The Lorentz Transformation and the Transverse Doppler Effect

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Abstract

According to the dy=dy' equation of the Lorentz transformation (LT) of the special theory of relativity (STR), the value of any distance interval measured on a moving object that is oriented *transverse* to the velocity of that object should be independent of its relative speed to the observer. It is known from experiments with the transverse Doppler effect, however, that the wavelength of light emitted from a moving source *increases uniformly in all directions* with its speed relative to the observer. When one combines the dy=dy' axiom from STR with the above experimental finding, the unavoidable conclusion is that the *in situ* value of the wavelength must also vary with the state of motion of the light source. Otherwise, it is impossible to explain how the laboratory observer could find that the wavelength of the light from the accelerated source changes even when it is measured in a direction which is transverse to its velocity

relative to this source. Experimental measurements indicate that this is not the case, however: the *in situ* value of the wavelength of light from a given source is always the same, regardless of the latter's state of motion. The Relativity Principle (RP) on which STR is based also leads to this conclusion. The only way to reconcile theory with experiment under these circumstances is to reject the dy=dy' claim of STR. Instead, one must assume *that the lengths of objects increase upon acceleration in the same proportion as the rates of clocks slow down*, independent of their orientation to the direction of relative motion to the observer.

I. Introduction

The special theory of relativity (STR) is based on two postulates [1]. The first of these is the relativity principle (RP) itself, which was simply restated from earlier mechanical theories dating back to Galileo. The second was more revolutionary, however, stating that the speed of light in a vacuum has the same constant value in all inertial systems independent of the states of motion of both the light source and the observer. Although Einstein did not mention it in his paper [2], the latter postulate is consistent with experiments that had been carried out earlier by Michelson and Morley [3]. Using their recently constructed interferometer, they were able to demonstrate to a high degree of accuracy that the speed of light is the same in all directions for a given observer, independent of the Earth’s orbital velocity relative to the Sun. The accuracy of this experiment has been improved over the succeeding years [4,5], so there is little room for doubt that this aspect of Einstein’s second postulate is correct.

It is interesting to note, however, that the Fitzgerald-Lorenz contraction effect (FLC) of STR states that two observers in relative motion with speed v may disagree on the length of an object by varying amounts depending on its orientation: distances measured along the line of relative motion are contracted by a factor of $\gamma = (1 - v^2 / c^2)^{-0.5}$ for one of them, whereas those perpendicular to it are the same for both.

When one applies this conclusion to the analysis of the Michelson-Morley experiment, however, an interesting result is found. Let us assume that one of the arms of the interferometer is pointed along the direction of motion of two observers, the other perpendicular to it. The arms are of equal length for the *in situ* (or "moving") observer M. On the other hand, the FLC states that the arm oriented in the parallel direction in the moving laboratory will be *contracted* for the "stationary" observer (O), while the other will be of the same length as found by M. Because of the time-dilation effect, O and M will also *disagree* on the amount of time it takes the light to make a round-trip passage in either arm.

The experimental results indicate that they will nonetheless agree that there is *no difference between the respective times they each measure for the two arms separately*. It therefore follows that since M measures the light speed to be the same in both directions, *O will not.* For him the light speed is γ times *slower* in the arm oriented parallel to the direction of relative motion due to the aforementioned contraction effect than in the other arm lying in the perpendicular direction. In the past one has attempted to resolve this *contradiction* by arguing that the two events in question, i.e. the round-trip passages of the light in the two arms of the apparatus, are only simultaneous for the *in situ* observer M.

Or does it indicate something more fundamental about the theory that has heretofore gone unrecognized? This question will be considered below by comparing the results of two other key experiments in relativity theory, the Kennedy-Thorndike experiment [6-7] and the transverse Doppler effect [8-10].

II. Relativistic Doppler Effect

In non-relativistic theory there is no transverse Doppler effect (TDE). The situation is different in STR because clocks are slowed when they are accelerated relative the observer. If its relative speed is v, the moving clock will run γ (v) times slower than that in the rest frame of the stationary observer. Because his clock is running slower, the moving observer will count γ times more wave crests in a given time interval than his counterpart who is at rest. Einstein predicted this effect in his original work [1], and it has subsequently been confirmed in many experiments [8-10]. If a light source moving at high speed emits radiation of known frequency ν, the stationary observer measures a (smaller) value of ν/γ in his laboratory. The TDE has been verified experimentally to an accuracy of better than 1 % [9].

In free space the speed of light is equal to the phase velocity, that is, the product of its wavelength and frequency. Since the frequency is known to be different for an observer (M) traveling with the light source than for his counterpart O at rest in the laboratory, the only way the measured speed of light could be the same for both is if they also *disagree* as to the corresponding wavelength of the radiation. Specifically, since O measures the wavelength to be $\gamma \lambda$, M must find it to be equal to λ.

It is important to note, however, that according to the Lorentz transformation (LT) of STR [1], there should be no difference in the values of distances measured perpendicular to the line of relative motion of the two observers (dy=dy'). Accordingly, since the radiation is being observed transverse to the direction of motion of the light source relative to the laboratory, an observer moving with the source (M) who also measures the wavelength along a transverse direction must therefore obtain exactly the same value as his counterpart at rest, i.e. $\gamma \lambda$, in order to be consistent with the LT/FLC. The product of the measured frequency (ν) and wavelength of the radiation would thus be γ times larger for M, which means that he would find a value for the (*in situ*) speed of light of γ c, in disagreement with Einstein's second postulate and also with the experimental data. Note that there is no way to resolve this *contradiction* by invoking non-simultaneity because the wavelength and frequency measurements can be made at any time by either observer without affecting the above result.

Nor is the situation improved if the laboratory observer O measures the wavelength of the radiation *along* the direction of motion of the light source. In this case, the FLC holds [1] that the wavelength, as all distances in M's rest frame, will be γ times smaller for him than if the source were at rest. Since O's measured frequency is also γ times smaller, this means that their product, the phase velocity, would be equal to γ^{-2} c for him, again in disagreement with Einstein's second postulate.

On the other hand, one could argue according to the FLC that the unit of length for M, i.e. his meter stick, is also smaller by a factor of γ than that used by O in his laboratory. Since both the wavelength and the standard device used to measure it have decreased in exactly the same proportion, there is no way for M to obtain a different value for the wavelength when the source is moving relative to O than when it is not.

There have been many unsuccessful challenges to STR since its introduction by Einstein in 1905 [1], so it is natural to conclude that since the theory has been able to overcome all such objections in the past, it is extremely unlikely that some new objection will be found that cannot be explained in a satisfactory manner. This attitude has to be tempered by the realization that no matter how successful a given theory is, it only takes one experiment to demonstrate its inadequacy. When this happens, the goal must be to try and find a suitable alternative formulation that does not sacrifice any of the satisfactory features of the previous theory, but one which is also capable of maintaining consistency with the new empirical findings.

In view of the previous discussion, it is clear that there are some inconsistencies in STR's description of the TDE. It is also clear how to probe this uncertainty experimentally: measure the wavelength of a known atomic line in a laboratory that is moving *at a high velocity relative to the Earth*. The key point is that the measurement must be carried out *in situ*, i.e. the radiation must both be generated and observed in a moving laboratory such as a satellite orbiting the Earth. For the sake of concreteness, let us consider the case when an atomic line with frequency ν_O is generated and observed in a stationary laboratory on Earth under vacuum conditions . According to the second postulate of STR, which has been verified on this point, the associated wavelength λ_O, must be equal to c/ν_O, that is, the phase velocity $\lambda_O \nu_O$ must equal *c*.

The experimental data for the TDE [8] show when the same atomic line is generated by a light source moving with speed v relative to the laboratory on Earth, the observed wavelength is

$$\lambda = \gamma \lambda_O , \qquad (1)$$

i.e. the wavelength λ of the radiation is thus γ times larger than when the source is at rest in the stationary laboratory on Earth. This result first received experimental verification in the study of Ives and Stillwell [8] reported in 1938.

The problem posed by the second postulate becomes evident when the same experiment is considered from the perspective of an observer who is *traveling with the above light source*. The available experimental evidence using rotors [9] finds that the frequency ν_M measured by him will be

$$\nu_M = \gamma \nu_O \qquad (2)$$

This latter relation is a consequence of Einsteinean time dilation [1]. The moving observer's standard clock runs γ times more slowly than that of his counterpart in the stationary laboratory on Earth, so he measures γ times more wave crests in a given amount of time.

As was discussed in Sect. II, however, according to the LT, the moving and the stationary observer must agree on the value of the wavelength of the atomic line when both are viewing it *transversely* to the direction of motion of the satellite. The wavelength λ_M must therefore satisfy the relation:

$$\lambda_M = \lambda_O \, . \tag{3}$$

Otherwise, one has to give up the $dy = dy'$ provision of the LT, *which also would mean a violation of STR*. Yet if one accepts eq. (3), the only possible conclusion is that the product

$$\lambda_M \, \nu_M = \gamma \;\; \lambda_O \, \nu_O \;\; = \gamma \, c, \tag{4}$$

thereby indicating that the measured speed of light is not equal to c for the moving observer, in direct contradiction of Einstein's second postulate.

It is important to recognize that *no matter what the outcome of the proposed experiment*, it is logically impossible for STR to claim that it has survived this challenge. If the measured wavelength on the satellite is the *same* as that found by the observer on Earth, this means that Einstein's second postulate is incorrect on this point. If it is *different*, it means that the LT does not apply in this instance, specifically that its $dy = dy'$ provision is not correct.

III. Kennedy-Thorndike Experiment

It is not necessary to carry out experiments on orbiting satellites to settle the issues discussed in the previous section, however. It is sufficient to measure the speed of light to high accuracy in a laboratory on the Earth's surface over the course of a half-year period. During this time the orbital speed of the Earth relative to the Sun changes from a maximum at perihelion of 30288 m/s to a minimum at aphelion of 29786 m/s. If the arguments given above were correct, the speed of light measured on Earth would have to change by about 0.05 m/s, i.e. since γ changes by 1.7 parts in 10^{10} over this period. In 1932 Kennedy and Thorndike [6] carried out experiments to test this possibility and their conclusion was that the speed of light is indeed independent of the Earth's orbiting speed, in agreement with Einstein's second postulate. The error bars for their measured value were only about 2 m/s, however, so the question was not actually resolved on this basis [11].

More recently, Braxmaier et al. [7] have reported more accurate measurements employing a cavity resonator technique [12]. On this basis, the authors were able to show that the speed of light varies by at most 6 parts in 10^{12} over a consecutive 190-day period, i.e. by only 0.0018 m/s. This level of accuracy is therefore more than sufficient to rule out the possibility that the light speed is proportional to the value of γ.

The key result of this experiment is that the *in situ* value *for the wavelength of light λ does not depend on the speed of the source*. The cavity resonator has a fixed length L and the experiments show that the resonance condition is maintained to the above degree of accuracy throughout the measuring period. Thus, the possibility that the *in situ* value for the wavelength varies in direct proportion to γ is ruled out definitively by this experiment.

As discussed in the previous section, however, this result needs to be reconciled with the fact that the wavelength of light emanating from an accelerated source is proportional to γ, as first demonstrated in 1938 by Ives and Stilwell [8] in their study of the relativistic Doppler effect. An observer O who stays behind at one point in the Earth's orbit must still find that there is an integral number of wavelengths in the cavity resonator located in the laboratory moving at high speed away from him. Since the value of the wavelength has changed to $\gamma\,\lambda$ for O, in accord with measurements of the transverse Doppler effect [8], this can only mean that *the length of the cavity resonator has also changed by the same factor*, i.e. to γ L.

Moreover, the resonance condition occurs in the laboratory for all orientations of the resonator. The wavelength of light varies with orientation of the source to the stationary observer, but this is only because he moves into the waves at a different rate depending on the source's radial velocity $\mathbf{v_r}$ toward him. To measure the wavelength inside the moving cavity using his meter stick, O must correct his Doppler-shifted value by multiplying it with a factor of $1 - v_r/c$. As a result, O is expected to obtain the same value of $\gamma\lambda$ for the wavelength inside the cavity *in all orientations*. However, this in turn can only be true if the length of the cavity measured by O is γ L in all orientations as well.

It needs to be emphasized that the expansion of the cavity's dimensions in the above example is a real effect. The ratio of the length of the cavity L to that of the stationary observer's meter stick or interferometer constantly changes as the Earth proceeds along its orbit. Whether it increases or decreases and by what factor depends on the point in the orbit from which the stationary observer (O) carries out his measurements. If the Earth is accelerating relative to him, then he finds that the cavity length is increasing. If it is in a decelerating phase, the opposite is true, however.

In effect, two different γ values are being compared in each case, one depending on the current orbital speed of the Earth, and the other on the speed it had at the point where the stationary observer has stayed behind. The faster observer O is moving relative to the Sun (or other objective rest system [13,14]), the larger is the meter stick on which he bases his length measurements. If O stays behind at perihelion, when the relative speed is greatest, he will find that the dimensions of the resonator are steadily decreasing because his meter stick is of maximum length at that point [15-17], whereas at aphelion O will find that they are steadily increasing because his meter stick has reached its minimum length at that position.

The simplest way to express these relationships is to assume that the *standard unit of length*, just as the corresponding unit of time, is constantly changing with the state of relative motion of the observer. If we define them to be 1 m and 1 s, respectively, when the observer is moving with speed u_O relative to the Sun, then they are F m and F s at another point in the orbit when the orbital speed is u_M, where $F = \gamma (u_M) / \gamma (u_O)$. This choice reflects the fact that the length of his meter stick has increased/decreased by this ratio, the same amount by which his standard clocks have slowed down/speeded up.

IV. The Alternative Lorentz Transformation

The above survey of key experiments makes clear that the only way for theory to avoid contradiction is to assume that the lengths of accelerated objects expand rather than contract as their clock rates slow. Moreover, the amount of the expansion must be the same in all directions. Otherwise, it is impossible for two observers in relative motion to both find that light propagates isotropically, as required by Einstein's second postulate of STR and experiment. As pointed out at the end of Sect. II, however, this finding is clearly incompatible with the dy=dy' provision of the LT [1], not to mention the FLC in general.

In order to investigate this point further, the equations of the LT are given below:

$$dt = \gamma (v) [dt' + (v/c^2) dx']$$

$$dx = \gamma (v) (dx' + v\, dt') \qquad (5)$$

$$dy = dy'$$

$dz = dz'$.

The LT is consistent with Einstein's second postulate, specifically that portion of it that requires that light propagate isotropically for all observers, independent of their state of relative motion. The problem comes when one attempts to use the LT to predict the way in which the measurements of two observers in relative motion will differ in their respective time and distance determinations. The contradictions that arise in trying to reconcile the FLC with the results of the TDE experiments are only part of the story. Because of the relationship between dt and dt' in eq. (5), one is forced to conclude that two observers can disagree on whether events occur simultaneously. As is often pointed out, the LT mixes time and space in such a way that the elapsed time dt for a given event is not simply proportional to dt'. Although authors have consistently seen this "non-simultaneity" feature as a key to the many successes of Einstein's theory [1], the fact is that this prediction is contradicted by modern-day experiments that have been carried out with atomic clocks [13]. They show unequivocally that the rates of clocks *simply change in a proportional manner upon acceleration,* i.e.

$$dt = Q\, dt', \qquad (6)$$

where Q is a constant proportionality factor.

As discussed in recent work [15-17], the success of the Global Positioning System (GPS) navigation technology demonstrates quite forcefully that this *strict proportionality between the rates of clocks in relative motion is a real effect.* Without it, it would be impossible to explain why the accuracy of the GPS technology is as high as it is. That being the case, one cannot claim that events that are simultaneous based on one clock will not also be simultaneous for another; if dt'=0, then eq. (6) shows necessarily that dt=0 as well.

Since the LT is contradicted by this result, there is no recourse but to eliminate it from the theory and replace it with a different space-time transformation, one that not only accepts the principle of absolute simultaneity of events, but which is also consistent with the experiments that have been carried out to demonstrate the TDE [8,9] which refute both the FLC and the symmetry principle of STR that claims that two clocks can each be running slower than the other at the same time.

The latter objective is satisfied by introducing an alternative Lorentz transformation (ALT [15-17] or GPS-LT) which eliminates the dy=dy' provision of the LT and instead replaces it with a proportionality condition, as shown below :

$$dx = \eta' (dx' + u\, dt')$$
$$dy = \eta' dy'/\gamma \qquad (7)$$
$$dz = \eta' dz'/\gamma,$$
$$dt = dt',$$

in which γ has the same meaning as in the FLC/LT used above and η is defined as:

$$\eta' = (1 + u\, dx'/c^2 dt')^{-1}. \qquad (8)$$

It is important to note that it is assumed in eq. (7) that *both observers use exactly the same set of units.* However, because of time dilation, this is not the usual case. If the clocks in the primed rest frame (S') run Q times slower than in the other (S), one has to alter *each* of the above equations by multiplying with Q on the right-hand side in order to insure that each observer uses his own set of proper units. For example, the last relation in eq. (7) is changed to eq. (6). In eq. (7) itself, it is understood that the respective primed and unprimed space-time quantities are based on the same set of units in each case. In the GPS application, for example, this means that elapsed times measured on a satellite must be adjusted relative to their uncompensated values so that they can be compared directly with the corresponding values read from clocks located on the Earth's surface.

Since the speed of light is the same for observers in both rest frames, this means that the unit of distance (m) employed in eq. (7) is also the same as in the rest frame of the Earth.

The GPS-LT does not predict time dilation, but it leaves open the possibility that such an effect exists for accelerated clocks. It assumes that the units of time and distance vary in a systematic manner from one inertial system to another, and in such a way as to be consistent with Einstein's postulate of the constancy of the speed of light in free space [15-17]. This means that *isotropic length expansion,* not the FLC, accompanies time dilation in the GPS-LT ; also that there is never any question in principle about which distance is longer and which elapsed time is shorter, independent of both the state of motion of the observer and also his position in a gravitational field [18].

V. Conclusion

Experimental tests of the transverse Doppler effect confirm that the wavelength of light increases in direct proportion to γ (v) when the source is accelerated to speed v relative to a stationary observer. The fact that standing waves in cavity resonators are stable to acceleration demonstrates that the ratio of the wavelength to the length of the cavity does not change, however. The conclusion must therefore be that the *lengths of objects expand upon acceleration.* Since one knows that the rates of clocks slow down by the same factor (γ), this result is consistent with the fact that the speed of light is constant for all observers (after gravitational corrections have been made). An observer traveling with the light source does not notice any change in its dimensions, just as he is also unable to detect any change in its frequency, consistent with the RP. Since light propagates isotropically for all observers as well, it follows that the length variations of accelerated objects must be in the same proportion in all directions.

The above results stand in clear contradiction to the Fitzgerald-Lorentz contraction effect (FLC), which is distinctly anisotropic by definition. They are also inconsistent with the dy=dy' provision of the LT, thereby eliminating it as a physically viable set of relativistic equations. One can nonetheless satisfy both of Einstein's postulates of relativity theory by introducing an alternative Lorentz transformation (ALT/GPS-LT) that insists upon both the principles of absolute simultaneity of events, by virtue of the proportionality relation between elapsed times, as well as the objectivity/rationality of measurement.

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