Simultaneity, Time Dilation and the Lorentz Transformation

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Abstract

The underlying basis for the predictions of non-simultaneity and time dilation in relativity theory is discussed. It is pointed out that Einstein based his conclusions entirely on the Lorentz transformation (LT). He claimed that once one assumes the constancy of the speed of light for all observers (at the same gravitational potential), there is no choice but to replace the Galilean transformation (GT) of classical physics by the LT. Lorentz pointed out as early as 1899, however, that the equations of the LT can only be specified to within a common factor ε based on this information alone. While Einstein mentioned this degree of freedom in his 1905 paper, he presented an argument that he felt settled the issue in favor of the LT (ε=1). The latter choice has the theoretical advantage of guaranteeing Lorentz invariance for the relativistic space-time transformation, but it also rules out *the principle of simultaneity of events* for observers in relative motion, a position that was revolutionary at the time he presented it. Another consequence of the LT is that it implies that *the ancient principle of the objectivity of measurement* no longer applies when two observers are in relative motion. For example, it becomes necessary to assume that each observer find that the other's clock is running slower than his own. It is pointed out that this predicted "symmetry" in the theory is actually contradicted by measurements carried out in the 1960s using high-speed rotors, as well as later with atomic clocks onboard circumnavigating airplanes in the 1970s. Moreover, the prediction of non-simultaneity of events is inconsistent with the basic assumptions employed for the Global Positioning System (GPS) technology that has become hugely successful in recent times. It is shown that relativity theory can be formulated in such a way as to remain consistent with both the objective measurement principle and simultaneity as well with the two relativity postulates simply by choosing a different value for ε than Einstein did in order to arrive at the correct relativistic space-time transformation. In agreement with Einstein, however, it does not require the existence of an ether, i.e. a unique reference frame in which the speed of light has a null value. The resulting version of relativity theory is consistent with time dilation and the modern definition of the meter, but not with the Fitzgerald-Lorentz contraction effect (FLC) derived from the LT. The revised theory also rules out the occurrence of time reversal and violations of Einstein causality, that is, that the ratio of the values measured by two observers for the respective elapsed times dt and dt' can be negative for a given event.

I. Introduction

A key aspect of the special theory of relativity (SR [1]) is its conclusion that events that occur at the same time for one observer may not be simultaneous for another. Poincaré [2] had already discussed this possibility in detail in 1898, seven years before Einstein's original work. The basis for non-simultaneity in SR is the Lorentz transformation (LT) and the fact that the speed of light is the same for two observers in relative motion to one another. However, Lorentz pointed out in 1899 [3] that the LT is not the only space-time transformation that satisfies the condition of the constancy of the speed of light for different observers in relative motion to one another. It is easy to see why this is so. Velocity is a ratio of distance traveled to elapsed time, so the equations for these quantities in the relativistic space-time transformation can be multiplied by a common factor without affecting the key light speed condition that Einstein [1] used as his second postulate. This aspect needs to be considered carefully before making any final decisions about whether events occur simultaneously for different observers or not.

II. Space-time Variables and the LT

The obvious place to begin the present discussion is the LT itself. Its precursor, the Galilean transformation (GT), was used to describe the motion of an object on a ship as perceived by two different observers. If the object moves in the same (x) direction as the ship, it was concluded that the following relationship holds:

$$dx = dx' + u\, dt'. \qquad (1)$$

The variables in the above equation are defined as follows: dx and dx' are the distances traveled by the object relative to two different origins, one fixed on the ship (dx') and the other fixed on the shore (dx); u is the speed of the ship relative to the shore and dt' is the elapsed time during which the two sets of observations are made. A key assumption was that the elapsed time observed on the shore is the same as on the ship. Hence, the second equation of the GT:

$$dt = dt'. \qquad (2)$$

Upon division of the above two equations, the corresponding relation between the two velocities in the GT results, namely,

$$dx/dt = dx'/dt' + u. \qquad (3)$$

In short, velocities are additive in the classical (Newtonian) theory.

In the present context, it is important to note that dt' must have a non-zero value for any of the above equations to be physically applicable to the problem at hand, that is, to describe the motion of an object on a platform that is itself moving relative to one of the observers. In other words, unless a finite amount of time passes, it is impossible to obtain anything meaningful from the GT. It is also clear that in this formulation it is perfectly immaterial who actually makes the measurements of elapsed time and distance traveled. The reason that dx differs from dx' in general is because the corresponding determinations are made with respect to different origins. An observer on the ship can make both measurements. Indeed, they could be made by someone else who is located neither on the ship nor on the shore. The same holds for the elapsed-time determination. The only thing that is important is that *the same units of time and distance be used* in any given comparison of the measured values of the two observers.

There is also another point that is often overlooked about the above equations. It is not necessary that the ship be traveling at a constant velocity relative to the shore (du/dt=0). The GT can be applied on an *instantaneous* basis to determine the values of dx and dt at any given time. The total distances traveled can then be obtained by appropriate integration. Neither the "ship" nor the "shore" must be an inertial system, that is, experience no outside forces. In that case, the corresponding acceleration values, d^2x/dt^2 and d^2x'/dt'^2, are not equal but simply differ by du/dt at any point in time.

Once it was demonstrated in the Michelson-Morley experiments [4] that the velocity of light is independent of the state of motion of the observer, it became necessary to alter the GT. Einstein [1] accomplished this by demanding that the speed of light be the same for both observers. He derived the LT on this basis, with the following revised equations for dx and dt:

$$dx = \gamma\,(dx' + u\,dt') \qquad (4)$$

$$dt = \gamma\,[dt' + (uc^{-2})\,dx'], \qquad (5)$$

where $\gamma = (1 - u^2c^{-2})^{-0.5}$. The other two equations of the GT (dy=dy' and dz=dz') were incorporated into the LT without change. The definition of the space-time variables themselves also remained the same as in the GT. In order to obtain his final result (the LT), however, it is necessary to impose an additional condition: $(dx^2+dy^2+dz^2 - c^2dt^2) = (dx'^2+dy'^2+dz'^2 - c^2dt'^2)$. Both sides of this equation vanish for a light pulse in free space, in accord with Einstein's postulate. This relation is referred to as the condition of *Lorentz invariance*. It is thought to be an essential component of any relativistic theory, but it also needs to be recognized that it constitutes an additional postulate in Einstein's original formulation [1], as will be discussed in the following section.

Einstein then went on to derive the time dilation and Fitzgerald-Lorentz length contraction (FLC) effects from the LT [1]. In order to do this, however, he had to depart somewhat from the traditional definition of the space-time variables. Instead of the distance traveled by an object in a given elapsed time, he referred to them as the distance and time intervals separating two unrelated physical events. For example, in order to derive the FLC it was necessary to assume that dt=0 in order to describe a situation in which the two termini of a line segment are measured simultaneously by an observer. However, it should be noted that this value for dt is excluded in the definition used to actually derive the LT as well as the GT, in which case the motion of an object is always considered over a finite period of time.

There is another problem with the derivation that affects both time dilation and the FLC, however. It is the symmetric nature of SR [1,5]. Einstein made a definite assignment for the observers implied in the LT. As mentioned above, it is not necessary to do this for the GT because it does not matter who carries out the two sets of measurements in eqs. (1,2). In Newtonian physics all observers have identical measuring rods and clocks, so their units of length and time are the same. Once the possibility presents itself whereby two observers in relative motion employ different standards for these quantities, it becomes necessary to make a definite choice as to which one of them carries out the measurements in a given case. Einstein's clear assumption was that one set is carried out by an observer located in the rest frame of the object (the primed variables in the above equations), while the other is made by a second observer moving at constant speed u relative to the first (unprimed variables). Since all inertial systems are equivalent, this means that time dilation and the FLC are not objective phenomena in SR [5,6]. Each observer thinks it is the other who is in motion and therefore both of the them must find that it is the clocks in the other's rest frame that are running slower, and that it is the measuring rods in the other's rest frame that are contracted.

Experiments with atomic clocks carried onboard circumnavigating airplanes [7] demonstrate that the above position in SR is not correct. After making appropriate gravitational corrections and taking into account the rotation of the Earth about its polar axis, it was shown that the accelerated clocks always run slower, and by predictable amounts, than their identical counterparts at rest on the polar axis. Measurements of the transverse Doppler effect with high-speed rotors [8,9] also demonstrate this lack of ambiguity as to which clock runs slower. These experiments speak against the hypothesis [10,11] that time dilation is symmetric during periods of constant motion. A more detailed discussion of these points may be found elsewhere [6].

III. Non-simultaneity, Time Dilation and Time Reversal

The LT, just as the GT before it, deals with the question of how the motion of an object is described with respect to two different origins that are themselves in relative motion. In order to have a valid comparison, it is important that both sets of results be made with reference to a common set of space-time units. For the sake of concreteness, let us assume that the object is moving along the x axis. One set of measurements relative to origin O finds that the object moved from $x=O_1$ to $x=O_2$. The difference between these two values is dx in the LT, and the corresponding elapsed time is dt. The other set is made relative to origin O' which is moving relative to O with speed u. As mentioned above, there is no need that u be constant in order to apply the LT at any given time. In the latter coordinate system the same object is observed to move from O_1' to O_2', with $dx' = O_2' - O_1'$.

The key point is that the corresponding elapsed time dt' is not generally equal to dt in the LT, as indicated by eq. (5), unlike the case in the GT where eq. (2) is valid. Let us define dx'/dt' =v', which is the speed of the object relative to O'. On this basis eq. (5) can be rewritten as

$$dt = \gamma\, dt'(1 + \mathbf{u}\cdot\mathbf{v}' c^{-2}). \qquad (6)$$

As long as both u and v' are small compared to c, this equation reduces to the non-relativistic limit of eq. (2) of the GT. The same conclusion holds if we assume that c is infinite. Eqs. (5-6) are the basis for claiming that events do not occur simultaneously for observers who are in relative motion to one another, that is, $dt \neq dt'$.

From eq. (6) it is clear that dt can be either greater than or less than dt' depending on the relative directions of **u** and **v'**. As a practical example, consider the case where two bullets are fired in opposite directions on an airplane with the same speed v'. If their respective targets are an equal distance away, it follows that the bullets will arrive simultaneously for an onboard observer. An observer on the ground will find, in accordance with eq. (6), that dt>dt' for the bullet traveling in the same direction as the airplane. Since v'>u in this example, the opposite ordering must hold for the other bullet, however. The conclusion is unequivocal. According to the LT, *the two bullets do not arrive simultaneously for the observer on the ground* even though the opposite is true for his counterpart on the airplane.

As discussed elsewhere [12], however, experience with the Global Positioning System (GPS) is not consistent with the above prediction. What it shows is that *the rates of clocks on a satellite/airplane are simply proportional to those on the ground* at all times. If we leave gravitational effects out of the picture, the conclusion is that an atomic clock on the GPS satellite runs $Q=\gamma>1$ times slower that its identical counterpart on the Earth (u is the speed of the satellite relative to the ground), i.e.,

$$dt = Q dt'. \qquad (7)$$

[Note that the value of Q is not always equal to γ based on the results of the studies of clocks on airplanes [7] and rotors [8,9], hence the use of a variable other than γ for the proportionality factor in eq. (7)]. Thus, if the above experiment with bullets is carried out on the satellite, the times measured for them to reach their respective targets will simply each be Q times larger based on the clocks located on the Earth's surface. As a consequence it is impossible for the latter two values measured on the ground to differ from one another if the corresponding times measured on the satellite are equal. In short, if the events on the satellite are simultaneous based on the local clocks there, they must also be simultaneous for the observer on Earth.

The experience with GPS demonstrates that time dilation is a real effect, but that it is perfectly compatible with the principle of simultaneity of events. One simply has to be aware of the fact that *the unit of time* is not the same for all observers because the respective clocks they use do not run at the same rate. *Since the LT rules out simultaneity in certain situations, it must be rejected as a physically valid space-time transformation.*

An alternative formulation of relativity theory therefore needs to be found which is consistent with simultaneity but which also is not contradicted by any other experimental evidence. The observation of Lorentz [3] mentioned in the Introduction shows that this objective can be realized by simply replacing the LT with a different space-time transformation [12,13] that still satisfies the requirement of the constancy of the speed of light for observers in relative motion. This is done by setting Lorentz's ε factor to a value that satisfies the simultaneity condition of eq. (7) when applied to the generalized version of eq. (6):

$$dt = \varepsilon\gamma dt' \left(1 + \mathbf{u} \cdot \mathbf{v}'/c^2\right) = Q dt'. \quad (8)$$

The desired value of ε is thus:

$$\varepsilon = Q\,[\gamma\,(1 + \mathbf{u}\cdot\mathbf{v}'/c^2)]^{-1}.$$

(9)

The alternative Lorentz transformation (ALT [12,13]) is then obtained by making the same "normalization" for eq. (4) as well as for the other equations of the LT. The ALT is thus:

$$dx = \varepsilon\gamma\,(dx' + u\,dt') = (1 + \mathbf{u}\cdot\mathbf{v}'c^{-2})^{-1}Q(dx' + u\,dt')$$

(10)

$$dy = \varepsilon\,dy' = [\gamma\,(1 + \mathbf{u}\cdot\mathbf{v}'c^{-2})]^{-1}Q dy' \quad (11)$$

$$dz = \varepsilon\,dz' = [\gamma\,(1 + \mathbf{u}\cdot\mathbf{v}'c^{-2})]^{-1}Q dz'\,. \quad (12)$$

As pointed out elsewhere [14], if the unit of time increases because of a change in the observer's state of motion, this automatically means that the unit of distance must undergo a strictly proportional change in order to satisfy the requirement that the speed of light retain the same value for him. This means that *isotropic length expansion* accompanies the slowing down of clocks due to time dilation, not the type of anisotropic length contraction foreseen in the FLC [1].

Another way to look upon the ALT is simply as a merging of the relativistic velocity transformation (VT) with the condition of simultaneity. Dividing eqs. (9-11) by $dt = Qdt'$ in eq. (7) leads directly to exactly the same velocity transformation (VT) as Einstein introduced in his original work [1], i.e. with $v_x = dx/dt$ and $v_x' = dx'/dt'$ and analogous definitions for the perpendicular directions. One difference between the VT and the ALT is that the unit of velocity does not change with the observer's state of motion and thus the proportionality factor Q defined in eq. (7) does not occur in the former equations [14]. It is important to note that the VT has received direct experimental verification from observations of the aberration of starlight and the Fizeau light drag phenomenon [15] as well as in the Michelson-Morley experiment [4]. These results are consistent with both the LT and the ALT, and thus in no way distinguish between the two space-time transformations. Only the ALT is consistent with the GPS observations of absolute simultaneity, however, thus eliminating the LT from further consideration in developing a completely viable theory of relativity. The ALT also does not have the “symmetry” problem of Einstein’s special relativity [1,5] because it recognizes the experimentally proven fact that it is always possible to say which of two atomic clocks in running slower than the other when they do differ. It makes a similar statement about length measurements and is perfectly consistent with the modern definition of the meter as the distance traveled by light in c^{-1} s, unlike the LT, which leads one to conclude that the periods of clocks are *inversely* proportional to the unit of distance.

Another subject where the interpretation of the LT space-time variables plays a decisive role is time reversal, that is, whether dt and dt’ can be of opposite sign for the same event. According to eq. (6), it is impossible for time reversal to occur since this would require that the product of the speed of the object v and the relative speed u of the two observers/origins be larger than c^2; v’ may not exceed c and u must also be less than c. Since time reversal has never been observed experimentally and would correspond to the seemingly absurd situation in which one observer would find that the object reached its final destination before it left its initial position, this result seems quite plausible from a purely theoretical point of view. The situation is left much more open when one relies on Einstein’s LT, however, as is clear from its eq. (5). In this case the condition for time reversal is easily fulfilled, at least in principle: $dx'/dt' > c^2u^{-1}$.

Nonetheless, it has been speculated that time reversal can occur in anomalously dispersive media. It is well documented that the speed of light can exceed c in the neighborhood of absorption lines [15,16]. When v>c (i.e., when the group refractive index $n_g<1$) it is clear from eq. (6) that dt and dt’ can have opposite signs when a) **u** and **v** are have opposite directions and b) $u>n_g$ c.

The above arguments about time reversal become moot once it is realized that the LT fails in its prediction of the non-simultaneity of events. The ALT of eqs. (10-12), by contrast, assumes the absolute simultaneity of events and thus precludes time reversal in any conceivable situation. Its equations are not Lorenz-invariant because this condition is inconsistent with simultaneity. That does not mean that the ALT makes no comparable statement about the energy-momentum four-vector, however, because different physical variables are involved in this case. In other words, the choice of the common Lorentz factor ε in the energy-momentum transformation does not have to be the same as its space-time counterpart. As discussed elsewhere [12], the condition in this case is that the relativistic energy/momentum transformation leads to the standard definition of the classical kinetic energy in its low-velocity limit. Consequently, Einstein's famous mass/energy equivalence relation ($E=mc^2$) is left unaffected by changes in the corresponding space-time transformation. A similar situation holds for the equations of quantum electrodynamics and thus the ALT is perfectly consistent with the latter theory as well.

IV. Conclusion

The non-simultaneity of events for observers in relative motion follows unequivocally from the Lorentz transformation (LT). Experimental evidence from the GPS technology indicates on the contrary that events always occur simultaneously for clocks on satellites and those on the Earth's surface. If this were not the case, it would be impossible to explain the high accuracy achieved by this technique. The key question is how to reconcile time dilation, which is also confirmed by the GPS technology and the Hafele-Keating experiments with circumnavigating airplanes [7], with simultaneity. The way to do this is to recognize that the units of time and distance and many other physical quantities vary with the state of motion of the observer as well as his position in a gravitational field. Measurement is objective in all cases but the numerical value obtained for a given quantity depends in a precisely defined manner on what unit is employed by a given observer. This conclusion is also inconsistent with the LT because of its "symmetry" principle [1,5], which claims that observers in relative motion can disagree as to which clock is slower or which measuring rod is longer. The main conclusion is therefore dictated by experiment: *the LT is not a* *physically valid space-time transformation*. The goal is then to find a replacement that is still consistent with Einstein's postulates of the constancy of the speed of light and the relativity principle, but *one that is perfectly consistent with both the principles of simultaneity and the absolute objectivity of measurement*.

The key to achieving the above objective lies is Lorentz's observation that there is a degree of freedom in the required relativistic space-time transformation that cannot be removed simply by adhering to the relativity principle and light speed constancy. Einstein insisted along with Poincaré that the transformation must be Lorenz-invariant, thereby removing the degree of freedom in favor of the LT. They both recognized that this additional condition on the transformation equations had the clear effect of denying the principle of absolute simultaneity of events. Only experiment can provide a solid basis for making this choice, however, and it shows conclusively that a different condition than Lorentz invariance is needed, specifically one that does not come in conflict with the principle of absolute simultaneity of events. This can be done by requiring that proper clocks in uniform relative motion run at strictly proportional rates [dt=Qdt', see eq. (7)].

On this basis, one obtains the alternative Lorentz transformation (ALT) of eqs. (10-12). The relativity principle needs to be restated as a consequence: the laws of physics are the same in all inertial systems *but the units in which they are expressed vary in a systematic manner depending on the observer's state of motion and position in a gravitational field*. In retrospect, it is clear that the belief in non-simultaneity encapsulated in the LT is based on the misguided conclusion that time dilation makes it impossible for observers in relative motion to agree on the elapsed time of a given event. Just because two clocks are running at different rates does not mean that events are not simultaneous for them. The success of the GPS technology not only proves that the latter conclusion is correct, it also shows how one can reformulate the theory so as not to violate either of the principles of simultaneity or the absolute objectivity of measurement.

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