

Faster-than-c Neutrinos and the Alternative Lorentz Transformation

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Abstract

Recent evidence for neutrinos moving faster than light in free space is consistent with earlier measurements of the speed of light in condensed media in wavelength regions of anomalous dispersion. Both findings are widely assumed to be ruled out theoretically because they would violate Einstein causality, that is, the principle that the time-order of events must be the same for all observers. The latter conclusion is based on the Lorentz transformation (LT) of special relativity (SR). A review of the assumptions made in Einstein's original derivation of the LT in 1905 shows that it is possible to avoid this conflict between theory and experiment while still satisfying his two postulates of relativity. An amended version of relativity theory is presented which is based on an alternative Lorentz transformation (ALT) that remains consistent with Einstein's velocity addition theorem (VT), which is compatible with super-luminal motion. At the same time, it is argued that faster-than-c neutrinos must have null rest mass, the same as photons, in order to satisfy other requirements of the original theory.

Keywords: faster-than-c neutrinos, Einstein causality, relativistic velocity transformation (VT), alternative Lorentz transformation (ALT), inertial mass dependence on velocity

I. Introduction

Soon after Einstein's original paper on the special theory of relativity (SR) [1], questions began to arise as to the maximum speed that particles can attain. It was pointed out that the group refractive index of light (n_g) is less than unity for condensed media in wavelength regions of *anomalous dispersion* (near absorption lines) [2]. Since the speed of light in condensed media was found experimentally [3] to be equal to cn_g^{-1} for normally dispersive liquids (with $n_g > 1$), it was argued that light speeds exceeding the value in free space ($c = 2.9979 \text{ ms}^{-1}$) might be possible in media with $n_g < 1$. Sommerfeld [2] vigorously denied the latter possibility on the grounds that it would contradict a basic tenet of SR: Einstein causality. He claimed instead that the speed of energy transport of the waves [4] was the only quantity of experimental significance, and that its value must necessarily be less than c in all conceivable situations.

This theoretical position has received wide-spread acceptance to the present day, but in 1993 new experimental evidence [5,6] emerged that appeared to demonstrate unequivocally that $u > c$ light speeds were indeed attainable in media with $n_g < 1$. However, even these results were not sufficient to dispel the general reluctance on the part of the physics community to accept as fact that single photons can indeed travel with faster-than- c speeds under the above conditions [5,7]. Yet less than 20 years later, there is again serious discussion about new experimental data that indicate that neutrinos also can travel with $u > c$.

The question that will be considered in the following discussion is whether the conclusion that faster-than- c speeds inevitably imply a violation of Einstein causality is actually correct. The basis for this position has been the Lorentz transformation (LT) of SR, specifically its prediction of the inextricable mixing of space and time coordinates. To begin this investigation, attention will be directed to Einstein's original derivation of the LT [1], particularly the assumptions on which it rests.

II. Alternative Lorentz Transformation

The relationship between measured time intervals Δt and $\Delta t'$ for the same two events as determined by observers in different inertial systems S and S' follows directly from the LT:

$$\Delta t' = \gamma (\Delta t - v\Delta x/c^2). \quad (1)$$

In this equation, Δx is the distance between the locations of the two events in S and v is the relative speed of S and S' [$\gamma = (1 - v^2/c^2)^{-0.5}$]. Consider the case when the two events refer to the passage of an object between two points, so that $u = \Delta x/\Delta t$ is the speed of the object. Rearrangement of eq. (1) this gives:

$$\Delta t' = \gamma \Delta t (1 - vuc^2) = \gamma\eta^{-1} \Delta t, \quad (2)$$

i.e. with $\eta = (1 - vuc^2)^{-1}$. As long as both v and u are less than c , the signs of Δt and $\Delta t'$ will always be the same and there will be no violation of Einstein causality. The situation changes fundamentally if the object moves with $u > c$, however. In that case it is theoretically possible that even if the relative speed of S and S' is less than c , which it is assumed will always be true, the product of u and v may still exceed c^2 with the result that $\eta < 0$ in eq. (2), thereby constituting a violation of Einstein causality. In other words, the observer in S would find that the object passes point A before it arrives at point B , whereas his counterpart in S' finds just the opposite time-order of the two events.

Eq. (1) is also the basis for the claim that two events that are simultaneous for one observer will generally not be simultaneous for the other. It is often claimed that this lack of simultaneity is an inescapable characteristic of the relativity postulates. Clearly, if $\Delta t = 0$ for the two events but both v and Δx are not equal to zero, it follows from eq. (1) that $\Delta t' \neq 0$. However, one has to carefully specify what is understood by “simultaneity.” It does not mean that the “news” of two simultaneous events simply does not reach the two observers at the same time, for example. It is important to see that there has never been any experimental verification of “remote non-simultaneity.” Indeed, there is a counter-example in the way the Global Positioning System (GPS) technology is designed [8]. It is assumed that the time that a light pulse is emitted on a satellite is exactly the same for an observer there as for his counterpart on the earth’s

surface. One simply has to correct for differences in the rates of clocks located at the two positions in order to determine the actual time of flight on a given clock. The key point is that one assumes that there is a *strict proportionality* between elapsed times measured in the two rest frames, and therefore ignores any effect of the space-time mixing implied by eq. (1).

The experimental indications of faster-than-c motion suggest strongly that one should take a closer look at the theoretical arguments that appear to rule it out. To this end it is helpful to take note of an intermediate relationship between the measured times of the two observers that Einstein used in his original derivation of the LT [1]:

$$t' = \gamma \varphi(t - vx/c^2). \quad (3a)$$

In order to arrive at the final version of the LT it is necessary to make an additional assumption other than the two postulates of relativity (the relativity principle and the constancy of the speed of light in free space). Einstein stated without further discussion that the function φ in eq. (3a) only depends on v . He then showed by symmetry arguments that $\varphi = 1$ is the only acceptable solution, which in turn leads directly to eqs. (1-2) for time intervals. The same function appears in the other three equations of the general space-time transformation, as was pointed out earlier by Lorentz [9]:

$$x' = \gamma\varphi(x - vt) \quad (3b)$$

$$y' = \varphi y \quad (3c)$$

$$z' = \varphi z. \quad (3d)$$

The reason that such a function needs to be considered at all is because the light-speed postulate is not sufficient by itself to completely determine the transformation. The value of φ is clearly immaterial if only a particular relationship between measured velocities is to be assured since they involve *ratios* of the spatial and time variables in which φ does not appear. There is thus an additional degree of freedom that needs to be eliminated to arrive at the LT, and Einstein merely proposed to do this by making the above assumption for the functional dependence of φ . The fact is that making another choice for φ leads to a different version of the LT *which still satisfies the two postulates of relativity* just as well as Einstein's original, but without demanding that eqs. (1-2) be satisfied.

In view of the above discussion, a good way to proceed is to avoid making a definite choice for ϕ at this stage of the derivation, and move directly to the corresponding velocity transformation (VT) by dividing each of eqs. (3b-d) by eq. (3a), i.e $u_x = xt^{-1}$ etc. The resulting set of relationships is the same as obtained in Einstein's original work [1] (actually only the first of these equations is given there):

$$u_x' = (1 - vu_x/c^2)^{-1}(u_x - v) = \eta (u_x - v) \quad (4a)$$

$$u_y' = \gamma^{-1} (1 - vu_x/c^2)^{-1} u_y = \eta \gamma^{-1} u_y \quad (4b)$$

$$u_z' = \gamma^{-1} (1 - vu_x/c^2)^{-1} u_z = \eta \gamma^{-1} u_z, \quad (4c)$$

where \mathbf{u}' and \mathbf{u} are the respective velocities of the object of the measurement for two observers moving along the common x, x' axis with relative speed v [note that $\eta = (1 - vu_x/c^2)^{-1}$ is the same quantity as in eq. (2) given above]. The VT is obviously relevant to the general question of faster-than- c motion of particles. Inspection shows that there are three possibilities. First of all, if $u=c$, then $u'=c$ as well, consistent with Einstein's light speed postulate. In the normal case in which $u < c$, it is found that u' is also less than c (with $v < c$). On the other hand, in the case of special interest, if $u > c$ then u' will exceed c as well. There is certainly nothing controversial or even surprising about that, although there is one aspect of the latter result that requires further consideration below.

Once the VT is in hand, one is free to impose an additional condition to arrive at a suitable space-time transformation. It is quite easy to avoid any conflict with Einstein causality by insisting that there is *a strict proportionality between elapsed times for the same event* measured by the two observers. Experience with clocks on circumnavigating airplanes [10] indicates that the proportionality constant (Q) can take on any value depending on how fast each of them runs relative to a standard clock located at one of the earth's poles (the faster the clock moves relative to this position, the slower will be its rate). This suggests that a suitable alternative to eq. (1) is:

$$\Delta t' = Q^{-1} \Delta t. \quad (5)$$

In order to obtain a corresponding space-time transformation, it is then only necessary to combine eq. (5) with the VT of eq. (4) by multiplying $t' = Q^{-1}t$ with each of its relations. The result is the alternative Lorentz transformation (ALT) given below:

$$x' = \eta Q^{-1} (x - vt) \quad (6a)$$

$$y' = \eta(\gamma Q)^{-1} y \quad (6b)$$

$$z' = \eta(\gamma Q)^{-1} z \quad (6c)$$

$$t' = Q^{-1}t. \quad (6d)$$

Because of the proportionality relation of eq. (5), clearly no violation of Einstein causality is possible as long as $Q > 0$; the elapsed times $\Delta t'$ and Δt always will have the same signs. Moreover, there is no need to assume remote non-simultaneity for the two observers since $\Delta t'$ must vanish whenever Δt does.

It is also not necessary to assume that the phenomenon of time dilation is *symmetric*, contrary to the case when the LT is used. The latter transformation forces one to believe *that two clocks in motion must each be running slower than one another*, whereas experiment indicates unequivocally that this is not the case [10,11]. When eq. (5) is used instead, the value of Q always determines unambiguously which of the two clocks is slower and by what fraction.

Another way of deriving the ALT is to assume that $\phi = \eta(\gamma Q)^{-1}$ in the general version of the LT given in eqs. (3a-d). Note that this choice of ϕ is not compatible with Einstein's assumption in his original paper that ϕ can only depend on v . The ALT version of ϕ depends on η , which is a function of both v and u_x , the component of the object's velocity that is parallel to the relative velocity \mathbf{v} of S and S' . One of the main consequences is that y and z are not equal to y' and z' , respectively, unlike the case for the LT. There is no *a priori* reason why y must always be equal to y' , any more than that u_y must always be equal to u_y' in the VT.

III. Tachyons and Photons

The dependence of kinetic energy W on velocity provides another constraint on the maximum speeds attainable [1]:

$$W = (\gamma - 1)\mu c^2, \quad (7)$$

where μ is the rest mass of the particle. Since $\gamma = (1 - v^2/c^2)^{-0.5}$ approaches infinity in the limit of $v=c$, Einstein concluded [1,12] that "velocities greater than light have ...no possibility of existence." Implicit in his statement is the assumption that photons are exempt from this constraint because they possess null rest mass

($\mu=0$). The possibility of faster-than- c speeds brings a new consideration into this discussion because γ then takes on imaginary values. Feinberg [13] coined the term “tachyons” to describe such hypothetical particles. They would have the interesting characteristic of having imaginary kinetic energies even though the corresponding rest energy μc^2 is real.

The above speculation about tachyons rests on the assumption that eq. (7) enjoys general validity. *It is easy to show that this is not the case by considering the very practical case of light in transparent media.* Planck’s relation,

$$E=h\nu, \quad (8)$$

allows one to determine the energy E of the photons in free space by measuring the frequency ν of the corresponding radiation (h is Planck’s constant, 6.625×10^{-34} Nms). When light enters a medium such as water with $n_g > 1$ (Fig. 1), its speed decreases to $u=n_g^{-1}c$ [3], causing the value of γ in eq. (7) to become finite. Accordingly, this would mean that the energy of the photons should vanish, but *experiment indicates that neither ν nor E is altered* as a result of the change of medium.

Newton has often been criticized for his prediction in *Opticks* [14] that the speed of light should increase as it enters water from free space. Closer inspection of the arguments that led him to this false conclusion reveals something interesting, however. As indicated in Fig. 1, Newton was using his Second Law of Kinematics to show that the momentum of “corpuscles” of light must be inversely proportional to the sine of the angle Θ of refraction:

$$p_1 \sin \Theta_1 = p_2 \sin \Theta_2. \quad (9)$$

He compared this theoretical result with Snell’s Law of Refraction,

$$n_1 \sin \Theta_1 = n_2 \sin \Theta_2, \quad (10)$$

concluding that *the momentum of photons is directly proportional to the index of refraction n of the medium.*

However, there is another empirical result of interest in this connection, namely that *the wavelength of light λ is proportional to n ; hence,*

$$\lambda_1 \sin \Theta_2 = \lambda_2 \sin \Theta_1. \quad (11)$$

Comparison of the latter relation to Newton’s eq. (9) therefore leads directly to the conclusion that *the momentum of the photons is inversely proportional to the*

wavelength of the associated radiation. This result in turn is clearly consistent with the de Broglie relation [15],

$$p = h \lambda^{-1}, \quad (12)$$

which is known to hold for electrons and other massive particles as well as for photons in free space [16]. Since $\lambda\nu=c$ in free space, the proportionality between p and n assumed by Newton thus leads one to conclude from eqs. (8,12) that

$$p = nh\nu c^{-1} = nEc^{-1} \quad (13)$$

holds on a general basis for photons in normally dispersive media, including the limiting case of $n=1$ in free space. Furthermore, it can be shown [17] that eq. (13) leads directly to the empirical relations between n and n_g and the speed of light:

$$n_g = n + \lambda dn/d\lambda \quad (14)$$

$$dE/dp = u = cn_g^{-1}. \quad (15)$$

These equations allow for a continuous change of the speed of photons from values less than c in normally dispersive media through a value of c in free space to higher values in media with $n_g < 1$. One simply has to conclude that eq. (7) is not valid for photons except in the limiting case of $n_g = n = 1$ in free space, even though there is considerable empirical evidence that it does hold universally for particles with non-zero rest mass.

The latter conclusion is also reinforced by a calculation of the inertial mass of photons in dispersive media based on the above equations. From the definition of momentum p as the product of inertial mass m and speed u , it follows from eqs. (13,15) that

$$m = pu^{-1} = nh\nu c^{-1} (cn_g^{-1})^{-1} = nn_g h\nu c^{-2} = nn_g Ec^{-2}, \quad (16)$$

which again is different from the classical Einsteinian relation [1] for massive particles ($\mu \neq 0$) of $m = Ec^{-2}$ except for the special case of $n = n_g = 1$ in free space.

IV. Adapting the VT to $v > c$ Motion

Replacing the LT by the ALT removes the supposed conflict between the Einstein causality principle and faster-than- c motion in general. It is helpful to consider a specific case in which the observer O' in S' views a particle that is moving in S with speed $u > c$ along the x -axis. His measured values for $\Delta x'$, $c\Delta t'$ and $u'c^{-1} = \Delta x'/(c\Delta t')$ are computed from eqs. (4-6) using the proportionality factor $Q = \gamma$ for the specific case of $u = 2c$ ($n_g = 0.5$ for light refraction), $\Delta x = 2$ and

$c\Delta t=1$. The results are plotted in Fig. 2 as a function of the relative speed v of S and S' . The sign convention is such that $\beta=vc^{-1}>0$ when S' is moving in the same direction as the particle. Qualitatively for each data point, observer O in S sees the particle moving with constant speed u and observer O' moving with constant speed v relative to his position.

First of all, it is seen that the $c\Delta t'$ curve has a very simple form because of eq. (5) of the ALT. The rate of the clock in S' is always γ times slower than that in S . This means that the elapsed time measured by O' has a maximum when $v=0$, i.e when S and S' are not moving relative to one another. More significantly as far as the causality principle is concerned, $\Delta t'$ and Δt always have the same sign. The situation is quite different when eq. (2) of the LT [1] is used (further discussion of this point may be found in earlier work [18]). In that case, $\Delta t'$ vanishes when $uv=c^2$, i.e when $\beta = 0.5$ in the present example, and it takes on negative values for still greater speeds of S' relative to S , thereby violating Einstein causality.

Changing from the LT to the ALT has no effect on the way the speed u' of the particle varies with the relative speed v of the two inertial systems. This is because both transformations are consistent with the VT of eqs. (4a-c), as discussed in Sect. II. The fact that $uv=c^2$ for $\beta = 0.5$ in Fig. 2 means that there is a singularity in u' at that point. In general, the manner in which u' varies with β according to the VT is counter-intuitive because it means that the faster O' approaches the particle, the more it appears to speed up from his vantage point. It is not a problem to understand this result from a purely arithmetical point of view. The denominator $(1-uv/c^2)$ in eq. (4a) simply decreases faster than the numerator $u-v$ as β increases. Analogous behavior occurs whenever $u>c$ in the region $\beta<cu^{-1}$.

Something truly unusual happens when β increases beyond the singular point in eq. (4a), however, namely the particle appears to change direction, with u' abruptly changing from $+\infty$ to $-\infty$. It is easy to avoid this situation, however. One simply has to define η in both eqs. (4,6) to be a positive definite quantity, i.e. $\eta = |1-uv/c^2|$. There is nothing in the derivation of either the VT or the ALT to preclude this possibility on a theoretical basis. The corresponding equations thus become:

$$u_x' = |1 - vu_x/c^2|^{-1}(u_x - v) = \eta (u_x - v) \quad (4a')$$

$$u_y' = \gamma^{-1} |1 - v u_x/c^2|^{-1} u_y = \eta \gamma^{-1} u_y \quad (4b')$$

$$u_z' = \gamma^{-1} |1 - v u_x/c^2|^{-1} u_z = \eta \gamma^{-1} u_z \quad (4c')$$

in the case of the VT, and

$$x' = |1 - v u_x/c^2|^{-1} Q^{-1} (x - vt) = \eta Q^{-1} (x - vt) \quad (6a')$$

$$y' = |1 - v u_x/c^2|^{-1} (\gamma Q)^{-1} y = \eta (\gamma Q)^{-1} y \quad (6b')$$

$$z' = |1 - v u_x/c^2|^{-1} (\gamma Q)^{-1} z = \eta (\gamma Q)^{-1} z \quad (6c')$$

in the spatial equations of the ALT. The variation in both u' and $\Delta x'$ with β in Fig. 2 has been computed on this basis. In this way, the “common-sense” expectation is maintained in the relativistic equations that an object does not appear to change direction until the speed of the observer exceeds that of the object. The latter condition can never occur for faster-than- c speeds of the object, although it is routinely found in every-day experience with corresponding sub- c speeds.

This still leaves open the question of whether the singularities in both u' and $\Delta x'$ can actually be observed. This would again seem to violate the principle that only finite values of physical quantities can occur. Since these are only point singularities, one can argue that such an observation is ruled out because of the impracticality of attaining *exactly* the speed at which the singularity should occur. In other words, as β is increased in this region, observer O' would first find u' for the object to be increasing to very large values, and then to suddenly start decreasing as the singular value is surpassed.

V. Neutrino Speeds

The discussion in the previous sections speaks in favor of faster-than- c speeds of particles on a general basis. The main point is that previous arguments against this phenomenon because of a perceived violation of Einstein causality overlook the possibility that there could be a problem with the LT. There is no reason to accept Einstein's position [1] that the undefined function ϕ in his original derivation of the LT can only depend on the relative speed of the relevant inertial systems, or, equivalently, that the corresponding observers in these rest frames must agree on the lengths of all line segments that are oriented perpendicularly to their relative velocity. Experiments that have been carried out on airplanes and satellites since Einstein's development of SR indicate instead

that the clock rates of such observers must be strictly proportional to one another so long as neither one of them experiences acceleration. Making this assumption leads to the ALT which a) removes any connection between faster-than- c speeds and Einstein causality and b) also supports remote simultaneity as a long-standing principle of physics.

When attention is turned to the question of what particles actually can attain faster-than- c speeds, it is necessary to confront another issue raised by Einstein in his original work, however. There is considerable experimental evidence to indicate that his formula for the kinetic energy given in eq. (7) is universally valid for particles *with non-zero inertial mass* such as electrons and protons. As a result, his conclusion that any $\mu \neq 0$ particle is necessarily restricted to sub- c speeds seems unassailable. However, it has been shown in Sect. III that this relation is not applicable to photons passing through dispersive media, which therefore leaves open the possibility that particles with null rest mass are generally exempt from this rule.

If neutrinos can move with faster-than- c speeds, as experiment now seems to indicate, two independent conclusions would appear to be justified. The first is that the LT is not valid and needs to be replaced with a modified version of relativity theory that insists on the strict proportionality of clock rates. The second is that the rest mass of the neutrinos must be exactly zero. The latter is at odds with most theories of particle physics, especially with the conclusion that there are different flavors of neutrinos.

The experience with photons indicates further that neutrinos always travel with $v=c$ in free space. Whenever photons are in any other environment, their speed is affected as well as their momentum. Experiments with single-photon counting [19] suggest that the speed of each photon is the same as long as the interacting medium is uniform. Whatever this interaction might be is not well understood because of the complexity of the media. This indicates that there also has to be some sort of interaction that affects the speed of neutrinos, despite their well-known penetrability characteristics. It is presumably of extremely short range, consistent with what is generally assumed for the weak interaction of particle physics.

VI. Conclusion

The recent experimental indications that neutrinos can travel faster than light in free space represent a crucial test of the validity of the Lorentz transformation (LT) of the special theory of relativity (SR). The space-time mixing that is an essential feature of the latter theory forces one to conclude that Einstein causality must be violated in order for such speeds to be attained by any particle. This includes photons passing through media with refractive index $n_g < 1$ (anomalous dispersion), even though there is also strong evidence that $u > c$ speeds do occur in this case as well.

The point that needs to be clearly understood is that the unequivocal verification of any or all of these empirical findings would indeed contradict the LT, thereby eliminating it from consideration as a valid physical theory. There is absolutely no way to avoid this conclusion, and this is ultimately the reason that physicists have been so reluctant to accept the experimental evidence for faster-than- c particles at face value. Nonetheless, the viability of the corresponding velocity transformation (VT) and the relativity postulates on which it is based would not be affected in any way. One simply has to assume that the rates of clocks in motion are strictly proportional to one another. This is indeed the result of numerous experiments that have been carried out in the last 50 years with atomic clocks located on airplanes and satellites. Making this choice leads directly to an alternative Lorentz transformation (ALT) that is every bit as consistent with the relativity postulates as the original LT. Ultimately, it is the much revered concept of *space-time mixing that is at odds with faster-than- c motion*, not the postulates of light-speed constancy in free space and Galileo's relativity principle.

Finally, the existence of faster-than- c motion of neutrinos has an important consequence for elementary particle theory. It forces one to conclude that neutrinos have exactly null rest mass ($\mu=0$), the same as for photons. This characteristic is essential for them to be exempt from Einstein's conclusion in his original work, based on the manner in which kinetic energy varies with speed in

eq. (7), of the impossibility of any particle with non-zero rest mass attaining $u=c$ and beyond.

(Nov. 15, 2011)

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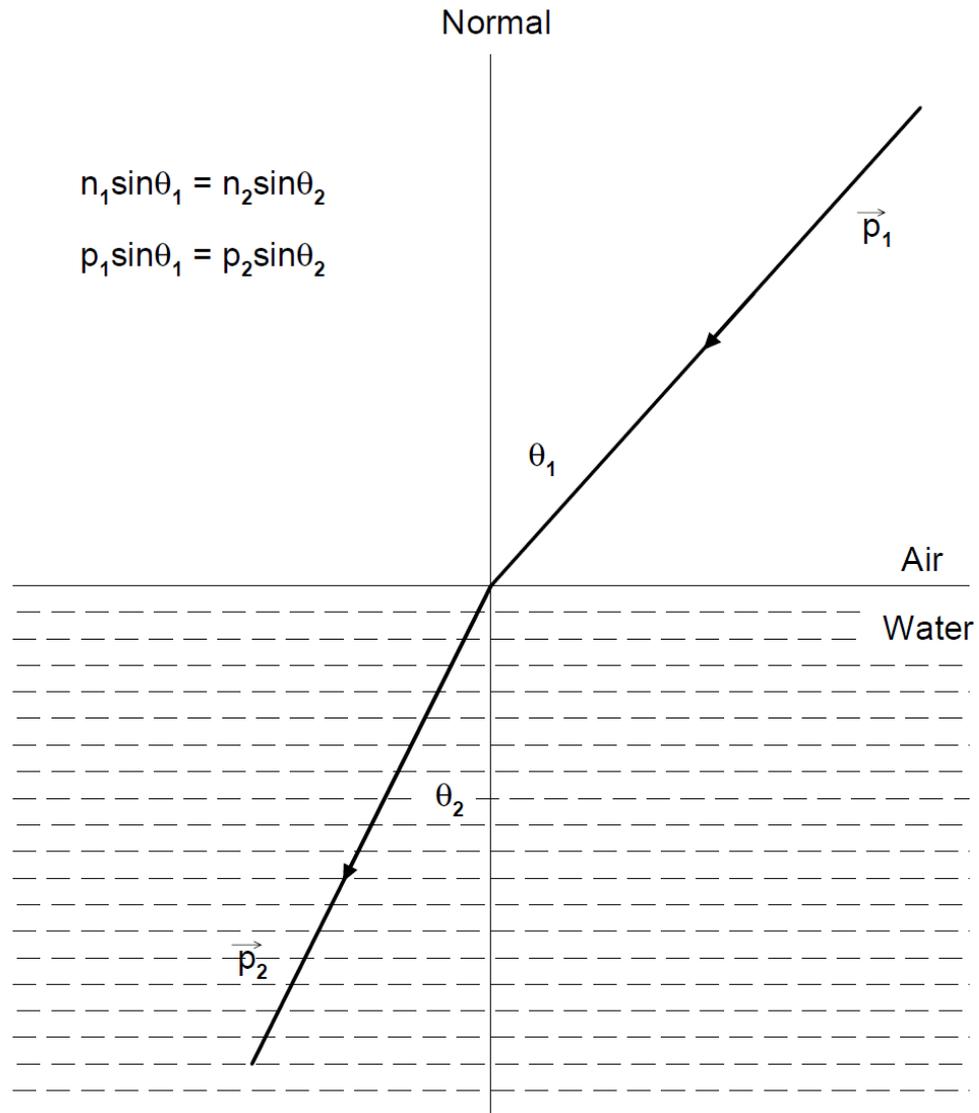


Fig. 1. Diagram showing the refraction of light at an interface between air and water. The relation between the angles of incidence θ_1 and refraction θ_2 in terms of the refractive indices n_i (Snell's Law of Sines) of the two media was viewed by Newton as a clear application of his Second Law of kinetmatics, according to which the component of the photon momentum p_i parallel to the interface must be conserved.

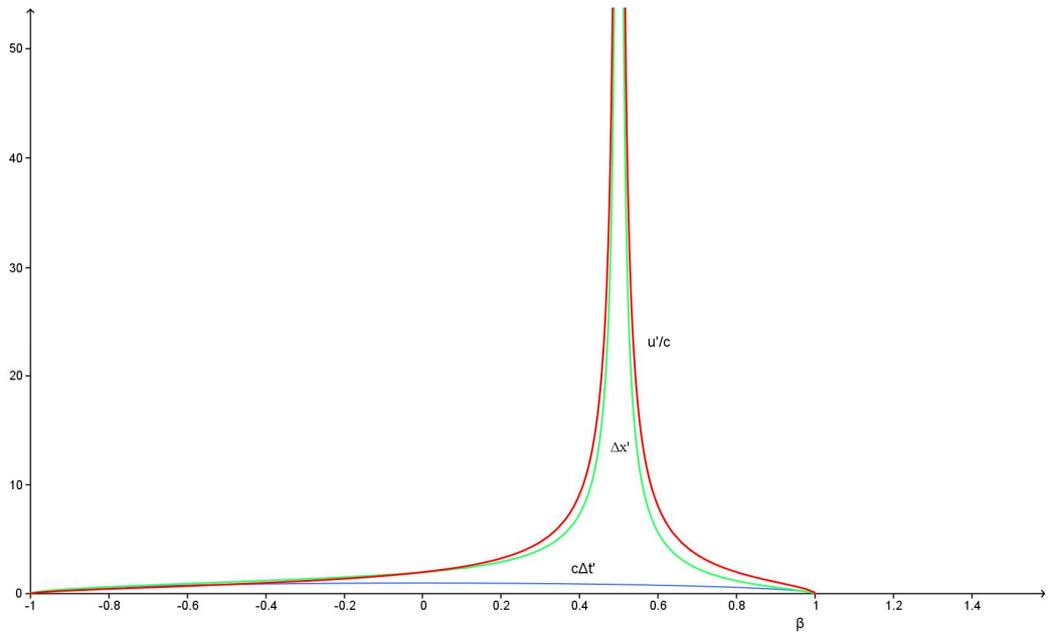


Fig. 2. Diagram showing the variation of the spatial and time intervals $\Delta x'$ and $c\Delta t'$ as a function of the relative speed $\beta = vc^{-1}$ of the observer for a particle travelling at $u=2c$ relative to the inertial system with $\beta=0$. Note that $\Delta t' > 0$ throughout, so that no violation of Einstein causality occurs. The variation of the particle speed $u'c^{-1} = \Delta x'(c\Delta t')^{-1}$ is also shown, as calculated from eq. (4a) of the VT. The values of $\Delta x'$ and $c\Delta t'$ are calculated using eqs. (6a,6d) of the ALT.