The Transverse Doppler Effect and the Relativity Principle: Experimental Test of Length Contraction

Robert J. Buenker

Fachbereich C-Mathematik und Naturwissenschaften,
Bergische Universität Wuppertal, Gaussstr. 20,
D-42119 Wuppertal, Germany

Abstract

The second-order or transverse Doppler effect (TDE) demonstrates that both the periods and wavelengths of radiation increase (after the non-relativistic first-order effect has been eliminated) when a light source is accelerated relative to the observer. It is pointed out that the fact that an observer traveling with the light source does not detect any change in either quantity, in accord with the relativity principle (RP), implies not only that there has been a corresponding increase in the periods of all naturally occurring processes in his rest frame but also a uniform *expansion* in the dimensions of all objects co-moving with him. These results constitute a key verification of Einsteinean time dilation, but it has not been recognized that they also stand in contradiction to another fundamental prediction of the special theory of relativity (SR), the Fitzgerald-Lorentz contraction effect (FLC). A survey of past claims for the validity of the FLC shows that they are either based on Gedanken experiments or on a specious interpretation of

lifetime measurements for accelerated metastable particles. The manner in which the FLC is derived from the Lorentz transformation (LT) is then discussed and it is noted that it is based on a generalization of the original interpretation of its space-time variables in terms of elapsed times and distances traveled by a given object, and therefore requires separate experimental verification than observations of the constancy of the speed of light. Experimentation with the Global Positioning System (GPS) is shown to be consistent with the TDE results and thus verifies the conclusion that isotropic length expansion accompanies time dilation, not the type of anisotropic length contraction (FLC) indicated by the LT.

I. Introduction

The transverse Doppler effect (TDE) has long been accepted as an experimental proof of time dilation. Ives and Stilwell [1] were the first to demonstrate this purely relativistic effect by employing excited hydrogen atoms with kinetic energies of up to 10 keV. They measured the displacement of the H_β line from its unshifted position to the center of gravity of the two lines recorded on the same photographic plate when the light source was in motion toward and away from it, respectively. Similar experiments were subsequently reported by Otting [2] and by Mandelberg and Witten [3]. Kündig [4] later employed the Mössbauer technique to demonstrate the TDE to even higher accuracy (1%). In this case the frequency of an ^{57}Fe x ray source was measured with a detector near the rim of an unltracentrifuge [5]. The common conclusion from all these experiments is that the frequency ν of light emitted from an accelerated source is smaller than the value ν_0 measured in the laboratory when the same source is at rest there.

The TDE is a purely relativistic effect that is second-order in v/c (v is the speed of the light source relative to the laboratory and c is the speed of light in free space). It is normally swamped by the first-order Doppler effect, but the latter has been carefully eliminated in each of the above experiments. The TDE might more properly be referred to as the second-order Doppler effect. As such, it is independent of the direction of the light source's velocity relative to the laboratory. The time-dilation effect proposed by Einstein [6] in his 1905 paper on the special theory of relativity (SR) becomes relevant because of an argument [7] based on the relativity principle (RP), as will be discussed below.

II. Time Dilation and Length Contraction

Two separate derivations of the relativistic Doppler effect can be found in the literature. One makes use of the invariance of the phase of a harmonic plane wave with respect to a Lorentz transformation [8]. The other makes direct use of the time-dilation effect [9] and is thus better suited for the present discussion. First, one shows that the period of pulses reaching the observer must be proportional to the factor $(1 + v_r/c) = (1 + v \cos \chi /c)$, where $\chi=0$ corresponds to motion of the source directly away from the observer. There is also a second-order contribution, however, due to the fact that clocks run slower by a factor of $\gamma (v) = (1-v^2/c^2)^{-0.5}$ in the rest frame of the source. If the *in situ* period for the light source is T_0, the value measured in the laboratory is therefore

$$T = \gamma T_0 (1 + v \cos \chi /c). \qquad (1)$$

The wavelength λ is equal to cT, so the corresponding expression for the Doppler-shifted wavelength is

$$\lambda = \gamma \lambda_0 (1 + v \cos \chi /c). \qquad (2)$$

For transverse radiation, $v_r = 0$ and $\chi= \pi/2$, so it is clear that there is no first-order effect in this case (TDE). For radiation observed in any other direction, however, the same value can be obtained by eliminating the first-order effect, as already mentioned in the Introduction.

The second-order Doppler effect is therefore independent of the direction of the light source's motion relative to the laboratory observer. It is a direct measure of the amount of time dilation in the rest frame of the light source. The values of T and λ in the above equations are given with respect to the units of time and distance in the rest frame of the laboratory. In other words, they can be looked upon as the values one *would* obtain if one could directly employ the clock and meter stick located in the laboratory rest frame to carry out measurements in the rest frame of the accelerated source. This is not actually possible for the simple reason that the clock rate and the length of the meter stick change as soon as they are accelerated with respect to the laboratory rest frame.

It is nonetheless possible to test this interpretation with the aid of other investigations not involving the TDE itself. In one such set of experiments [11-15], the spontaneous disintegration of accelerated pions and muons has been measured in both the upper atmosphere and in the laboratory. It was shown to quite high accuracy that the lifetimes of these particles increase in direct proportion to their γ value relative to the rest frame of the laboratory. One can also look upon the muons and pions as natural clocks. Their lifetimes are measured relative to a standard clock in the laboratory and they are indeed found to be larger than for identical particles that have not been accelerated.

A different type of experiment that leads to the same conclusion is that carried out by Hafele and Keating [16] with circumnavigating airplanes. Identical clocks were located on the Earth's surface and on two airplanes traveling in opposite directions around the globe. After correcting for gravitational effects, it was found that the clock on the plane traveling in an easterly direction was slower than the one left behind at the airport. The westbound clock actually gained time relative to the latter, but this could be explained as the consequence of the Earth's rotation about its polar axis. The observed differences in elapsed times for these clocks were found to agree with Einstein's predictions to within an error of about 10%.

Taken together, these two sets of experiments exclude any possibility that the periods of the radiation in the TDE experiments simply change in going between the light source and the observer in the laboratory. They show instead in an unequivocal manner that light frequencies and wavelengths do change when the light source is accelerated.

Nonetheless, there is no evidence to suggest that an observer traveling with an accelerated source will detect these changes. This is the result that one expects from the RP, since otherwise it would be a rather simple matter for the local observer to detect that he has changed his state of motion, even after the acceleration phase has been completed and the rest frame of the light source is again an inertial system (the ultracentrifuge experiments show that the time-dilation formula also holds when the light source is subject to a very high degree of acceleration [4,5]). An obvious question arises from this state of affairs: why doesn't the local observer detect a change in the period and wavelength of radiation emitted from the accelerated source? The simple answer is that all local clocks have slowed by exactly the same proportion and so the observer in the rest frame of the light source must continue to measure the same value for the frequency no matter how great his speed relative to the laboratory from which he departed. Another way of expressing this point is to say that the unit of time has increased from its initial value of 1 s in the laboratory rest frame to γ s in that of the light source [7]. In absolute terms both observers obtain the same result for the period of the radiation, but the one moving with the light source obtains the smaller value of T_0 because his result is given with respect to the larger unit of time.

The latter argument can be readily accepted because it is perfectly consistent with Einsteinean time dilation [6]. The same line of reasoning is used to explain why observers at different gravitational potentials disagree on the magnitude of a given light frequency [17]. In that case the unit of time is shorter on top of the mountain than it is in the valley below, but the *in situ* value for a given light source is the same at each location.

There is still one issue to resolve for the TDE, however. The observer co-moving with the light source also does not detect any change in the wavelength of the radiation, even though his counterpart in the laboratory finds that it has increased by the same factor of γ that the period has. Again, the object is the same for both observers, namely light waves of exactly the same frequency in absolute terms. The explanation must be completely consistent with what has already been discussed for periods of the radiation. Not only must the unit of time increase in the accelerated rest frame of the light source, but also the unit of length. Moreover, the latter must change in exactly the same proportion in all directions. Otherwise, it is impossible to explain why the second-order Doppler effect for wavelengths is the same for each angle of approach χ in eq. (2), that is, $\lambda = \gamma \lambda_0$. The observer co-moving with the light source has no means of detecting this change in wavelength because any and all devices that he might use to make this determination have increased in length by the same factor of γ. It is exactly the same argument that has been accepted for many decades for radiative periods [7].

If the unit of length did not change in direct proportion to the unit of time, regardless of orientation of the measuring device, it would also be impossible to explain why both observers measure exactly the same value for the speed of light in every direction. Speed (or velocity) is a ratio of the length traveled by an object to the corresponding elapsed time. If the unit of length is changed from 1 m to 1 cm, the numerical value of the distance traveled must increase by a factor of 100. The value of the speed can nonetheless remain the same as long as a corresponding decrease in the unit of time is made, that is, from 1 s to 0.01 s. Clearly, the same situation holds for the numerical values of wavelengths and periods of radiation, with the result in this case that their ratio, the phase velocity of light, is also left unchanged when such a proportional change in the units of length and time is introduced.

As straightforward as the above interpretation of the TDE is, it stands in direct conflict with another tenet of SR [6], the Fitzgerald-Lorentz contraction effect (FLC). The latter holds that the lengths of objects contract upon acceleration, and that the amount of this change depends on orientation. Specifically, there is no contraction at all when the length is measured along a direction that is transverse to its velocity relative to the stationary observer, whereas the maximum effect (contraction by a factor of γ) occurs along the radial direction. The TDE in combination with the RP indicates quite the opposite, namely that objects expand uniformly in all directions in the rest frame of the accelerated light source.

III. Other Tests of the Length Contraction Hypothesis

The FLC has a long history [18] that predates Einstein's original paper on STR [6]. Fitzgerald [19] introduced it in 1889 in an attempt to reconcile the results of the Michelson-Morley experiments [20] within the framework of Newtonian mechanics and the existence of an ether as a medium for transmitting electromagnetic radiation. Einstein later derived the FLC from the Lorentz transformation (LT) equations. He saw it as an objectively real effect [18] and speculated at some length as to how it might be observed [6]. He expected this to be especially difficult in view of the fact that the FLC is of second order in v/c.

Since that time there have been numerous discussions in the literature concerning the FLC, and many so-called paradoxes have been suggested and analyzed in great detail [21]. A key point in these discussions is the requirement that the measurement of both termini of the rapidly moving object must occur simultaneously (dt=0) in the observer's rest frame in order to obtain a valid result. There is a broad consensus that the theory is internally consistent and that all objections to the FLC can be removed by careful attention to detail. In the present context, it is important to add that there has never been a confirmed experimental verification of the FLC, however. The main argument for its validity continues to be that it is an integral part of SR, which has otherwise withstood any challenges to its authority over the last 100 years. It is also noteworthy, however, that the relevance of the TDE to the FLC has been absent in all such discussions, at least those known to the author. Instead, the tendency has been to rely on Gedanken experiments involving the transmission of light signals between the observer and the object of the hypothetical measurement. These discussions inevitably assume that the FLC is valid and that no contradictions arise as a consequence, but no actual experiment is ever carried out.

As remarked above, the general attitude is that any apparent conflict regarding the FLC can be removed by paying strict attention to the requirement that the locations of both ends of the object must be determined simultaneously. This is not a problem for the TDE measurements under discussion, however. The observer in the laboratory records a wave pattern from a moving light source on a photographic plate, similarly as in the original experiments carried out by Ives and Stilwell [1]. No light signals are required other than those emitted from the light source itself. The other observer co-moving with the light source carries out an identical wavelength measurement with his own photographic plate or equivalent device. Both results are completely reproducible as long as the conditions of measurement remain the same (for example, that the light source always be moving at a particular velocity relative to the observer in the laboratory and that the *in situ* frequency of the light emitted from the source be the same in each instance). Under the circumstances, it does not matter if the two observers make their respective wavelength determinations at anything like the same time.

There is another claim (see, for example, Refs. [22,23]) of a verification of the FLC that needs to be considered, however. It is based on the experiments with accelerated meta-stable particles [11-15] already mentioned. The argument goes as follows. The observer in the laboratory O measures the elapsed time for the particles to travel a certain distance at speed v to be Δt. The corresponding value for the distance traveled is thus $L(O) = v\Delta t$. Because of the time-dilation effect, however, an observer M who is co-moving with the particles must find the elapsed time on the clock in his rest frame to be only $\Delta t\gamma^{-1}$. Since it is assumed that the speed of the particles is the same for both observers, it follows that the corresponding distance measured by M is $L(M) = v\Delta t\gamma^{-1}$, that is, M measures a shorter distance than O by a factor of γ. It is then concluded that this result is consistent with *length contraction in M's rest frame.*

There are a number of problems with this conclusion, however. According to the FLC, the amount of length contraction is not the same in all directions. If the particles are moving transverse to their velocity vector relative to O, for example, the FLC states that there should be no difference in the distances they measure, contrary to what has been found above. The conclusion that L(O) = γ L(M) is based solely on the time-dilation effect. This ratio is therefore independent of the direction that M travels relative to O. The other problem is more basic. If the FLC were correct and the particles' motion was radial, this would imply that the measuring rod M uses to make his length determination *would be contracted by a factor of* γ. That would mean in turn that he must obtain a *larger* numerical value for the distance traveled in this case, that is, L(M)= γL(O), which is also in contradiction to what is deduced on the basis of time dilation.

The distance traveled by the particles in the above example is actually the same for both O and M in absolute terms. The only reason that their measured values are not the same is because their unit of distance is different. The time-dilation effect cannot be disregarded in this discussion because it has been experimentally verified on a quite general basis. There is thus no question that M must measure a smaller value for the distance it travels than does O. In addition, the ratio of their respective length determinations must be *independent* of the direction of their relative motion as long as the speed v is the same in all cases. For this to happen, M's measuring rod must be larger than O's by exactly a factor of γ, so this example actually is perfectly consistent with what has been deduced in the previous section on the basis of the TDE and the RP. It confirms that objects in the accelerated rest frame are subject to *isotropic expansion*, not the type of anisotropic length contraction predicted by the FLC.

In more recent times there has been a development that underscores the soundness of the above conclusions. Since 1983 the meter has been redefined as the distance light travels in a certain amount of time. The key point is that the units of length and time are no longer determined independently as a result of this decision. It is therefore interesting to see how this definition is related to the present discussion of changes in the lengths of objects that occur solely because of their acceleration.

The basic idea is illustrated by the following Gedanken experiment. Assume that observers O and M in the previous example are not in relative motion at the beginning of the experiment. They both measure the elapsed time it takes for a light pulse to travel between two fixed points A and B in their laboratory on a rocket ship. They each find it to have a value of Δt since their respective clocks run at the same rate. According to the above definition, this means that the distance between A and B is equal to $c\Delta t$ for both of them. Then the rocket ship is accelerated with M on board until it reaches a constant velocity $\mathbf{v}$ relative to O, who stays behind at the original position. The experiment with the light pulse is repeated, and in accordance with the RP, M obtains the same value Δt as before for the time it takes the light pulse to travel between A and B on the rocket ship. Because of the time-dilation effect, it is known that the clock that remains in O's rest frame runs faster than M's by a factor of $\gamma(v)$. On this basis, it is not necessary for O to carry out a measurement of the elapsed time on the rocket ship at the same time that M does. Indeed, he doesn't have to carry out a second measurement at all. He knows with certainty that the elapsed time for the light pulse to travel between A and B is $\gamma\Delta t$. Because of the definition of the meter, he therefore finds that the distance between these two points has also changed as a result of the rocket ship's acceleration. It now has a value of $\gamma c\Delta t$. Just as in the previous example, the value of this length is completely independent of the direction the rocket ship moves relative to O because the time-dilation effect is the same for all orientations. Since O references all his measurements to the same clock/meter stick, there is only one explanation for his observations: the distance between A and B must have physically increased by a factor of γ. The fact that M does not detect this change is consistent with the assumption that the lengths of all objects in the rest frame of the rocket ship have increased in exactly the same proportion upon acceleration to their current speed relative to O.

The Global Positioning System (GPS) is actually based on exactly the same principles as are used above in the Gedanken experiment. As discussed elsewhere [24], the key element underlying the success of the GPS technology in making predictions of distances on the Earth's surface is the existence of atomic clocks on satellites that have been "pre-corrected" so that they run at exactly the same rate as identical clocks at rest on the Earth's surface. The amount of the rate corrections is determined in a manner that is completely consistent with that employed in the Gedanken experiment. After accounting for gravitational effects, one therefore must conclude on this basis that the time for a light pulse to traverse a rod of L m on the satellite will be Lc^{-1} s on the local clock but γLc^{-1} s on the corresponding clock on Earth. Since the speed of light is equal to c (again after accounting for gravitational effects) in both locations, it follows that the observer on Earth finds that the length of the rod is *longer* (γL m) than does his counterpart on the satellite. This result is consistent with isotropic length expansion, but not with the contraction effect predicted by the FLC and the LT. Because of the RP, one assumes that both observers must have measured the length of the rod to be L m *before* it was sent into orbit because no change in its length could have been detected by the observer traveling with it on the satellite. Since his counterpart on Earth does not change his state of motion throughout the experiment, his unit of length (meter stick) must have remained the same. The only plausible explanation for the above results is that both the rod and the meter stick carried on the satellite have increased by a factor of γ as a consequence of being put into orbit.

IV. The Lorentz Transformation and the FLC

Whenever a prediction from a physical theory is contradicted by experiment, it is necessary to revisit the logical premises on which it was based. Einstein [6] derived the FLC directly from the LT, starting from the equation given below:

$$dx' = \gamma (v) [dx - v\, dt]. \qquad (3)$$

The definition of the variables in this expression is crucial. In his original derivation of the LT, Einstein introduced two inertial systems, S and S', in constant relative motion to one another. Observers in each of them record their measurements for the elapsed time and distance traveled by a light pulse. In his subsequent derivation of the FLC, a more general definition was used, however. In this case, dx and dx' are the respective distances between two events as measured by the two observers along the direction of relative motion. To obtain the FLC, one simply sets dt=0 in eq. (3) and equates dx and dx' with the respective length measurements of a given object that is located in the rest frame of S'. The result is that $dx' = \gamma dx$, that is, the length measured in the rest frame of the accelerated observer in S' is larger than for the other observer in S. Since the corresponding LT equation when the object is measured along a perpendicular direction is simply $dy = dy'$, it follows that there is no difference in this case.

There is a qualitative difference between the above two interpretations of the LT, however. This is most easily seen by considering the condition, dt=0. This value is not physically relevant in the first case. If no time elapses, there is no motion to consider and thus no conclusions can be made on this basis. On the other hand, when one defines dt to be the time difference between two unconnected events, a null value simply refers to the case when the respective measurements were carried out simultaneously in the rest frame of one of the inertial systems. While either interpretation is possible in theory, it needs to be recognized that each of them requires *a separate experimental verification.* In the case of the light pulse, experiment has shown that its speed is the same for both observers, as assumed in Einstein's original derivation of the LT [6]. This only shows that the interpretation of its variables as distances traveled by an object in a given elapsed time is tenable. As such, the LT represents a critical advance over the Galilean transformation, which uses the same interpretation for its space-time variables. There is no compelling reason to conclude that the same experimental evidence justifies the other more general interpretation, however, because it relies on an assignment for one of these variables which has no physical meaning for the special case actually tested, namely dt=0.

Finding independent evidence that the FLC actually does not occur in nature does not in itself constitute a violation of the LT. It simply proves by contradiction that the interpretation of its variables as distance and time intervals separating two general physical events is incorrect. The oft-stated argument that the FLC must be satisfied in actual measurements because the rest of STR is so strongly founded on experimental observations is therefore unjustified. By restricting the definition of the LT variables to measured values for the distance traveled by an object in a given elapsed time, one preserves the integrity of the LT without coming into conflict with any of the experimental evidence mentioned above.

There is another point that needs to be discussed in the present context, however. The interpretation of the LT variables as distances and times between events led Einstein to successfully predict the time-dilation effect, which has received unequivocal experimental verification. There is no contradiction in this. Experiment can never be taken as proof of a theoretical assumption, however. Once one finds that a given assumption is invalid, it is also necessary to go back and find a different explanation for why it appears to work in other applications, such as for time dilation in the present case. In this connection it is important to note that quantitative experimental verification of this effect, that is, where one clock runs slower than the other by exactly a factor of γ, only occurs under quite specific conditions, namely when the former is accelerated directly from the rest frame of the other. The Hafele-Keating experiments with circumnavigating airplanes [16] show that one cannot generally compute the ratio of two clock rates solely on the basis of their relative speed to one another [25].

In a companion paper [24], it is shown that the LT has another problem in explaining the GPS method, however, namely with its prediction of the non-simultaneity of events. The success of the GPS navigation system rests on a clear assumption, namely that the rates of atomic clocks on the satellite are simply *proportional* to those left behind on the Earth's surface. The magnitude of the proportionality constant in a given case is consistent with Einstein's SR, *but not with its prediction of non-simultaneity*. The whole idea of the pre-corrected clock on the satellite is that it runs at the same rate as its counterpart on the Earth at all times, so if two times are equal for one of them, they also have to be equal for the other. The way out of this apparent dilemma is to recall that the experimental evidence generally cited in favor of the LT is invariably a verification of the *relativistic velocity transformation*, not of the LT itself. There is a degree of freedom in fixing the corresponding space-time transformation for the obvious reason that when one multiplies the right-hand side of eq. (3) for dx' by the same factor ε as in the corresponding expression for dt', *no change in velocity occurs*. Lorentz [26] pointed this out as early as 1899 as he was grappling with the problem of finding a suitable relativistic transformation in Maxwell's theory of electromagnetism. The LT can therefore be replaced by another space-time transformation, the alternative Lorenz transformation (ALT [24]) that satisfies both of Einstein's postulates of SR without coming into conflict with the GPS method's assumption of the remote simultaneity of events.

V. Conclusion

The transverse or second-order Doppler effect (TDE), when taken in conjunction with the relativity principle (RP), shows unequivocally that all natural processes slow down uniformly in an object's rest frame when it is accelerated. Because the speed of light is independent of the state of motion of the observer, this means that the lengths of objects in the same rest frame must have *expanded uniformly* by the same fractional amount in all directions as the periods of co-moving clocks have increased. Experiments with circumnavigating airplanes have confirmed time dilation and thus there is no reason to doubt that length expansion is a real effect as well. One can express these results quite succinctly by stating that the units of time and distance change in strict proportion in the rest frame of an accelerated observer. As a consequence, it is impossible for the latter to detect these changes in either the dimensions of co-moving objects or the rates of their natural processes.

These considerations prove that the Fitzgerald-Lorentz contraction effect (FLC) does not occur in nature, since its prediction of a decrease in the lengths of accelerated objects, in varying proportions depending on their orientation to the direction of motion relative to the observer, is contradicted by the above experimental results. A survey of the extensive literature on the FLC shows that none of the previous claims for its validity has been verified by actual experiment. In particular, an argument based on experiments with accelerated meta-stable particles is shown to be specious in that it overlooks the anisotropic nature of the FLC and also that it confuses the numerical value of the measured quantity with the unit in which it is expressed. It is also pointed out that the definition of the meter in terms of the amount of elapsed time it takes a pulse of light to travel this distance is only consistent in other inertial systems if one assumes that objects expand uniformly upon acceleration.

Examination of the original derivation of the FLC shows that it requires a particular interpretation of the space-time variables in the Lorentz transformation (LT). In deriving the LT itself, Einstein associated these variables with the distance traveled by an object in a certain elapsed time, similarly as Newton had done in his derivation of the Galilean transformation. To arrive at the FLC, Einstein had to deviate from this assignment by looking upon dx and dt in the LT respectively as the distance and time difference separating any two physical events. This allowed him to use the value of dt=0 needed to obtain the FLC, even though this choice is not physically relevant for the description of the motion of objects such as light pulses. Nevertheless, this broader interpretation of dx and dt requires separate experimental verification than the light speed measurements used to demonstrate the superiority of the LT over the Galilean transformation. The fact that the TDE observations actually contradict the FLC shows unequivocally that the LT could only retain its validity if the original interpretation of its variables in terms of elapsed times and distances traveled by a given object is employed.

It is a simple matter to bring relativity theory into full agreement with all known experimental results, however, but not without eliminating the LT [24]. In addition to Einstein's original assumptions of the RP and the constancy of the speed of light, it is necessary to make the following postulate: the units of time and distance (as well as those of energy and inertial mass [27]) increase by a factor of γ (v) = $(1-v^2/c^2)^{-0.5}$ in the rest frame of an object that has been accelerated to speed v relative to its original position. This addition to the theoretical foundation implies something that could never be achieved in the original version of SR, namely that the ratio of the lengths of any two objects is the same for all observers, independent of their state of motion. Moreover, the same state of affairs holds for any physical quantity. This is in agreement with our everyday experience with measurements carried out in the rest frame of the Earth's surface, which show that all such ratios are independent of the system of units employed to determine them [28].

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