The Triplet Paradox and the Alternative Lorentz Transformation

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Abstract

An experiment with cesium clocks carried onboard two aircraft as they circumnavigated the globe in opposite directions provided an important confirmation of the time-dilation effect of the special theory of relativity (STR). A key element in the discussion of this event was the fact that since a clock located on the Equator is not at rest in an inertial system because of the Earth's rotation, it could not be used directly as a reference in applying Einstein's formula. It is pointed out that this line of argumentation implies that if the Earth were *not* rotating, the onboard clocks would both be at rest in inertial systems after the airplanes had reached their cruising altitude and had stopped accelerating. On this basis it could be concluded that a person traveling on one of these airplanes would be aging more rapidly than his twin on the other because the latter is in relative motion to him. One can make exactly the same argument for the other twin, however,

and this leads to an obvious contradiction since they cannot both be aging faster than the other. To resolve this issue (Triplet Paradox), it is necessary to distinguish between different inertial systems based on their state of motion. In particular, it is argued that there is a uniquely defined objective rest system (ORS) in both cases (i.e., with and without the Earth rotating) from which the prescriptions of the theory must be applied. This interpretation is put on a sound basis by introducing an alternative Lorentz transformation (ALT) that not only satisfies both of Einstein's original postulates of STR but also assumes that the rates of clocks are always strictly proportional to one another regardless of their position in space or state of relative motion. The success of the Global Positioning System (GPS) technology provides detailed experimental verification of the latter assumption and therefore rules out the original Lorentz transformation (LT) as a physically valid set of equations relating the space-time measurements of observers in different rest frames.

I. Introduction

In his original work on the special theory of relativity (STR [1]), Einstein predicted the time-dilation effect, but it remained a subject of much debate for many years thereafter [2, 3]. An important turning point in the discussion came with the publication by Hafele and Keating [4] of timing results for cesium beam clocks carried onboard circumnavigating aircraft. It was demonstrated not only that time dilation does occur, but also that the changes in clock rates are predicted quantitatively (to within the experimental error) by STR. The timing comparisons were made relative to a "hypothetical coordinate" clock located in a non-rotating inertial frame at one of the Earth's poles.

A key assumption in this general discussion is that two clocks in relative motion can be distinguished on the basis of their acceleration relative to an inertial system [5, 6]. These considerations provided the basis for what has come to be known as the "Twin Paradox" of relativity theory. Accordingly, it is asserted that a twin who goes on a high-speed journey over an extended period of time will return to find that the other who has stayed behind has aged much more quickly than he has. The idea that their respective atomic clocks should have a symmetric relationship because each is in relative motion to the other is rejected by this argument. The Hafele-Keating experiment provided support for the view that only the stay-at-home twin can apply the theory from his vantage point because he alone is always in an inertial system.

The basis for Einstein's prediction of the time-dilation effect is the Lorentz transformation (LT) [1] of space-time coordinates, whose applicability is claimed to be restricted to measurements carried out in inertial systems. The latter assumption provides the justification in the Hafele-Keating experiment for *not* employing a clock on the Earth's surface as a reference in computing the relative rates of the clocks carried onboard the moving airplanes. In some ways it might seem paradoxical itself that Einstein could have predicted an effect that is clearly based on one clock being accelerated relative to another by employing a theory whose range of applicability is claimed to be restricted to inertial systems. In the following discussion, another example will be considered which gives further insight into the relationship between the Twin (or Clock) Paradox and the LT.

II. The Triplet Paradox

A straightforward modification of the Hafele-Keating experiment [4] is illustrated in Fig. 1. Again one has two airplanes departing simultaneously from the same point in space. In the present case, however, it is assumed that the airport is located on a non-rotating surface, so that it also constitutes an inertial system. The airplanes fly off in opposite directions, but at all times their speed u relative to the airport changes at the same rate. At some point the airplanes go into a cruising mode and each proceeds at a constant velocity **u** from then on, so that they also constitute inertial systems. This change in circumstances alters the situation dramatically relative to the original experiment with circumnavigating airplanes. It means that an observer located at the airport can apply the STR time-dilation formula directly. Accordingly, he should find that clocks E and W onboard the two aircraft are running slower by a factor of $\gamma(u) = (1 - u^2/c^2)^{-0.5}$ than his own.

There is a more critical point, however. The relative speed of the two airplanes is (approximately) 2u. Since each of the airplanes is an inertial system, it should be possible for an observer on either one of them to apply the time-dilation formula from his perspective, unlike the case in the original Hafele-Keating experiment. Accordingly, the observer on the airplane moving in the westerly direction must conclude that the clock E located on the other airplane is running $\gamma(2u)$ times slower than his own (W). On the other hand, the observer on the airplane traveling in the easterly direction must conclude that clock W is running $\gamma(2u)$ times slower than his. Obviously, they can't both be right. Moreover, from the symmetry of their respective trajectories, it seems certain that both clocks E and W are actually running at exactly the same rate, as already concluded above, i.e. $\gamma(u)$ times slower than the clock at rest at the airport.

To find out what's wrong in the above argumentation, it is necessary to go back and consider the assumptions that are implied in the above timing procedures. The starting point is clearly the Lorentz transformation (LT), as given below:

$$dx = \gamma(u)(dx' + u\,dt') \qquad (1a)$$

$$dt = \gamma(u)(dt' + u\,dx'/c^2) \qquad (1b)$$

$$dy = dy' \qquad (1c)$$

$$dz = dz'. \qquad (1d)$$

The equations themselves contain two sets of space-time variables. The standard interpretation for these quantities, going back to Einstein's original work [1], is that they refer to measurements carried out *for the same event* in two different inertial systems moving with speed u relative to one another along the x direction. The time-dilation formula is obtained quite simply by assuming that one set of measurements is made at the same location in the "primed" inertial system, which means that $dx'=0$. Substitution in eq. (1b) then leads directly to the time-dilation formula employed above, namely:

$$dt = \gamma (u)\, dt' . \qquad (2)$$

It is assumed in one case that for a given event the elapsed time measured by the observer moving in the westerly direction with clock W is dt, whereas the corresponding elapsed time measured on clock E is only dt', which in this case is γ (2u) times less than dt. The problem with this interpretation is that one can just as well apply the arguments from the perspective of either of the above observers, since each of the airplanes is a *bona fide* inertial system.

Does this mean that STR is wrong? It might seem extremely unlikely that this would be the case given the fact that the theory has withstood numerous challenges over the past 100 years and that it has provided quite accurate predictions for a wide range of effects, including time dilation. On the other hand, because of the fundamental nature of this theory it is not judicious to let the matter drop so simply. The very success of STR strongly indicates that the contradictory behavior discussed above can be eliminated in a straightforward manner by carefully examining each and every assumption that has gone into its derivation. The goal is to preserve the essential character of the theory with regard to its past predictions, while at the same time making it possible to resolve the Triplet Paradox in a thoroughly consistent manner.

III. The Alternative Lorentz Transformation

The main question that needs to be explored is under what conditions does application of eq. (2) lead to an accurate prediction of elapsed times. To obtain a suitable answer it is helpful to recall how the LT came about in the first place. Its precursor, the Galilean transformation, was introduced at the close of the 17^{th} century by Newton in order to clarify the problem of relative motion: if a ship is moving in a given direction relative to the shore, how far does an object moving a given distance on the ship appear to have traveled to an observer at rest on the shore? The main point that is expressed by the resulting equations is that the origin from which distances are measured is fixed on the ship but is moving (in the x direction) with speed u relative to an observer on the shore:

$$dx = dx' + u\, dt'. \tag{3a}$$

In his derivation of the LT, Einstein [1] assumed that the above equation be satisfied for $u \ll c$, as is the case for eq. (1a) since $\gamma(u)$ becomes nearly equal to unity under this condition.

In the present context it is important to note that Newton took for granted the fact that the unit of time is the same for all observers. This is most obvious from the equation for dt in the Galilean transformation, namely

$$dt = dt'. \tag{3b}$$

The elapsed time for a given event was thought to be invariant, unlike the case in STR, i.e. in eq. (1b). There is also no indication that Newton considered the possibility that the meter stick onboard the ship was not of exactly the same length as that employed to carry out the measurements onshore. The question of who actually made the measurements of length and time was completely immaterial in the classical physicist's view.

It was necessary to change that seemingly indisputable proposition once it was realized that the speed of light is independent of the motion of the source, as was shown in the Michelson-

Morley experiments of 1887 [7]. Einstein [1] clearly assumed that the primed variables in the latter equations correspond to values measured by an observer in the rest frame of the ship, whereas the onshore observer in the present example is responsible for the other (unprimed) values; in other words, two observers in different inertial systems carrying out measurements for the same event. If the observer on the ship carries out his measurements at the same location, i.e. so that dx'=0 in eq. (1b), the time-dilation formula of eq. (2) results directly.

The Triplet Paradox (Fig. 1) demonstrates that the above analysis is incorrect in the general case. The symmetry in this example leaves no room for doubt that the clocks on both airplanes run at exactly the same rate, making eq. (2) inoperable in this case. It is important to see that the latter result is not just "paradoxical." It demonstrates beyond any doubt that Einstein's theory is contradicted by experiment. A paradox is something that is actually true but only appears to be questionable because of a false premise or otherwise faulty point of logic in a particular argument. There is never a need to change a theory on the basis of a paradoxical result. A contradiction on the other hand is completely unequivocal. Once a theory has actually been contradicted by an experimental result, there is no recourse but to abandon it in favor of an alternative that both succeeds in explaining the new data and also avoids any difficulty with earlier empirical findings with which the previous theory has been consistent. In the present case, this means at a minimum that a revised theory be found that does not violate either of Einstein's two postulates of STR but at the same time does not claim that two clocks can both be running faster than the other at the same time.

As discussed elsewhere [8, 9], the above objective is obtained by altering the LT so as to be consistent with eq. (3) rather than either eq. (2) or eq. (1b). This can be done while still insisting on the constancy of the speed of light in free space for all observers at the same gravitational potential by simply multiplying the right-hand sides of each of the LT equations by the same factor $\varepsilon = \eta/\gamma$, with $\eta = (1 + u\, dx'/c^2dt')^{-1}$, the same factor that appears in Einstein's relativistic velocity transformation (VT [1]). The result is the alternative Lorentz transformation (ALT [8]) given below:

$$dx = \eta\, (dx' + u\, dt') \tag{4a}$$

$$dy = \eta\, dy'/\gamma \tag{4b}$$

$$dz = \eta\, dz'/\gamma, \tag{4c}$$

$$dt = dt'. \tag{4d, 3b}$$

The above equations assume that the respective units of time and distance are the same for both observers and therefore incorporate the phenomenon of time dilation explicitly in relativistic theory without insisting that eq. (2) hold under all circumstances. In the example of Fig. 1, the unit of time is the same for both observers, whereas in the Hafele-Keating experiments [4] it varies depending on the speed of a given airplane relative to the reference clock located on the Earth's polar axis. In practical terms, this means that one compares the time dt' read on the "pre-corrected" clock on the GPS satellite with the corresponding value dt for the same event measured on the Earth's surface [8,9]. The assumption is that the rates of clocks are always strictly proportional to one another even though they may differ because of their acceleration histories or positions in a gravitational field. Accordingly, events that are simultaneous for one of the observers must also be simultaneous for the other, contrary to what is claimed by the LT based on its eq. (1b).

It is important to note that it is assumed in eqs. (4a-d) that both observers use exactly the same set of units. However, because of time dilation, this is not the usual case. If the clocks in the primed rest frame (S') run Q times slower than in the other (S), one has to alter *each* of the above equations by multiplying with Q on the right-hand side in order to insure that each observer uses his own set of proper units. For example, eq. (4d) becomes dt=Qdt'.

The example of a light pulse moving along the y axis shown in Fig. 2 demonstrates that the ALT is still consistent with Einstein's second postulate. The observer for whom the light source is moving with speed u finds that the pulse has traveled a distance of $dx = udt = uA/c$ along the x axis, whereas $dy = A/\gamma$ in the perpendicular direction. The vector sum is thus $dr=A$, exactly the same value ($dr' = dy'$) measured *in situ*. Because of eq. (4d), this means that the speed of light is the same for both observers, i.e. $dt = dt' = A/c$. This result assumes that both observers use the same respective units of time and distance on which to base their measured values (see the above remark about how to change the transformation when the clocks in S and S' are not running at the same rate). If the uncompensated clock on the GPS satellite mentioned above were used to measure the elapsed time there, the value would be smaller by a factor of γ. The corresponding unit of length would also be greater by the same factor because of the definition of the meter, however, so that the ratio of elapsed time to distance traveled by the light pulse would remain the same. The key point is that the conversion factor for the two sets of units does not always have a value of γ. How it is determined in a given case is the subject of the following section.

IV. Objective Rest System and the Unit of Time

The Hafele-Keating experiment with circumnavigating airplanes [4] not only demonstrates the reality of the time-dilation effect predicted by Einstein [1], it also provides clear direction as to how the rates of clocks can be computed from the theory. The above discussion emphasizes, however, that there is something wrong with the argumentation used in STR to compute the amount of the effect quantitatively. The experiments [4] show that a clock located at the center of the Earth can be used as reference, whereas one on the surface or on either of the airplanes cannot. The reason is *not* because such a clock is at rest in an inertial system while the others are not, however. If the Earth's rotation is eliminated (Fig. 1) and the airplanes are both moving at constant velocity so that they also constitute inertial systems, it still is not possible to apply eq. (2) by using clocks located on the airplanes as reference.

It is possible to resolve these issues by making a few basic assumptions. First, it seems clear that clocks that are not in relative motion and are located at the same gravitational potential all have the same rate. Secondly, *acceleration* of any one of these clocks relative to the above rest system causes a change in its rate (after appropriate gravitational corrections are applied). A third point is critical for obtaining quantitative predictions, however. In order to apply the time-dilation formula of eq. (2), it is necessary to designate what shall be referred to as an objective rest system (ORS). In the Hafele-Keating experiment [4] this rest system is located at the center of the Earth or anywhere on its axis of rotation.

The reason that they qualify as ORS but not a point on the Equator, for example, needs to be understood, however. The simplest assumption, consistent with the arguments of ref. [4], is that a clock located at the Equator is under constant acceleration (moving with constant speed u) relative to the Earth's axis, and thus it must be running γ (u) times slower than one located at the latter position. The center of the Earth itself is under constant acceleration, however, because of its motion around the Sun. The rate of a reference clock located there is most probably greater when the Earth is at perihelion than at aphelion as a result. The experimental data [4] are not precise enough to allow for verification of such a conjecture, however.

What is clear is that there is a definite east-west effect and this cannot be explained by taking a clock on the Earth's surface as a reference. When one assumes as in Fig. 1 that the Earth is no longer rotating, this effect must disappear and one can then conclude that the airport itself becomes an ORS. The rates of clocks on both airplanes are equal as long as they are moving at the same speed relative to the airport. The key point is that the airplanes themselves do not qualify as ORS in this example, despite the fact that they have reached constant velocity relative to the airport. The time dilation occurs because an object has been accelerated relative to an ORS and so the clocks aboard the airplanes slow down, not the reference clock at the airport.

To remain true to the above principles, it is necessary to distinguish between eq. (2) and the actual time-dilation relation, which is:

$$dt\ (M) = dt\ (ORS) / \gamma\ (u). \qquad (5)$$

The latter equation assumes that two different observers have measured the elapsed time for the same event. One (O) is located in an ORS and the other (M) has been accelerated relative to it. Since the rate of M's clock is slowed by a factor of γ (u), it follows that the elapsed time measured by him will be smaller by this factor than the corresponding value measured by O (always assuming that gravitational effects have been suitably taken into account). As already discussed above, a simple way to express this relationship is to state that M's unit of time is γ times greater than O's.

The Triplet Paradox in Fig. 1 is resolved by applying the much more specific eq. (5) rather than Einstein's eq. (2). The unit of time must be the same on both airplanes because of the symmetry of their trajectories, and neither qualifies as an ORS. If the goal is to measure the speed of light on the other airplane, or the motion of some other object there, then the ALT of eqs. (4a-d) is directly applicable. One simply has to know the values of dx', dy', dz' and dt' that are measured by the observer on one of the airplanes as well as its speed u with respect the other. If the second airplane is accelerating at the time of the measurement, the ALT can still be used on an *instantaneous* basis, i.e. one must always use the current value of the relative speed u in the computations. This point was made most effectively in the centrifuge experiments of Hay et al. and Kuendig [10].

The distinction in the unit of time embodied in eq. (5) and the ORS interpretation in general underscores another very basic point. In order to be consistent with the results of experiment, any system of physical units must be *rational* [11]. The numerical values measured for a given quantity by two different observers must always be in the same ratio. To illustrate this, it is helpful to consider the experiments that have been carried out for metastable particles originating from cosmic radiation in the upper atmosphere [12]. Consistent with eq. (5), the lifetime of muons is found to increase with relative speed u to the observer on the Earth's surface. Since the muons have been accelerated in this example, their lifetime is found to be γ (u) times larger than when they are at rest on the Earth's surface ($\tau = 2.2\ \mu s$). It is common to read that this relationship is symmetric, i.e., that an observer co-moving with the accelerated particles will find that the lifetime of muons at rest on the Earth is also $\gamma\ \tau$, but this conclusion is inconsistent with eq. (5) and therefore incorrect. Instead, such an observer must find the lifetime of the Earth-bound muons to be τ / γ because his unit of time is larger than that in the Earth's rest frame. The results of the Hafele-Keating experiment [4] provide unambiguous, albeit indirect, proof of this prediction [13]. The accelerated clock is known to run γ times slower than that in a laboratory on the Earth's surface and thus the muon lifetime must be shorter by this factor when determined on the basis of this clock.

The situation with energy measurements is wholly analogous [14]. The energy E of the accelerated muons is γ times larger that those at rest in the laboratory (ORS). The relevant equation is

$$dE\ (M) = dE\ (ORS) / \gamma\ (u), \tag{6}$$

where M is the observer co-moving with the muons. When M measures the energy of an equivalent sample of muons at rest in the laboratory on Earth, he finds that the value is *less* than for the muons in his rest frame because his unit of energy is γ times larger. The relationship is not symmetric but rather rational/objective [11], i.e., O finds that M's muons have more energy than his while M finds that O's muons therefore have less energy than his. The fact that M's rest frame is accelerated relative to the ORS is decisive in this respect, just as it is in the case of the lifetime measurements [13].

At the same time, there is also an equation for energy that is analogous to eq. (2):

$$dE = \gamma\ (u)\ dE' . \qquad (7)$$

In this case, dE is the energy of an object in motion and u is its speed relative to the observer located at the respective ORS. The latter equation stems from the energy-momentum four-vector relation:

$$dE = \gamma\ (u)\ (dE' + u\ dp'), \qquad (8)$$

for the case when the object is not moving relative to one of the observers (dp'=0). When combined with the corresponding relation for dp, the following inverse relationship is found:

$$dp' = \gamma\ (u)\ (dp - u\ dE/c^2). \qquad (9)$$

Since dp' = 0 in this case, an expression for the inertial mass of the system dμ is obtained, namely

$$d\mu = dp/u = dE /c^2 = \gamma\ dE'/c^2, \qquad (10)$$

which is Einstein's famous mass/energy equivalence relation [1].

The above discussion needs to be extended to deal with the unit of distance in accelerated frames. This topic is judiciously avoided in standard texts. A probable explanation lies in the fact that the Fitzgerald-Lorentz contraction (FLC) effect of STR [1] is *anisotropic*: lengths in accelerated systems are thought to be contracted along the line of relative motion to the observer but to be equal in a perpendicular direction. This conclusion is based on the LT of eqs. (1a-d) and is therefore inconsequential. According to the relativity principle, free space must be isotropic regardless of the observer's state of relative motion, so at the very least, defining the unit of length to vary with orientation would be quite impractical. In order for the speed of light to retain a value of c in the accelerated system, as foreseen in Einstein's second postulate of STR [1], it is necessary that the unit of length also vary with γ (u) in exactly the same manner as the unit of time [15, 16]. This means *isotropic expansion*, not anisotropic contraction.

In the last analysis, the only way to decide how the unit of length varies with relative motion is through experiment. If the unit of length were the same in every inertial system, for example, this fact would manifest itself through an increase in the light speed to a value of γ (u) c, i.e., the speed of light would necessarily be inversely proportional to the rates of standard clocks as given by eq. (6). This possibility has been ruled out definitively, however, on the basis of modern highly accurate measurements [17] of the speed of light employing an updated version of the Kennedy-Thorndike experiment [18]. More detailed discussion of this point is given in a companion publication [19].

V. Conclusion

The experiment with circumnavigating airplanes carried out by Hafele and Keating in 1971 demonstrated in a clear and convincing manner that atomic clocks slow down upon acceleration in a way that is quantitatively consistent with Einstein's time-dilation conjecture. The observed disparity in the rates of clocks moving in easterly and westerly directions was explained by assuming that Einstein's formula must be applied directly from the vantage point of an observer who is in the rest frame of an inertial system. In the present case this meant that a reference clock must be located somewhere on the Earth's axis of rotation, not on the surface at the Equator and not on the airplanes themselves. The Triplet Paradox illustrated in Fig. 1 shows that there is something deficient in this line of argumentation. If the Earth were not rotating, it follows that the airplanes become inertial systems at the moment that they level off at constant velocity relative to their starting point. Applying the time-dilation formula from the vantage point of observers located on each airplane leads to a clear contradiction, however. One must conclude that in each case the clock traveling with the observer runs slower than that on the other airplane. This is a logical impossibility, however, one that can be demonstrated at a later time by bringing the respective clocks back together at the same point on the Earth's surface. It simply cannot be that each one has run slower than the other.

There is a way out of this impasse while still remaining consistent with the results of the actual experiment carried out with the Earth rotating beneath the airplanes. It is merely necessary to assume that the time-dilation formula can only be successfully applied from the vantage point of an objective rest system (ORS). By definition, a clock located in the rest frame of an ORS does not undergo acceleration during any time in the measuring period. Since the airplanes must be accelerated relative to the starting point on the Earth's surface, they do not qualify in either case as ORS, i.e. whether the Earth is rotating or not. A clock at the Equator also cannot be used as reference when the Earth is rotating, but may be used as such in the hypothetical case of Fig. 1 when it is not.

Since the special theory of relativity (STR) should only apply to inertial systems, it is not surprising that an effect that can only occur when clocks undergo acceleration is not completely understandable without going outside the domain for which it was originally intended. This observation raises the question as to whether the time-dilation effect can actually be inferred directly from the Lorentz transformation (LT), as is conventionally assumed. On that basis, there is no reason to distinguish between different inertial systems, for example, even though a contradiction arises when this is not done in the Triplet Paradox. One is forced to conclude from the LT that the clocks on the two airplanes do not run at the same rate, even though it is clear from symmetry that this cannot be true.

This is not an isolated case by any means. The Hafele-Keating experiment demonstrates first and foremost that the rates of atomic clocks are always in strict proportion to one another and therefore that events that are simultaneous for one of them are also simultaneous for the other. This is the basis of the Global Positioning System for determining distances on the Earth's surface. Eq. (1b) of the LT leads unequivocally to the lack of simultaneity for observers in relative motion and therefore needs to be discarded in favor of another relationship that takes explicit account of the experimentally observed proportionality of clock rates and still remains consistent with Einstein's postulates of STR. This objective is accomplished with the ALT of eqs. (4a-d), specifically with its Galilean-like dt=dt' relation of eq. (4d), or alternatively, using dt=Qdt' when differences in local clock rates are to be taken into account explicitly in the space-time transformation. The latter equation assumes that both observers employ the same unit of time in which to express their measured results, requiring that a correction be made to the natural rates of clocks that explicitly accounts for the effects of time dilation. This procedure is followed in the GPS methodology through the use of "pre-corrected" clocks on satellites that run at exactly the same rate as their counterparts on the Earth's surface.

The Triplet Paradox is easily resolved on the basis of the ALT. Because of the inherent symmetry in the trajectories of the two airplanes in Fig. 1, one simply assumes that the unit of time is the same for both observers in this case and thus that their respective clock readings can be compared directly in eq. (4d) without adjustment. More generally, one needs to identify an objective rest system (ORS) from which to apply Einstein's time-dilation formula in order to define the conversion factor required for the necessary change in units. In the example of the Triplet Paradox, the ORS is simply the airport of departure for the two airplanes. In the Hafele-Keating experiments, it is a point on the non-rotating axis of rotation of the Earth. The conversion factor for the units of time of different airplanes is then obtained as the ratio of the γ

(u) values based on their respective speeds relative to the ORS. In this way, it is possible to avoid the lack of objectivity inherent in the symmetry principle of STR, as well as its prediction of non-simultaneity, while still ensuring that both of Einstein's original postulates are rigorously satisfied through the use of the ALT space-time transformation.

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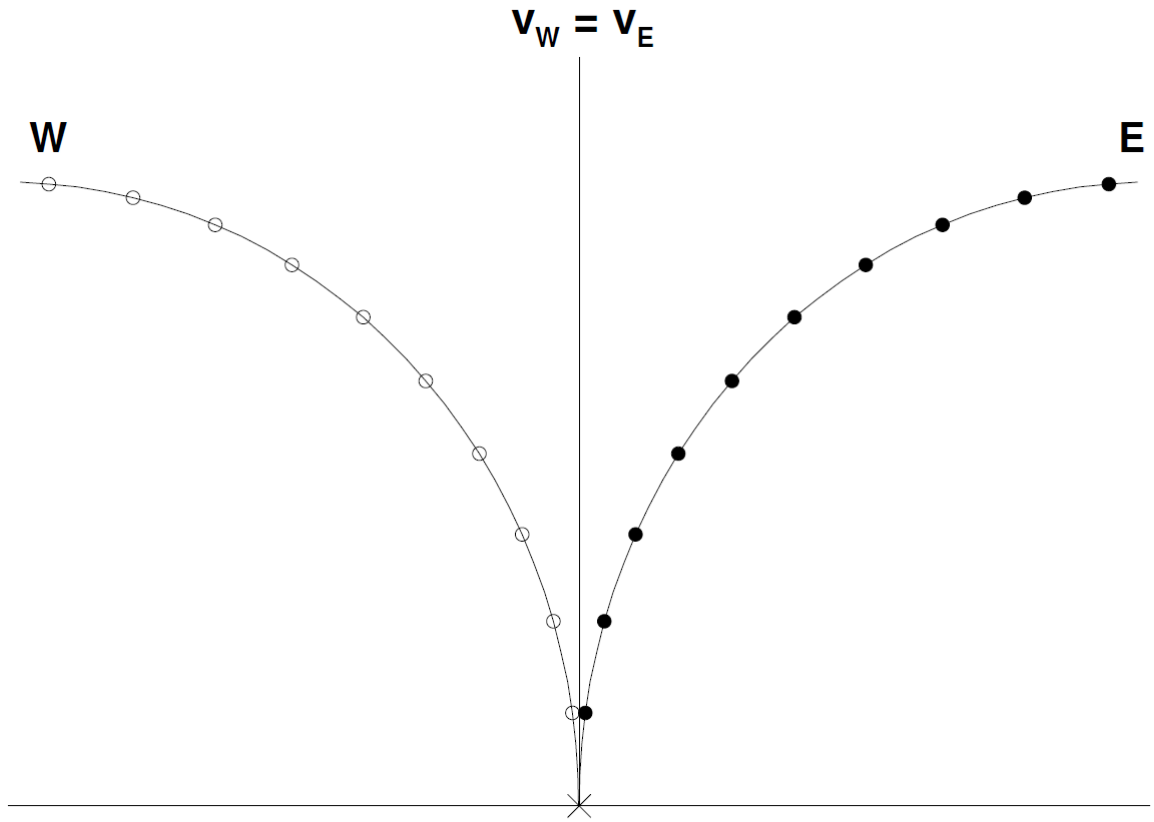

Fig. 1. Diagram showing two rockets leaving the same position in a gravity-free region of space. Their speed relative to the departure position is the same for both at all times, even though the respective directions of velocity are always different. The symmetric relationship of their trajectories indicates that the rates of their respective onboard clocks are always the same. This remains true even for the termini of the trajectories shown, in which case the rockets are both a)inertial systems (each traveling at constant velocity) and b) in relative motion to one another at that point.

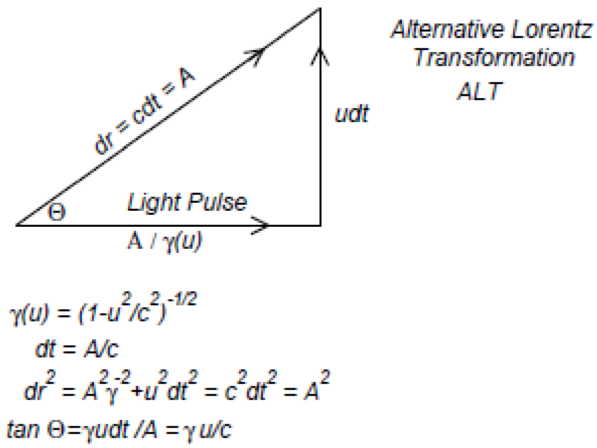

Fig. 2. Diagram showing a light pulse traveling in a transverse direction on a satellite that is moving with speed u relative to observer O. The distance dr = A traveled by the light pulse is computed from O's vantage point by employing the alternative Lorentz transformation (ALT) discussed in the text [8,9]. He finds that the light pulse travels at an angle $\Theta = \tan^{-1} (\gamma\ u/c)$ and with speed equal to c, so that his measured elapsed time is dt = A/c. Another observer (M) at rest on the satellite, who agrees to use the same unit of time as O, i.e. the same clock rate, also finds the light pulse to travel the same distance from his perspective (dr'=A) at the same speed c, but in a different direction ($\Theta = 0$). The elapsed time for this event measured by M is therefore dt' = A/c, the same value as for O. The light pulse therefore arrives at the same time at its detector on the satellite for the two observers, despite the fact that they are in relative motion to each other.

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