Mass Dilation and the Lewis-Tolman Conjecture

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Abstract

The arguments of Lewis and Tolman that predict mass dilation are critically reviewed. It is pointed out that their conclusion that two observers in relative motion would measure different *relative* velocities of a given object in their model collision system is incompatible with Einstein's second postulate of the special theory of relativity, in particular for the case when the object is a light pulse. It also contradicts assumptions that have been made in succeeding years about the respective measurements of the relative velocities of meta-stable particles of different observers. Momentum can still be conserved in the Lewis-Tolman collision model if one assumes that the *in situ* masses of the two objects *are not the same* but differ in such a way as to exactly compensate for any mass dilation effect caused by the acceleration of one of the observers.

I. Introduction

In 1909 Lewis and Tolman [1] introduced a Gedanken experiment that suggested that the inertial mass of an object increases with its speed relative to the observer. Bucherer was able to confirm their prediction experimentally [2] by studying the motion of charged particles in a transverse magnetic field. Their arguments were based on Einstein's special theory of relativity (STR), which had been published several years earlier [3], particularly on the phenomenon of time dilation that had first been enunciated in that paper. In the following discussion the assumptions that were employed to arrive at the prediction of mass dilation will be reviewed and compared with others that have since been used to describe other aspects of relativity theory.

II. Assumptions of the Lewis-Tolman Model

The basic content of the theoretical arguments presented by Lewis and Tolman can be understood with the help of the diagram in Fig. 1. An elastic collision between two identical objects A and B is considered from the vantage point of two observers O_1 and O_2, respectively. The observers are located in inertial systems S_1 and S_2, and the direction of their relative motion is along the x axis. Object A is initially at rest in S_1 before the collision, whereas object B is initially at rest in S_2. The collision process is designed to be perfectly symmetrical for the two observers. Each one sees one of the objects moving along the y axis in his rest frame, A by O_1 and B by O_2, making a collision with the other object after it travels a distance y relative its initial rest position. That object then returns to its initial position with exactly the same speed in each case. The other object appears to be traveling at (high) speed u along the x (horizontal) direction, but with a small (vertical) component so that it makes a glancing collision with the former object. After the collision, the other object continues moving along the x axis in the same direction as before, but returns with a vertical component in the opposite direction. The underlying idea is that object A will simply appear to move up and down in the y direction by O_1, whereas object B will appear to do the same for O_2.

The top part of Fig. 1 shows how the collision plays out for O_1. For him, object A moves upward with speed v_{1y}, makes the collision with B, and then returns with the same speed. The time for the object to move before the collision occurs is T_0, and hence $(v_{1y})_A = y/T_0$. The bottom part shows the collision from the perspective of O_2. He finds that object B moves downward with speed $(v_{2y})_B = y/T_0$ before returning to him in the opposite direction. The clear assumption is therefore that $(v_{1y})_A = (v_{2y})_B$ in Fig. 1.

Let us now consider how the motion of object B appears to the observer in S_1. It is here that Lewis and Tolman [1] begin to make use of some of Einstein's conclusions in his original work [3]. First, they assumed that because of time dilation in S_2, the time T of the downward flight of object B must be longer than T_0 by a factor of $\gamma = (1-u^2/c^2)^{-0.5} > 1$. Secondly, because of the Fitzgerald-Lorentz contraction effect (FLC [3]), the distance traveled by B over this period is equal to y from the observer's vantage point in S_1, that is, *the same distance as measured by the observer in* S_2. This is because the direction of the object's motion is *perpendicular* to that of the relative motion of S_1 and S_2. As a result, Lewis and Tolman concluded that, from the viewpoint of the observer in S_1, the speed of object B is γ *times less* than that of object A, that is, $(v_{1y})_B = y/T = \gamma^{-1}\, y/T_0 = \gamma^{-1}\, (v_{1y})_A$.

It is at this point in the discussion that conservation of momentum is brought in. If we assume that the respective y components of the momentum of the two objects must be equal in order for object A to return to its starting position (as measured by O_1) with the same speed as before $[(v_{1y})_A]$, it follows that the inertial mass of object B (also as measured by O_1) must be *larger* than that of object A by the same factor of γ. In short, according to the predictions of STR [1], the two objects must not have the same inertial mass, even though it has been assumed that the *in situ* mass of A measured by O_1 is exactly the same as the *in situ* mass of B measured by O_2. This conclusion led Lewis and Tolman to predict [1] that *the inertial mass of any object must increase by a factor of* γ *(u) when it is accelerated from a rest position to speed u relative to the observer.*

III. Lack of Consistency of the Lorentz Transformation

The fact that the above argument led to a successful experimental verification of "mass dilation" does not prove, however, that the underlying theory is correct. Careful inspection of the Lewis-Tolman justification for their prediction shows clearly that it is fundamentally flawed. It relies on the assumption that the relative velocity of object B to its rest position in S_2 is different for the two observers, i.e. $(v_{2y})_B \neq (v_{1y})_B$. If one takes the special case that particle B is a photon, this means that O_1 and O_2 do not agree that the speed of light is equal to c for both of them. This assumption therefore contradicts Einstein's second postulate of relativity [3], which states that the speed of light in free space is independent of the state of motion of the observer. Moreover, the same objection holds for any relative velocity of the particle [4,5]. The Lorentz transformation of STR satisfies this postulate, but is also responsible for the other two assumption of the Lewis-Tolman model, namely time dilation and the FLC, whereby the latter states that O_1 and O_2 must agree on the distance y traveled by B since it is in a direction which is perpendicular to their separation velocity **u**.

The problem in this regard ultimately goes back to the simple fact that distance, time and velocity are not completely independent variables, whereas the Lorentz transformation treats them as such. Specifically, if one claims that two observers *agree* on the distance traveled by an object and also on its speed relative to some starting point, they cannot also claim that they disagree on the amount of elapsed time it took for it to arrive at its destination.

There is an even easier way to demonstrate that the Lorentz transformation is inconsistent. Consider the example of two lightning strikes occurring in the rest frame S_2. According to the arguments employed by Lewis and Tolman, if the time difference between the two strikes observed by O_2 is ΔT_2, the corresponding time difference observed by O_1 will be $\Delta T_1 = \gamma \, \Delta T_2$ because of time dilation in S_2. The Lorentz transformation also predicts something else [3], however, namely that ΔT_2 can be equal to zero, i.e. the two lightning strikes can occur simultaneously for O_2 whereas the time difference ΔT_1 can be different than zero for O_1. This prediction of the Lorentz transformation is referred to as "remote non-simultaneity." Yet substitution of $\Delta T_2 = 0$ in the above equation leads one directly to the conclusion that $\Delta T_1 = \gamma\Delta T_2 = 0$, from which one must conclude that the lightning strikes do indeed occur simultaneously for O_1 as well as for O_2 [6] . There is thus a clear contradiction, proving that the Lorentz transformation is not valid since both assumptions are derived from it. Once again, it is seen that the Lorentz transformation does not provide a sound basis for the Lewis-Tolman prediction of mass dilation.

IV. Correcting the Lewis-Tolman Model

The prediction of mass dilation and its experimental verification was one of the first successful applications of Einstein's 1905 theory. As discussed above, however, it is ironic that the basis for the prediction is itself faulty. The question that will be discussed below is whether the Lewis-Tolman model can nonetheless be reformulated to give a proper understanding of the role of momentum conservation in high-energy dynamics.

To begin with, it is essential that one do away with the assumption that the two observers O_1 and O_2 of the collision system can disagree on the relative speed of either of the objects A or B with respect to its starting point in Fig. 1. In other words, as discussed in detail in Sect. III, $(v_{1y})_B=(v_{2y})_B$ and $(v_{1y})_A=(v_{2y})_A$, not the inequality assumed by Lewis and Tolman in their discussion. When the objects in the former equations are photons, it is clear that the above equalities are essential in order to maintain consistency with the light-speed postulate on which the Lorentz transformation is based, but they also hold for any type of massive particle [5].

In order to illustrate how the conservation-of-momentum principle should operate under the Lewis-Tolman conditions, it is helpful to consider the case where the speeds of both particles are the same, i.e. $(v_{1y})_A = (v_{1y})_B$. Because of the above general equalities this means that $(v_{2y})_A = (v_{2y})_B$ as well. The masses of A and B must therefore also be equal, i.e. $m_{1A}=m_{1B}$, in order for momentum to be conserved.

It is helpful to consider the Bucherer experiment [2] with accelerated electrons in order to understand how the above condition can be met. It is known from experiment that the laboratory observer (O_1) finds that the inertial mass of the electron increases in direct proportion to γ (u), where u is the speed of the electron in the laboratory. If μ is the rest mass of the electron, it therefore follows that O_1 measures the mass of the accelerated electron to be $m_1=\gamma(u)\ \mu$. It is therefore clear that momentum will not be conserved in the example of Fig. 1 if the *rest masses* of A and B are the same because then $m_{1A}\neq m_{1B}$.

It is therefore clear how to guarantee momentum conservation in the Lewis-Tolman model when $(v_{1y})_A = (v_{1y})_B$. One simply has to choose the rest mass of B (μ_B) to be smaller than that of A (μ_A), specifically, so that $\mu_B = \gamma^{-1}\ \mu_A$. In that way, it is guaranteed that the masses of A and B will be equal when they begin the collision process.

Such an approach is analogous to the "pre-correction technique" employed [7,8] in the Global Positioning System navigation procedure. It is used to adjust the rates of atomic clocks *prior to launch* so that after attaining orbit, they run at the same rate as clocks on the ground. In this case, it is the effect of time dilation which slows the rate of the satellite clocks to produce the desired equality, whereas mass dilation accomplishes the desired equalization of masses A and B in the Lewis-Tolman model.

As with non-relativistic collisions, however, it is not necessary that the speeds of the two particles be equal in order to satisfy the conservation-of-momentum principle. Any ratio of the two speeds is possible, not just that indicated by the Lewis-Tolman assumption based on time dilation.. If one chooses $(v_{1y})_A = X\ (v_{1y})_B$, for example, momentum can be conserved by having the relationship between the two rest masses of the two particles be $\mu_B=X\gamma^{-1}\mu_A$. After mass dilation occurs in S_2, the mass of B will then be X μ_A, thereby cancelling out the difference in A and B's relative speed. Clearly, the same cancellation occurs regardless of whether $X<1$ or $X<1$.

It is obvious from this discussion that Lewis and Tolman's original argument [1] is specious; it is not true that the amount of mass dilation is solely determined by the supposed effect of time dilation in S_2 on the ratio of the relative speeds of A and B . Instead, the ratio X of the two relative speeds can be determined completely at random. Time dilation has nothing whatsoever to do with this choice. The key point is that the ratio of rest masses must be chosen to exactly compensate for this difference in relative speeds. Nonetheless, Lewis and Tolman were able to deduce correctly that *the factors* for time and mass dilation are exactly the same in the model shown in Fig. 1, namely γ (u). In the last analysis, this is the result of primary significance in their investigation as a whole.

An important aspect of the Lewis-Tolman model is its relevance to interactions with light. According to both the Lorentz transformation and the overwhelming evidence available from experiment, the speed of light in free space is the same for every observer, independent of his state of motion. Therefore, it is inconsistent to argue that the speeds of particles A and B in Fig. 1 cannot be equal. The question that needs to be discussed in the present context is how conservation of momentum can be assured when the particles in question are photons (incidentally, Lewis coined the word "photon" [9] in a famous argument supporting Newton's assertion that light consists of particles). As before in the previous discussion, it is essential in that case that some adjustment be made to insure that mass dilation causes the necessary equalization of the two masses in the collision. This can be done by choosing the frequency ν_0 (B) of the light emanating from a source at rest in S_2 to be *lower* than the standard value ν_0 (A) originating from the corresponding source at rest in S_1. Specifically, the condition must be ν_0 (B) = ν_0 (A)/γ. Because of time dilation in S_2, it can be assumed that the frequency measured in S_1 will be smaller by a factor of γ, so that its value will be ν_0 (A)/γ^2. At the same time, both the energy and momentum of the photon measured in S_1 will be larger than the corresponding values for the photon in S_2 by a factor of γ, just as are masses of particles that are stationary there. These seemingly contradictory relationships are resolved by noting that the unit of angular momentum and therefore also of Planck's constant h is γ^2 larger [9] in S_2 than in S_1. In this way, the measured values of the energy and momentum of photon B in S_1 will be $h\nu_0$ (A) and h ν_0 (A)/c, respectively, as required to conserve both momentum and energy in the collision.

Finally, there is another key area where Lewis and Tolman were misled by the Lorentz transformation. They assumed that the amount of time dilation and therefore the ratio of the elapsed times measured by O_1 and O_2 is always equal to γ (u). *Each observer* was assumed to measure a smaller elapsed time by this factor than his counterpart in the other rest frame. This means that one must believe that the clock at rest in the other rest frame must run slower by this factor than that at rest in his own rest frame (Einstein's symmetry principle [3]).

Experiment tells an entirely different story, however. In their study of circumnavigating atomic clocks, for example, Hafele and Keating [11] found that the elapsed time registered on a given clock *decreases as its speed u increases relative to the earth's center of mass (ECM).* This means that the clock on the eastward-flying airplane ran slower than that moving in the opposite direction in their study. If the speeds of the two clocks are u_1 and u_2, respectively, it was found that the ratio of the two elapsed times $\Delta T_1/\Delta T_2$ for the same portion of the journey (after suitable correction for gravitational effects on the clock rates [11]) was equal to Q =

$\gamma(u_2)/\gamma(u_1)$. The measured ratio is not γ (u) (where u is the relative speed of O_1 and O_2; see Fig. 1), contrary to what is assumed by Lewis and Tolman [1] in their model. The same inverse proportionality between elapsed times and γ (u) was found in an earlier study employing x-ray detectors and absorbers [12]. In that case, Sherwin pointed out [13] that this result was inconsistent with the *symmetric* prediction based solely on the Lorentz transformation. Because the above ratio Q applies in the description of all known timing studies as yet carried out experimentally, it is appropriate to refer to the corresponding equation as the Universal Time-dilation Law (UTDL) [14]. Note also that the conversion factor for O_2 [$\gamma(u_1)/\gamma(u_2) = 1/Q$] when carrying out measurements for clocks at rest in S_1 is the reciprocal of that for O_1 when the reverse comparison is made. The symmetry that Einstein envisioned is therefore contradicted by the experimental data underlying the UTDL.

V. Conclusion

The success of the Lewis-Tolman model in predicting the phenomenon of relativistic mass dilation is a textbook example showing that an experimental confirmation does not constitute proof of the theory on which it was based. The authors formulated their arguments under the assumption that the Lorentz transformation is completely valid. They concluded on this basis that two observers can differ on the speed v_{iy} of a given particle relative to its starting point in the collision system they proposed. Years later, the experimental confirmation of time dilation in muon decay experiments [4] was firmly based on the opposite conclusion, namely that observers in different rest frames must be in complete agreement on the speed of the accelerated particles.

Applying their model for photons also makes it clear that something is wrong with the Lorentz transformation, since it forces one to overlook the light-speed postulate on which the transformation is clearly based. In addition, its prediction of remote non-simultaneity does not mesh with the proportionality characteristic of time dilation, both of which effects are derived squarely from the Lorentz transformation.

Perhaps the most insightful aspect of the Lorentz-Tolman model is its assertion that time dilation and mass dilation change in direct proportion to one another. The quantitative factor that governs these changes has been found from numerous timing experiments to increase with the speed u of the object relative to a definite rest frame. In the Hafele-Keating study [11] employing circumnavigating atomic clocks, this rest frame has been shown to be the earth's center of mass. The elapsed time for a given portion of the journey was always found to be inversely proportional to γ (u), in accordance with the Universal Time-dilation Law (UTDL) [12]. The ratio Q of elapsed times is determined to be equal to $\gamma(u_2)/\ \gamma(u_1)$ when the object clock moves with speed u_2 while the observer's moves with speed u_1. A simple way to look at Q is as a conversion factor between different units of time in the two rest frames. The same factor holds for the ratios of units of inertial mass and also for energy and momentum. A consistent picture emerges for the conversion factors of units for all other physical properties. Each factor turns out to be an integral power of Q, as determined by the composition of each property in terms of the standard quantities of time, distance and inertial mass [8].

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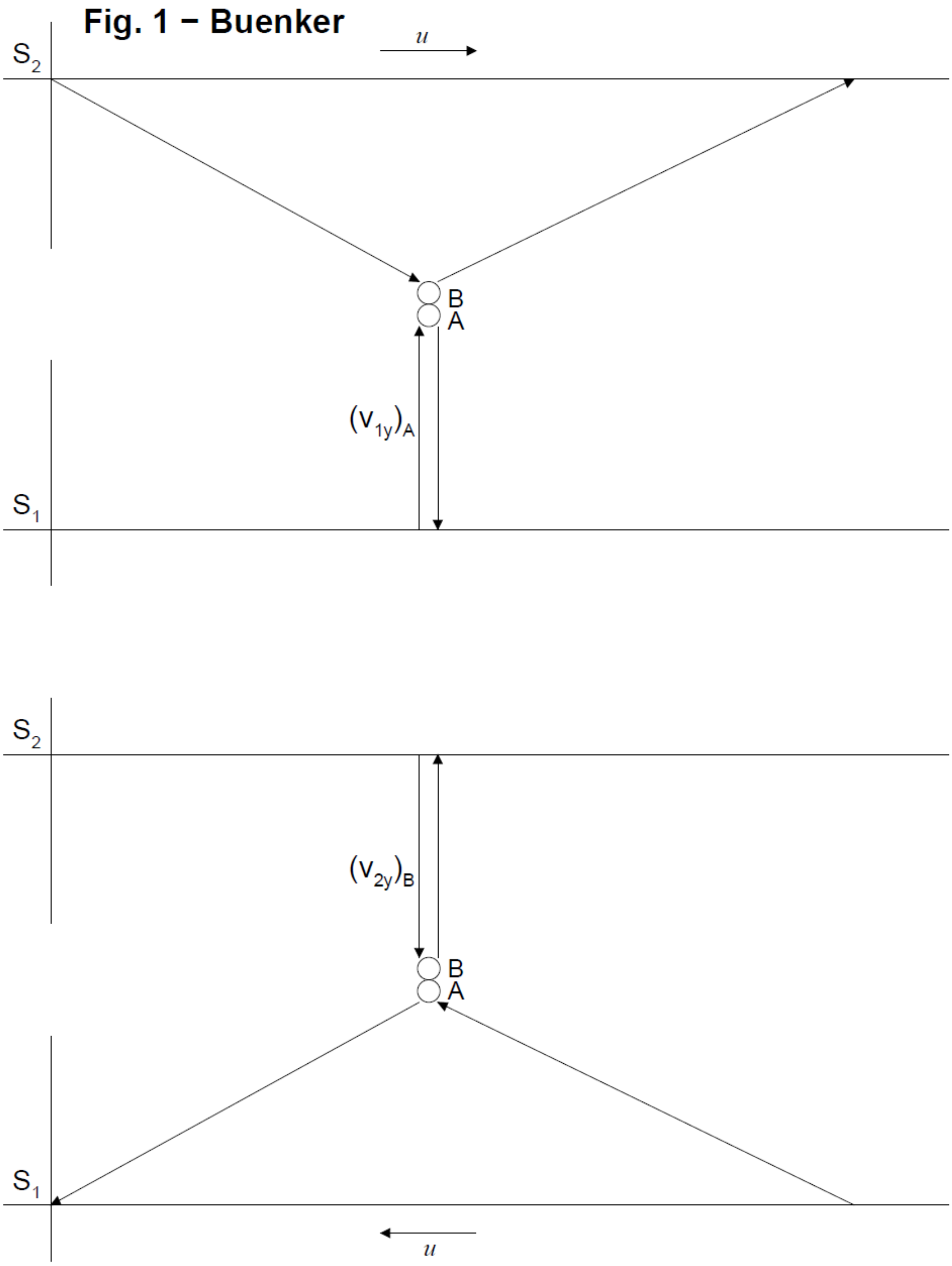

Fig. 1. Lewis-Tolman model for the elastic collision of two objects that were originally at rest in different inertial systems S_1 and S_2 that are moving with speed u relative to one another. The upper half of the diagram shows the collision as viewed by an observer at rest in S_1 (note that S_2 is moving to the right for him). The lower half shows the same collision as viewed by an observer at rest in S_2 (he views S_1 moving to the left).