first well to note how eq. (3) differs from the well-known time-dilation relations of STR [1] for inertial systems:

$$\Delta t' = \gamma \Delta t \,, \tag{4a}$$

$$\Delta t = \gamma \Delta t' \,. \tag{4b}$$

In this case, *two equations* are given to emphasize that time dilation is *symmetric* in Einstein's theory. A stationary observer in S must find that his clock runs faster than its identical counterpart at rest in S', but another observer at rest in S' must find that it is his identical clock which runs faster. Eq. (3) on the other hand corresponds to *asymmetric* time dilation. Whether the clocks in S run slower or faster than those in S' depends solely on whether the constant Q is less than unity or greater than unity.

It also should be clear that the γ proportionality factors in eqs. (4a-b) depend directly on v, the relative speed of S and S'. The same is not true for eq. (3). In order to calculate Q, more information is needed than just the value of v. The experiments of Hafele-Keating [2,3] indicate a definite method for accomplishing this objective. First, one needs to identify a particular reference frame [the earth's center of mass (ECM) is this case] with respect to which the relative speeds v_{i0} of the various clocks are determined. In previous work [7,8] this reference has been designated as the objective rest system (ORS). The results of the HK study [2,3] then indicate that the rates of the clocks are inversely proportional to the respective values of $\gamma(v_{i0})$. It is assumed that the elapsed times Δt_1 and Δt_2 measured on two clocks for a given event satisfy the following relation:

$$\Delta t_1 \gamma(v_{10}) = \Delta t_2 \gamma(v_{20}) \quad . \tag{5}$$

In the notation of eq. (3), one therefore finds that

$$Q = \frac{\gamma(v_0')}{\gamma(v_0)} \quad , \tag{6}$$

where v_0 and v'_0 are the speeds relative to the ORS of identical clocks at rest in S and S', respectively. Note that the relative speed v of S and S' does not occur in eq. (6). As a result, the constant Q is a ratio that can take on values that are greater than unity as well as less than unity, thereby determining whether $\Delta t > \Delta t'$ or $\Delta t < \Delta t'$ in eq. (3). HK [2] found, for example, that a clock moving eastward returned with less elapsed time than the clock left behind at the airport from which the flight originated, whereas the clock flying in a westerly direction showed more elapsed time than the airport clock.

Analogous results were found in transverse Doppler studies carried out with high-speed rotors in 1960 [9-11]. The empirical formula for second-order frequency shifts $\Delta\nu$ is:

$$\frac{\Delta \nu}{\nu} = \left(R_a^{\,2} - R_s^{\,2}\right) \frac{\omega^2}{2c^2} \quad , \tag{7}$$

where R_a and R_s are the radial locations on the rotor axis of an x-ray absorber and source, respectively, and ω is the rotational speed. This equation agrees to within first-order in ω^2 / c^2 with eq. (5) [8, 12, 13]. The rotor axis serves as the ORS in this case. The same equation is applicable to Einstein's example [1] of an electron moving in a closed path and returning to the point (the ORS in this case) at which it experienced acceleration due to an applied force.

Consequently, eq. (5) can be looked upon as the Universal Time-Dilation Law (UTDL) [8]. It is directly related to Newton's Law of Inertia and clock-rate proportionality.

The inverse of eq. (3) is given below as:

$$\Delta t = Q\Delta t' = Q'^{-1}\Delta t' . \tag{8}$$

This equation is obtained by algebraic manipulation of eq. (3), unlike the situation for the STR relations in eqs. (2a-b). As a consequence, the measurement of elapsed times is seen to be *objective* when clock-rate proportionality is assumed in eqs. (3, 8), i.e. all observers agree on which clock is running slower.

This condition makes it possible to think of the constant Q as a conversion factor between the units of time in S and S'. The corresponding conversion factor Q' in eq. (8) in the opposite direction is simply the *reciprocal* of Q, similarly as for conversion factors between cm and m or between one currency and another. By contrast, eqs. (2a-b) rule out any possibility of defining a unit of time in one inertial system which is consistent with that in another. Measurement is *subjective* in STR because of its prediction of symmetric time dilation, i.e. the results depend on the perspective of the observer.

The fact that the unit of time is not the same in each rest frame suggests an amended form [14] of the relativity principle (RP): The laws of physics are the same in all inertial systems *but the units in which they are expressed can and do vary in a systematic manner* from one rest frame to another. When an observer changes his state of motion, he is incapable of detecting any variation in physical units based on *in situ* measurements alone because all properties are subject to a *uniform scaling*. An observer in another rest frame can notice the changes in the other, however, and this is the basis for the quantitative measurements of time dilation already mentioned [2, 3, 9-11]. The uniform scaling of physical properties other than time is discussed elsewhere [15,16] and will be taken up in Sect. IV. There is also a separate scaling of properties due to gravitational forces [15].

It is evident from eq. (3) that remote non-simultaneity of events is ruled out by clock-rate proportionality. Since Q is a constant, it follows that if two events occur simultaneously in one rest frame ($\Delta t = 0$), they also must occur simultaneously in the other ($\Delta t' = 0$). The example of lightning flashes on a moving train has famously been used in STR [17] to justify its prediction of remote non-simultaneity, but *a different outcome which is consistent with absolute remote simultaneity* has also been shown to result when the analysis is based on the relativistic velocity transformation (RVT) and the light-speed postulate [18], both of which are consistent with eq. (3), as discussed in the following section. Again, in order to believe in remote non-simultaneity it is necessary to argue that the rates of clocks can change while they are in uniform translation, which is tantamount to saying that the velocities of objects do not conform to Newton's Law of Inertia.

It is also obvious from eq. (3) that it rules out space-time mixing [19, 20], contrary to what is deduced on the basis of the LT's eq. (1). Newton and classical physicists in general were of the strong belief that time is fundamentally different from space. Einstein [1] changed that view among modern-day physicists solely on the basis of eq. (1), which indicates that whenever $v \neq 0$, the difference in the times of two events as measured by one observer (Δt') will depend on where the events occurred (Δx) as well as on the corresponding time (Δt) for the other. The concept of "spacetime" as a single entity, which is quite important in modern-day theories of cosmology (for example in string theory [21]), is derived solely on the basis of eq. (1) and the belief that *only* the LT can satisfy both of Einstein's postulates of relativity [1]. However, the discussion in the following section will show that there is another space-time transformation that satisfies these postulates, one which is also compatible with the clock-rate proportionality expressed by eq. (3) and therefore also with Newton's Law of Inertia.

III. The GPS-compatible Version of the Lorentz Transformation

Einstein's original derivation [1] of the Lorentz transformation was closely related to that given in earlier work of Lorentz [22] in which he defined a general form for a space-time transformation that leaves Maxwell's equations invariant. This set of four equations will be referred to as the General Lorentz Transformation (GLT). The equations are given in terms of space and time intervals for two events as observed in different inertial systems S and S' (see the discussion after eq. (1) for definitions of the various constants and variables that appear therein):

$$\Delta t' = \gamma\varepsilon\left(\Delta t - v\Delta x c^{-2}\right) = \gamma\varepsilon\eta^{-1}\Delta t \tag{9a}$$

$$\Delta x' = \gamma\varepsilon\left(\Delta x - v\Delta t\right) \tag{9b}$$

$$\Delta y' = \varepsilon\Delta y \tag{9c}$$

$$\Delta z' = \varepsilon\Delta z\ . \tag{9d}$$

Einstein gives an equivalent set of equations in Ref. 1, but uses a specific definition of coordinate systems for S and S' such that the respective origins coincide when $t = t' = 0$. The right-hand side of each equation contains a common factor (normalization function) referred to as ε above (φ is used instead in Ref. 1). The GLT satisfies Einstein's second postulate of relativity, the constancy of the speed of light (LSP), but not necessarily his first postulate, the relativity principle (RP). In order to have a uniquely defined relativistic space-time transformation to replace the classical Galilean transformation, it is clearly necessary to have a definite value for the normalization function. To achieve this end, Einstein argued *without proof* that ε could only be a function of v, the relative speed of S and S'. He then showed [1] that the choice of $\varepsilon = 1$ satisfied this condition. Substitution of this value in eqs. (9a-d) then leads to the LT. The same choice also satisfies the RP. Note that eq. (9a) with $\varepsilon = 1$ is identical with eq. (1), however, which has been shown to be incompatible with both Newton's First Law and the observation of asymmetric time dilation in experimental tests [2-6].

What is needed instead is a space-time transformation that is compatible with the clock-rate proportionality relation in eq. (3) as well as with both of Einstein's postulates of relativity. The GLT in eqs. (9a-d) offers a clear first step. Any version of the GLT is guaranteed to satisfy the LSP for any choice of the normalization function ε. This degree of freedom enables one to include eq. (3) in relativity theory as a *third postulate* [23, 24]. One simply needs to combine it with eq. (9a) of the GLT as follows:

$$\Delta t' = \gamma\varepsilon\left(\Delta t - v\Delta x c^{-2}\right) = \gamma\varepsilon\eta^{-1}\Delta t \ = Q^{-1}\Delta t \quad , \tag{10}$$

from which follows

$$\varepsilon = \eta\left(\gamma Q\right)^{-1} \quad . \tag{11}$$

Substitution of this value for the normalization function in the GLT of eqs. (9a-d) then gives the desired alternative space-time transformation given below:

$$\Delta t' = \frac{\Delta t}{Q} \tag{12a}$$

$$\Delta x' = \eta\left(\Delta x - v\Delta t\right) \tag{12b}$$

$$\Delta y' = \frac{\eta\Delta y}{\gamma Q} \tag{12c}$$

$$\Delta z' = \frac{\eta\Delta z}{\gamma Q} \quad . \tag{12d}$$

In the following discussion this transformation will be referred to as the GPS-LT because of its compatibility with the clock-rate proportionality assumption employed in the GPS technology. Note that eq. (12a) is identical with eq. (3). It also can be seen that η only appears in the three space-type relations in eqs. (12b-d), thereby avoiding the space-time mixing that occurs in eq. (1) of the LT.

There are two major points to be considered in detail regarding the GPS-LT. First of all, the same relativistic velocity transformation (RVT) is obtained as in Einstein's original work [1] when Δx', Δy' and Δz' in eqs. (12b-d) are each divided by Δt' in eq. (12a). The result in each case is given below, with the velocity components defined as $u_x' = \Delta x' / \Delta t'$, $u_x = \Delta x / \Delta t$, etc.:

$$u_x' = \left(1 - v u_x c^{-2}\right)^{-1} \left(u_x - v\right) = \eta \left(u_x - v\right) \tag{13a}$$

$$u_y' = \gamma^{-1} \left(1 - v u_x c^{-2}\right)^{-1} u_y = \eta \gamma^{-1} u_y \tag{13b}$$

$$u_z' = \gamma^{-1} \left(1 - v u_x c^{-2}\right)^{-1} u_z = \eta \gamma^{-1} u_z . \tag{13c}$$

The normalization factor ε in the GLT of eqs. (9a-d) is simply cancelled out in each of the corresponding divisions, so the RVT is compatible with any transformation of this type, including the LT with $\varepsilon = 1$. Therefore, ε does not appear at all in the RVT, whereas η appears in all three equations by virtue of eq. (9a) of the GLT.

A number of the most important results of relativity theory actually result directly from the RVT, and thus do not rely in any way on Einstein's erroneous assumption about the normalization factor. These include the aberration of starlight at the zenith [25] and the Fresnel light-drag experiment [26], both of which were quite important in Einstein's thought process [27]. The RVT also guarantees compliance with the LSP. It is used directly in the derivation of the Thomas precession of a spinning electron [28, 29] and thus the LT is not essential in this case either. Moreover, the proof that Maxwell's equations are invariant to the GLT in eqs.(9a-d) demonstrates that the value chosen for ε is inconsequential for this purpose as well. Indeed, it was this fact that caused Lorentz to introduce the normalization factor ε in the general transformation in the first place [22]. Note also that eqs. (12b-d) of the GPS-LT can be obtained somewhat more directly by combining eq. (3) with the RVT of eqs. (13a-c), i.e. by multiplying each of the velocity components with the corresponding times in S and S', respectively.

The second crucial point to be considered is whether the GPS-LT satisfies Einstein's first postulate, the RP. Simply choosing a definite value for the normalization function ε in the GLT of eqs. (9a-d) is sufficient for conforming to the LSP but not for satisfying the RP. For example, the first space-time transformation that is consistent with the GLT was reported by Voigt in 1887 [30, 31], but it does not satisfy the RP, as will be shown below. In order to establish a definite mathematical condition to achieve this goal, it is useful to form the squares of the GLT equations and sum them. The result is

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = \varepsilon^2 \left(x^2 + y^2 + z^2 - c^2 t^2 \right) . \tag{14}$$

The inverse of the GLT is obtained in the usual way by interchanging the primed and unprimed symbols for the two rest frames and changing the sign of their relative speed v. Note that in this way a new function ε' must be defined that is obtained by performing the above operations on the original normalization function ε. The inverse of eq. (14) is thus obtained as

$$x^2 + y^2 + z^2 - c^2 t^2 = \varepsilon'^2 \left(x'^2 + y'^2 + z'^2 - c^2 t'^2 \right) . \tag{15}$$

Dividing both sides of eq. (15) by ε'^2 and comparing with eq. (14) shows that there is a clear condition for satisfying the RP, namely ε'^2 must be equal to ε^{-2}. In order for the GLT to be valid in the low-velocity regime (Galilean transformation), it is necessary that both ε and ε' are real and positive, from which one concludes that the condition for satisfying the RP is

$$\varepsilon\varepsilon' = 1. \tag{16}$$

The LT, with its value of $\varepsilon = 1$, meets this criterion since coordinate inversion leads to the corresponding value of $\varepsilon' = 1$. It also obviously ensures the desired symmetry between the two rest frames required by the RP. This fact has led to a strong belief in the physics community that the LT is *unique* in this respect. For example, the value of $\varepsilon = \gamma^{-1}$ originally proposed by Voigt [30] does not satisfy eq. (16) because $\varepsilon' = \gamma^{-1}$ as well.

In order to satisfy the RP and eq. (16), the GPS-LT in eqs. (12a-d), with the value of ε given in eq. (11), requires instead that

$$\eta\eta' = \gamma^2 \mathrm{QQ}'. \tag{17}$$

However, comparison of eq. (12a) with its inverse, $\Delta t = \Delta t' / Q'$, shows that $QQ' = 1$, consistent with eq. (8), so the condition for satisfying the RP reduces to:

$$\eta\eta' = \gamma^2. \tag{18}$$

The definition of η has been given above in connection with eq. (1) in terms of the ratio (speed) $u_x = \Delta x / \Delta t$. The corresponding value for the inverse function η' is obtained as ($1 + vc^{-2}\Delta x' / \Delta t'$)$^{-1}$. Hence, with $u'_x = \Delta x' / \Delta t'$,

$$\eta\eta' = \left[\left(1 - vu_x c^{-2}\right)\left(1 + vu'_x c^{-2}\right)\right]^{-1} = \left(1 - vu_x c^{-2}\right)^{-1}\left[1 \ + \ v\eta\left(u_x - v\right)c^{-2}\right]^{-1}$$

$$= c^4\left(c^2 - vu_x\right)^{-1}\left[c^2 + v\left(u_x - v\right)\left(1 - vu_x c^{-2}\right)^{-1}\right]^{-1}$$

$$
\begin{aligned}
&= c^4\left(1 - vc^{-2}u_x\right)\left[\left(c^2 - vu_x\right)\left(c^2 - vu_x + vu_x - v^2\right)\right]^{-1} \\
&= \left(c^4 - vc^2u_x\right)\left[\left(c^2 - vu_x\right)\left(c^2 - v^2\right)\right]^{-1} \\
&= \left(c^2 - vu_x\right)\left[\left(c^2 - vu_x\right)\left(1 - v^2c^{-2}\right)\right]^{-1} \\
&= \left(1 - v^2c^{-2}\right)^{-1} = \gamma^2 \; .
\end{aligned}
\tag{19}
$$

The condition of eq. (18) is indeed satisfied. Note that eq. (13a) of the RVT is used in the first step of eq. (19) to eliminate u_x' so that only the unprimed velocity component u_x remains. The GPS-LT of eqs. (12a-d) is thus shown to satisfy the RP as well as the other of Einstein's relativity postulates, the LSP. Unlike the LT, however, the GPS-LT is also consistent with Newton's Law of Inertia, which demands that the rates of identical clocks in different inertial rest frames have fixed ratios. It is therefore consistent with all tests of time dilation as yet carried out. They show unequivocally that the clock-rate proportionality relation of eqs. (3, 12a) is essential for describing space-time relationships.

IV. Isotropic Length Expansion and Uniform Scaling of Properties

It has already been mentioned in Sect. II that there is merit in regarding asymmetric (but not symmetric) time dilation as a change in the unit of time in the two rest frames of interest (S and S'). Accordingly, the parameter Q in the GPS-LT in eqs. (12a-d) can be looked upon as a *conversion factor* between these two units, as indicated explicitly in eq. (3). In the following, it will be shown that the same approach applies to all other physical properties.

To begin this discussion, it is well to consider the consequences of continuing to satisfy the LSP when the unit of time changes. Since stationary observers in both rest frames agree on the value of the light speed, it can be concluded that, despite the change in the unit of time, the unit of velocity remains constant. It is important in this connection to make sure that the two observers are measuring the same quantity when they compare their measured values. An example is useful in this regard. Consider a satellite (S') moving along the x axis with speed v relative to a stationary observer on the earth (S). An object on the satellite is measured by the local observer there to move from A to B with speed w along the x axis. From his perspective the stationary observer in S measures the corresponding speed of the object *relative to him* to be $w_S = (v + w) / (1 + vwc^{-2})$, in agreement with eq. (13a) of the RVT. This (w_S) is not the speed of the object moving on the satellite relative to its starting point A, however. To obtain this value in his units, the stationary observer in S must take into account that point A is moving with speed v relative to him. He then must use the RVT to compute the difference between w_S and v. That value is found to be w [32], in agreement with the measured speed of the object from the vantage point of the stationary observer on the satellite. To distinguish between this value and w_S, it is useful to refer to w as the "relative speed" of the object to its starting point (both of which are moving relative to S), whereas w_S is the speed of the object relative to the stationary observer in S.

This result holds for any other direction of the object's motion on the satellite [33]. The required generalization of the above example is discussed below in which an object is observed to travel at an angle θ relative to **v**, the velocity of separation of S and S'. The stationary observer is S' finds that the object moves with speed w, with components $u'_x = w\cos\theta$ in the parallel (x) direction and $u'_y = w\sin\theta$ in the perpendicular direction (y). The RVT of eqs. (13a-c) can be used to obtain the corresponding velocity **u** for the object's motion *relative to the S'* *observer* from the vantage point of the stationary observer in S. There are two steps to be followed for each component of **u**: first, the velocity $\mathbf{w}_S$ of the object relative to S is computed; secondly, the desired relative velocity **u** of the object to S' is obtained based on the value of $\mathbf{w}_S$. The RVT eq. (13a) gives the following value for the x-component of w_S, namely $w_{Sx} = (w\cos\theta + v)(1 + wvc^{-2}\cos\theta)^{-1}$. Next, the difference u_x between w_{Sx} and v needs to be computed, again using eq. (13a), and not simply subtracting v from w_{Sx}:

$$u_x = (w_{Sx} - v)(1 - vw_{Sx}c^{-2})^{-1}$$

$$= \left[(w\cos\theta + v)(1 + wvc^{-2}\cos\theta)^{-1} - v\right]\left[1 - vc^{-2}(v + w\cos\theta)(1 + wvc^{-2}\cos\theta)^{-1}\right]^{-1} \quad (20)$$

$$= (w\cos\theta + v - v - wv^2c^{-2}\cos\theta)(1 + wvc^{-2}\cos\theta - v^2c^{-2} - vwc^{-2}\cos\theta)^{-1}$$

$$= w\cos\theta(1 - v^2c^{-2})(1 - v^2c^{-2})^{-1} = w\cos\theta$$

$$= u'_x .$$

The corresponding calculation of u_y using the RVT eq. 13b in two steps is given next, whereby

$$w_{Sy} = w\sin\theta(1 - v^2c^{-2})^{0.5}(1 + wvc^{-2}\cos\theta)^{-1}$$

$$u_y = w_{Sy}(1 - v^2c^{-2})^{0.5}\left[1 - vc^{-2}(v + w\cos\theta)(1 + wvc^{-2}\cos\theta)^{-1}\right]^{-1}$$

$$= w\sin\theta(1 - v^2c^{-2})(1 + wvc^{-2}\cos\theta - v^2c^{-2} - vwc^{-2}\cos\theta)^{-1} \quad (21)$$

$$= w\sin\theta(1 - v^2c^{-2})(1 - v^2c^{-2})^{-1} = w\sin\theta$$

$= u'_y$.

It is therefore seen that both components of **u** and **u'** are equal, as was to be proved.

In other words, the RVT leads to the conclusion that two observers always agree on the *relative velocity (speed and direction) of an object to its starting point (or to another object).* This finding is thus consistent with the above conclusion based on the LSP that the unit of velocity is the same for the respective stationary observers in S and S', even though their units of time differ by a factor of Q.

The above conclusion about the unit of velocity determines the value of the corresponding conversion factor for distance. The fact that speed is defined as a ratio between distance travelled in a given elapsed time implies that the conversion factor for the unit of distance is exactly the same as for the unit of time, namely Q. A useful way of expressing these relationships is to say that both time and distance *scale* as Q whereas the unit of speed is constant, i.e. scales as Q^0.

It is important to see that the above conclusion about the way distance scales is at odds with the conventional interpretation based on STR. In that case it is predicted that FitzGerald-Lorentz length contraction (FLC) [34] occurs for stationary objects in a rest frame in which time dilation has also taken place. It is possible to use the LSP to obtain the opposite result, however. Suppose that a local observer on a satellite (S') measures the length of an object by sending a light pulse between its two ends. He finds that the elapsed time on his clock is Δt', so the LSP allows him to conclude that the length of the object is $L' = c\Delta t'$. An observer on the earth can use the GPS-LT to make the corresponding length measurement from his perspective. In accordance with eq. (12a), he concludes that the elapsed time on his clock is $\Delta t = Q\Delta t'$. Since the speed of light on the satellite is also equal to c for him because of the LSP, he can use the above result to obtain the length L of the object on the satellite without actually being there, namely by multiplying his value for the elapsed time by c. His result is therefore $L = c\Delta t = c\left(Q\Delta t'\right) = Q\left(c\Delta t'\right) = QL'$. If there is time dilation on the satellite, then $Q > 1$, from which he concludes that $L > L'$. Therefore, the GPS-LT is seen to lead to the following general result: *length expansion accompanies time dilation in a given rest frame*. Moreover, the amount of the expansion is independent of the orientation of the object because the LSP assures him that the speed of light is the same in all directions. The overall conclusion is that isotropic length expansion accompanies time dilation, not the type of asymmetric length contraction predicted by the FLC of STR.

The ultimate judge of whether lengths expand isotropically or contract anisotropically is experiment. It is more difficult to deal with distance variations than with time dilation for the simple reason that clocks have "memories" whereas length-measuring devices do not. A clock that has been in motion for a period of time can be brought back to its original position and its elapsed time can be compared with that of stationary clocks left behind there. The only way to obtain useful information about length variations is to *combine experiment with a deduction process based on hopefully reliable theory*. Fortunately, the LSP and the RP qualify in this respect, so all one needs is experimental information that can be uniquely interpreted using these two theoretical tools. The Ives-Stilwell study of the transverse Doppler effect [35, 36] is a case in point. The experiment consists of measuring the wavelength of light emanating from an accelerated source. Results are obtained for opposite directions of the light source moving relative to the laboratory. Averaging of these two measured values for the wavelength eliminates the first-order Doppler effect and therefore enables the determination of the shift caused by relativistic effects. It is found that the average wavelength λ is greater than the standard value λ_0

for a given light source, specifically that $\lambda = \gamma\lambda_0$. Ives and Stilwell [35] then used the LSP to conclude that the frequency of light ν coming from the accelerated source was less than the standard value ν, i.e. $\nu < \nu_0$.

This result was interpreted as verification of time dilation in the rest frame of the light source, as indeed seems justified. A corresponding conclusion about length variations in the same rest frame was not made, however, even though the inference is quite straightforward: the experiment indicates unequivocally that the wavelength of light is greater in the rest frame of the light source than in the laboratory, thereby providing a clear verification of *length expansion accompanying time dilation.* The reason for ignoring this conclusion in the past seems quite clear. It is because the conclusion stands in contradiction to the FLC prediction of length contraction.

Subsequently, it has been argued that the increase in wavelength should not be construed as a violation of STR because this theory supposedly only applies to material objects. It is interesting that the same reservation was not made for light frequencies, but there is a much stronger argument against this assertion [37]. According to the RP, an observer co-moving with the accelerated light source must measure the standard wavelength value λ_0, whereas the laboratory observer finds a larger value, i.e. $\lambda > \lambda_0$, for the same light source. The only rational explanation for this difference is that the measuring devices (spectrometers) employed in the two rest frames *are also not the same*. In order for the standard wavelength value to be obtained *in situ*, the measuring device there must also have increased by the same fraction in all directions as the wavelength. The spectrometer is made of metal so the increase in its dimensions cannot be put off as an exception to the FLC prediction for material objects. Indeed, the RP indicates that the observer himself has increased in size as a result of the change in rest frame [38].

Analysis of another experiment leads to the same conclusion. Rossi et al. [39] showed that the range of decay of meta-stable particles such as muons *increases* when they are accelerated in the upper atmosphere. Because of the RP, the corresponding range must be smaller for observers moving with the particles. Although the original authors did not mention it, their results have been hailed as a confirmation of the FLC [40-42]. The truth is that this experiment tells us just the opposite. The reason the observer co-moving with the muons measures smaller distances is because *the length of his meter stick has increased* as a result of the acceleration, and in exactly the same proportion as the lengths of everything else that is stationary in the rest frame of the muons. *The numerical value of a measurement is inversely proportional to the unit in which it is expressed.* When the meta-stable particles are produced in collisions, the rates of all clocks in their rest frame slow down and the corresponding length-measuring devices expand in the same proportion (giving both smaller distances and smaller elapsed times) so that the measured speeds of other objects are unaffected by these changes. The Rossi et al. experiment [39] is therefore another confirmation of isotropic length expansion accompanying time dilation, not anisotropic length contraction as the FLC and LT predict.

Examination of previous claims of length-contraction observations [40, 43] shows that they involve distributions of a large ensemble of particles such as electrons. As such, the claims ignore the effects of de Broglie wave-particle duality [44], which is known to produce a decrease in the wavelength of the distribution in inverse proportion to the momentum of the particles ($\mathrm{p} = \mathrm{h}\lambda^{-1}$) [45]. It should be noted that the FLC has a substantially different dependence on the speed of particles than does the above de Broglie relation. For example, doubling v in the latter case leads to a reduction in the de Broglie wavelength of the particles by 50%, whereas if the FLC is assumed, a much smaller decrease is expected, namely by a maximum factor of $\gamma(2v)/\gamma(v) \approx 1+1.5v^2c^{-2}$. Since vc^{-1} is never greater than 10^{-6} in these experiments, it is seen that the actual effect of the FLC would not be detectable in this experiment.

The question then arises as to how the concept of uniform scaling applies to other physical properties. Experiments carried out shortly after Einstein′s original paper [1] indicate that the conversion factor for inertial mass is the same as for time and distance [46]. It was found that the relativistic mass m of an electron depends on its speed v relative to the laboratory according to the empirical formula: $m = \gamma(v)\, m_0$, where m_0 is the corresponding rest mass. Since the analogous dependence on v holds for elapsed times, it can be concluded in general that $m' = Q^{-1}m$, where Q has the same value as defined in eq. (6). In other words, Q is also the conversion factor for inertial mass in going from S to S'.

Once one knows the conversion (also known as scale) factors for the three fundamental physical quantities, inertial mass, time and distance, it follows that the corresponding values for all other physical properties can be deduced from their respective compositions in terms of these three quantities [15]. The conversion factors are *always equal to integral powers of Q*. Each such power is obtained by adding the individual exponents for each of the fundamental units contained in the total unit for a given property. For example, energy E has units of kgm^2s^{-2} in the mks system, so the exponent for its conversion factor is equal to $1(\text{for kg}) + 2(\text{for m}) - 2(\text{for s}) = 1$; the conversion factor is thus Q^1. The conversion factor for frequency is Q^{-1} because it has units of s^{-1}. Momentum p also has a conversion factor of +1 because it has units of kgms^{-1}. Note that this factor can only be applied for *relative* velocities and momenta according to the definition given at the beginning of this section.

A detailed discussion of the general topic of uniform property scaling is given in Ref. 16 (see Table 1). The scaling of angular momentum $l = mrv$ is particularly informative. From its mks composition it is concluded that its conversion factor is Q^2 according to the above prescription. This also means that Planck's constant h, which has the unit of angular momentum (Js), also scales as Q^2 [47]. This conclusion goes against conventional wisdom, which holds that h has the same value in all inertial systems. It needs to be kept in mind, however, that the standard assumption is that the energy E of a photon emitted from a light source increases by a factor of $\gamma(v)$ when it is accelerated to speed v relative to the laboratory, whereas the corresponding light frequency ν decreases by the same factor. If one assumes that the value of Planck's constant is unchanged in the process, the conclusion would necessarily be that the $E = h\nu$ radiation law only holds when the light source is at rest ($v = 0$) relative to the observer. However, the scaling arguments presented above say on the contrary that the value of h increases by a factor of $Q^2 = \gamma^2$ as the speed of the light source is increased and therefore that Planck's radiation law continues to hold, i.e. E varies as γ and $h\nu$ does as well (Q^2 for h times
Q^{-1} for ν).

Finally, it has been shown elsewhere [48] that the uniform scaling procedure also can be applied to electromagnetic quantities. This is possible because of an ambiguity in the standard definition of electric charge. The latter quantity can be assigned a definite composition in terms of the fundamental units in the mks system as long as a consistent choice is made for the permittivity of free space ε_0 as well. For example, one can assign a unit of Nm to electric charge e and N to ε_0, so that $e^2\varepsilon_0^{-1}$ has a unit of force (N) times the square of the distance (m^2) between two charges, thereby fulfilling Coulomb's Law [48].

V. Conclusion

Although Newton's First Law deals specifically with the motion of inertial systems, it also implies something quite important about the properties of objects in uniform translation. In the absence of unbalanced forces there is no reason why the rates of clocks should vary over time. It therefore follows that the rates of two such clocks must be strictly proportional to one another as they move at constant velocity through space. Experiments with high-speed rotors and circumnavigating airplanes have confirmed this prediction quantitatively. The clear inference from both theory and experiment is thus that time dilation is asymmetric, i.e. it is always possible in principle to say which of two clocks is running slower than the other. In the Hafele-Keating study [2, 3] it was found that the eastward-flying clock runs slower than its counterpart at the airport, for example, whereas the opposite relationship holds for the westward-flying clock. All observations are accurately described by an empirical formula given in eq. (5) which has been referred to in the present work as the Universal Time-Dilation Law (UTDL). It is the logical consequence of Newton's Law of Inertia.

These results stand in direct contradiction to the predictions of the Lorentz transformation (LT) of Einstein's STR since it predicts that time dilation is symmetric in character, whereby two clocks in motion can each be running slower than another at the same time. The LT is therefore a physically invalid space-time transformation. Its predictions of space-time mixing and remote non-simultaneity lose all credibility as a consequence. The problem with the LT can be traced directly to a false assumption Einstein made in his 1905 paper [1]. He claimed without proof that the normalization function in the general form of the Lorentz transformation (GLT) given in eqs. (9a-b) can *only* be a function of the relative speed v of the two inertial systems S and S' under consideration. This assumption is directly responsible for the mixed space-time relation of eq. (1) that leads to the erroneous prediction of symmetric time dilation and remote non-simultaneity.

Newton's First Law indicates on the contrary that all measurements of elapsed time in two different rest frames satisfy the clock-rate proportionality relation in eq. (3). Combining this with eq. (9a) of the GLT leads to an alternative value for the normalization function given in eq. (11). Inserting this value in the GLT then leads to the GPS-LT, so-named because of its compatibility with the clock-rate proportionality relation assumed in the methodology of the Global Positioning System. This version of the Lorentz transformation also satisfies both of Einstein's two postulates of relativity [1] and is compatible with the relativistic velocity transformation (RVT), which has been used successfully to describe the aberration of starlight at the zenith and the Fresnel light-drag experiment. Unlike the LT, however, it predicts that time dilation is asymmetric and is valid for all states of motion regardless of their current state of acceleration.

The GPS-LT is consistent with a perfectively objective view of the measurement process. The parameter Q contained in each of its equations can be looked upon as a conversion factor between the different units of time in the two inertial systems of interest. This characteristic again stands in stark contrast with the LT, which must eschew the definition of units because of the inherent ambiguity connected with its prediction of symmetric time dilation and other properties. The Relativity Principle (RP) can be amended on this basis to assert that the laws of physics are the same in all inertial systems *but the units in which they are expressed can and do vary from one rest frame to another*.

The conversion factors for all other properties are integral powers of Q, as determined by their respective composition in terms of the fundamental quantities of inertial mass, distance and time. The value for distance variations is also determined to be Q by virtue of the fact that the unit of velocity must be the same in all inertial systems in order to satisfy Einstein's light-speed postulate.

As a consequence, it is concluded that length expansion accompanies time dilation in a given rest frame. The latter result is confirmed on the basis of the Ives-Stilwell experimental study of the transverse Doppler effect, as well as by observations of the average range of decay of meta-stable particles. Inertial mass has the same conversion factor, based on experiments with accelerated electrons carried out shortly after Einstein's original work. Accordingly, the conversion factor for angular momentum is found to be Q^2, which is therefore the same as for Planck's constant h. On this basis, it is concluded that the energy/frequency radiation law holds for accelerated light sources and not simply for *in situ* measurements, as would otherwise not be the case if Planck's constant had the same value in all rest frames.

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