Kinetic Energy and the Relativistic Symmetry Principle

by

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Abstract

According to the special theory of relativity (STR) the rates of clocks and energies of objects should change in direct proportion as their state of motion is varied. The rates of clocks should slow down by a factor of $\gamma = (1-v^2/c^2)^{-0.5}$ when they reach a speed of v relative to the observer (c is the speed of light in a vacuum) and the energies and inertial masses of objects should increase by the same factor. There is ample experimental evidence for both predictions of STR, but there is another aspect of the theory that has yet to be verified conclusively. According to the Lorentz transformation (LT) of STR, a symmetry principle must exist whereby two observers in relative motion each think that it is the other's clock that has slowed down or that it is objects in the other's rest frame whose energy has increased. Probably the most credible evidence for this symmetry principle is found in the measurement of kinetic energies in elastic collisions. Measurements of the rates of clocks onboard airplanes are not consistent with this principle, however, and one is forced to explain this

negative result by restricting the validity of the LT to inertial systems, even though no comparable restriction appears to be necessary in the case of measured kinetic energies. A plausible explanation for this apparent distinction between the relationships of measurements of clock rates and kinetic energies for different observers is discussed in the present work. Finally, an alternative relativity theory is presented that is consistent with the latter results and also requires strict adherence to the principles of simultaneity of events and the rationality and objectivity of measurement.

I. Introduction

On the basis of their experiments with circumnavigating airplanes 40 years ago, Hafele and Keating [1] were able to obtain an empirical formula for the dependence of the rates of onboard clocks on relative speed and altitude. Experiments with a rocket launched from the Earth [2] are quite consistent with the latter result. The latter formula is consistent with Einstein's prediction of time dilation [3], but only with an important proviso: the speed v that is used in the appropriate relativistic formula must be measured relative to a standard clock located on the Earth's polar axis. Hafele and Keating [1] justified this procedure by stating that the reference clock must be located in an inertial rest frame, in accordance with the standard interpretation of the special theory of relativity (STR), and that only a clock on the Earth's polar axis satisfies this condition. As a consequence, it is not possible to compute the amount of time dilation by simply inserting the speed v of the clock relative to a given observer on the Earth's surface into Einstein's formula: $dt = (1-v^2/c^2)^{-0.5}dt' = \gamma\, dt'$.

The question arises, however, as to why a similar situation apparently does not exist for the measurement of kinetic energy K. In non-relativistic theory, $K= mv^2/2$ for an object of mass m moving with speed v relative to the observer. The corresponding relativistic formula, $K = (\gamma-1)\, \mu c^2$ (μ is the proper mass of the object, which is equal to m in the classical formula), is well known to reduce to the latter result in the low-velocity regime. So why is it apparently always possible to compute the kinetic energy of particles in laboratories on the Earth's surface by simply inserting their speed v into the classical formula instead of its corresponding value relative to the polar axis? Relativistic theory (STR) is quite clear in predicting a strictly analogous relationship between the amounts of time and energy-mass dilation, so an explanation needs to be found for this apparent distinction. The answer to this question will be seen to offer new insight into the conditions that must exist before the relativistic formulas can be successfully applied.

II. Applying the Hafele-Keating Formula to Kinetic Energy Measurements

The empirical formula for the difference in elapsed times for two clocks located respectively on an airplane (τ) and on the ground (τ_0) at the Equator is given below (eq. (2) of ref. [1]):

$$\tau - \tau_0 = [gh/c^2 - (2R\Omega\, v + v^2)/2c^2]\, \tau_0. \qquad (1)$$

In this equation, R is the Earth's radius, Ω is the Earth's angular frequency of rotation about the polar axis, g is the acceleration due to gravity, h is the altitude of the airplane and v is its speed relative to the ground. This equation expresses the fact that there are two causes for the difference in τ and τ_0, the gravitational red shift and time dilation. This fact is made more apparent by rewriting it in the more general (relativistic) form:

$$\tau = [(1 + gh/c^2)\, \gamma\, (R\Omega)]/\, \gamma\, (R\Omega+v)]\, \tau_0, \qquad (2)$$

which reduces to eq. (1) when γ (v) is replaced by its first-order approximation, $1 + v^2/2c^2$, in each case. The first term expresses the fact that the rates of clocks increase with altitude and therefore this effect can be easily separated from that of relativistic time dilation. The latter is computed as follows. It is assumed that Einstein's formula can only be applied with reference to a standard clock located at one of the Poles *as long as the same conditions are present as in timing experiments [1]*. For this purpose one needs to know the speed u_P of a given clock relative to the polar axis. It is equal to RΩ+v for the airplane clock and to RΩ for its counterpart on the ground, whereby v is taken to be positive when the airplane moves in the easterly direction at the Equator, and negative when it moves in the westerly direction (more generally, one has to take account of the latitude of a given clock and the angle of flight relative to the Equator in computing u_P [1]).

If we assume that the energy E of an object changes in direct proportion to the periods of clocks upon acceleration, as predicted by relativity theory, it is straightforward to deduce an analogous relationship to eq. (2) for this quantity. We just have to know that energies increase with speed and altitude, whereas the rates of clocks *decrease* with speed and increase with altitude. On this basis one obtains:

$$E = [(1 + gh/c^2)\, \gamma\, (R\Omega + v)]/\, \gamma\, (R\Omega)]\, E_0, \quad (3)$$

where E refers to the energy of an object on the airplane and E_0 is the corresponding value for an identical object located on the ground. In experiments employing highly accelerated particles with $v >> R\Omega$ and $h \approx 0$, the above equation simply reduces to $E = \gamma\,(v)\, E_0$, so it is not surprising that the effects of the Earth's rotation can be neglected under these circumstances. In the low-velocity (non-relativistic) limit when this is not the case, however, eq. (3) becomes:

$$E - E_0 = [gh/c^2 + (2R\Omega\, v + v^2)/2c^2]\, E_0. \quad (4)$$

This in turn gives the following relation for the energy difference ($E_0 = \mu c^2$):

$$E - E_0 = [gh/c^2 + (2R\Omega\, v + v^2)/2c^2]\, \mu c^2$$

$$= \mu gh + \mu R\Omega v + \mu v^2/2. \quad (5)$$

The latter equation contains both the classical term for gravitational energy (μgh) and that for kinetic energy, $K = \mu v^2/2$, as one would normally expect. *Yet it also has a term that is linear in v* that requires careful consideration, especially since it is also proportional to the rotational speed of the Earth ($R\Omega$). The question that needs to be answered is whether eq. (5) can be reconciled with the experimental fact that the kinetic energy is normally thought to be a function of only the speed v of the object relative to the observer, i.e., $K = \mu v^2/2$, but not to be dependent in any way on the rotational speed of the Earth at a given location.

The formula for kinetic energy is verified experimentally in elastic collisions. In this case one expects the following relation from the principle of energy conservation for two particles of rest mass μ_1 and μ_2:

$$\mu_1 v_{1i}^2 + \mu_2 v_{2i}^2 = \mu_1 v_{1f}^2 + \mu_2 v_{2f}^2, \quad (6)$$

where v_{1i} and v_{2i} are their initial speeds before the collision and v_{1f} and v_{2f} are the corresponding values after the collision has taken place. According to eq. (5) with h=0, however, the corresponding relations should be:

$$2\mu_1 R\Omega v_{1i} + \mu_1 v_{1i}^2 + 2\mu_2 R\Omega v_{2i} + \mu_2 v_{2i}^2$$
$$= 2\mu_1 R\Omega v_{1f} + \mu_1 v_{1f}^2 + 2\mu_2 R\Omega v_{2f} + \mu_2 v_{2f}^2. \quad (7)$$

It needs to be recalled that momentum is also conserved in the collision, however, so that

$$\mu_1 v_{1i} + \mu_2 v_{2i} = \mu_1 v_{1f} + \mu_2 v_{2f}. \quad (8)$$

Multiplication of the latter equation with the constant factor 2RΩ and subtraction from eq. (7) therefore leads back to the standard kinetic energy relation of eq. (6). This cancellation occurs quite generally, independent of the direction of the individual particles, as can easily be shown.

The above example assumes that the object and observer are at the same latitude. If this is not the case, additional terms must be added to eq. (7) that are proportional to the difference of the squares of the Earth's rotational speeds at the two latitudes and also the mass of the particle in each case, *but are independent of the speed v of the object relative to the observer*. Since the former values are constant and the masses of the individual particles do not change in the collision (at the non-relativistic level under discussion), *a cancellation also occurs for these quantities* in the analogous energy conservation equation under these circumstances.

In short, what one sees from this analysis is that including the extra terms implied by eq. (5) for the kinetic energy of a particle that are either proportional to its velocity or independent of it do not in any way affect the standard formulas that one conventionally uses to describe collisions. The fact that these relationships hold experimentally when one assumes that the kinetic energy of a particle is always equal to $mv^2/2$ is therefore in no way inconsistent with eq. (5). The indication is simply that there must be additional terms in the kinetic energy (i.e., in the energy difference $E-E_0$) that have no effect on the outcome of collisions and thus cannot be verified by direct experiment. As a result, there is every reason to believe that energy and time do change with the speed of an object in an exactly parallel manner, exactly as predicted by STR [1]. *At the same time, it needs to be emphasized that in* *laboratory work, the key rest frame is that in which the particle is accelerated, and so the above considerations do not apply.*

The fact that there is such a close analogy between variations in energy and elapsed times in relativity theory indicates quite strongly that both effects have the same origin. The standard argument from STR is that one can compute the amount of time and energy-mass dilation simply by knowing the speed of the object relative to a given observer, but the Hafele-Keating experiments [1] indicate something quite different. More insight into this question can be obtained by examining the derivation of the dependence of kinetic energy on relative speed in classical mechanics. The starting point is the definition of work/energy and Newton's Second Law. An applied force **F** is assumed in order to compute the change in energy dE of a given object as it moves a distance **dr**:

$$dE = \mathbf{F}\cdot\mathbf{dr} = (\mathbf{dp}/dt)\cdot\mathbf{dr} = \mu v dv = d(\mu v^2/2). \qquad (9)$$

This derivation emphasizes that the speed v used to define the kinetic energy is defined **relative to a distinct rest system**, namely *that in which the force is applied to the object*. It is inconsistent with the assertion that the reference point is arbitrary for computing a particle's kinetic energy, although this is invariably assumed. As we have seen, collision experiments lend support to the above conclusion, but certainly do not rule out the possibility that the more complicated eq. (5) is actually required to compute the actual increase in the particle's energy as a result of the applied force **F** in eq. (9). There are no additional constraints such as momentum and mass conservation to contend with in determining the way in which elapsed times vary with acceleration, however, and experiment [1] shows quite clearly that there is a unique rest system from which the speed of the clock must be measured in order to obtain correct results from Einstein's time dilation formula [3]. In previous work [4] this has been referred to as the objective rest system (ORS), and, in view of eq. (9), it can be concluded that the reason for its unique position is intimately tied up with the fact that a force has been applied to the object in this rest system that is directly responsible for its acceleration. This can be the laboratory rest frame in conventional experiments, but it can be the Earth's polar axis under the conditions of the Hafele-Keating investigation [1,4].

One of the main consequences of the existence of a definite rest frame from which to compute time dilation, and by extension, also energy-mass dilation, is that one can employ a **rational set of units** in order describe the changes that occur upon acceleration. This point is discussed in detail in a companion paper [5]. The relative rates of clocks are expressed as fixed ratios on the basis of eq. (2). There is no "symmetry" principle on this basis, contrary to what is claimed in STR [3], according to which two clocks can both be running slower than one another at the same time. Instead, one has the principle of the rationality of measurement (PRM [6]), which states that the ratio of any two measured values for quantities of the same type is the same for all observers. This principle holds quite generally for all physical quantities and is completely consistent with Einstein's first postulate of STR [3], the principle of relativity. Accordingly, the laws of physics are indeed the same in all inertial systems, as Galileo first claimed, *but the units in which they are expressed can and do differ* depending on the state of motion and position in a gravitational field of a given observer. Clocks run at different rates depending on how fast they have been accelerated relative to a common ORS. Yet an observer co-moving with the clocks notices no change in these rates because they have all slowed down by the same fraction as a result of their mutual acceleration. The same argument also holds for energy and inertial mass, and indeed for all other physical properties. An independent observer at the ORS does notice such changes, however, and in this sense can distinguish one inertial system from another, contrary to what is often assumed [7], without violating the relativity principle in any way.

III. Postulates of Relativity

The above discussion has indicated that acceleration plays a key role in relativity theory and that it is responsible for both the slowing down of clock rates and energy-mass dilation. This possibility was already pointed out in Einstein's original work [3], namely by stating that an applied force destroys the symmetry that otherwise exists between inertial systems. The two postulates he enunciated on which to base his new relativity theory do not actually reflect the role of acceleration, however. They simply state that the laws of physics are the same in all inertial systems and that the speed of light is independent of the state of motion of the observer and light source.

In a companion paper [8] it has been shown that there is actually a **hidden** postulate in Einstein's formulation of the theory that is also essential in order to derive the Lorentz transformation (LT). The main consequence of the latter assumption is that it rules out the principle of simultaneity of events, since the LT leads to the conclusion that measured times (dt and dt') depend on the relative position (dx') of a given observer to the object:

$$dt = \gamma\,(u)\,(dt' + u\,dx'/c^2). \qquad (10)$$

While this result has been hailed as one of the key advances of relativity theory, the fact remains that there has never been any experimental verification of non-simultaneity. Indeed, in GPS navigation technology [8,9] exact simultaneity for all observers is assumed for the time of emission of a light pulse from a satellite, regardless of the observer's location or state of motion. Thus, there is a need to reconsider the underlying structure of conventional relativity theory (STR [3]) to bring it into line with the results of experiments that have been carried out since its inception in 1905, as will be done below.

One can best start this exercise by restating Einstein's two postulates:

1) **The laws of physics are the same in all inertial systems (Relativity Principle), but the units in which they are expressed vary systematically depending on their state of motion and position in a gravitational field.**
2) **The speed of light in free space is a constant, independent of the inertial system, the source and the observer.**

As discussed at the end of Section II, an addendum is required for the Relativity Principle to emphasize the experimental finding that clocks slow down upon acceleration and also that other physical quantities are changed as well. The resultant variation in properties is uniform within any rest frame, and so it amounts to a simple change in units in each case. Since the laws of physics are mathematical equations in every instance, such a **uniform scaling** clearly does not alter the laws themselves [5,10]. The effect is the same as if one converts from one system of standard units to another, such as going from feet to meters of length or from s to ms of time. One of the primary goals of relativity theory is to establish how these units change upon acceleration and change of position in a gravitational field. Two additional postulates are required to specify these relationships for the "kinetic" scaling of units [5].

As mentioned above, the second postulate needs to be augmented with an additional assumption before it leads unambiguously to the LT. Lorentz pointed out in 1899 [11] that without this additional constraint the LT can only be specified to within a scale (or normalization) factor. Rather than give up the principle of simultaneity of events, as Einstein did in 1905 [3], it is possible to insist upon it as the condition for completely specifying the required space-time transformation that is consistent with his second postulate:

3) **Every physical event occurs simultaneously for all observers, independent of their state of motion and position in a gravitational field (Simultaneity Principle).**

When this postulate is combined with that of the constancy of the speed of light in free space, the result is the **Alternative Lorentz Transformation (ALT [9,10]):**

$$dx = \eta\,(dx' + u\,dt') \quad (10a)$$

$$dy = \eta\,dy'/\gamma \quad (10b)$$

$$dz = \eta\,dz'/\gamma, \quad (10c)$$

$$dt = dt', \quad (10d)$$

with $\eta = (1 + u\,dx'/c^2dt')]^{-1}$. (11)

It is important to note that it is assumed in eqs. (10a-d) that both observers use exactly the same set of units. However, because of time dilation, this is not the usual case. If the clocks in the primed rest frame (S') run Q times slower than in the other (S), one has to alter *each* of the above equations by multiplying with Q on the right-hand side in order to insure that each observer uses his own set of proper units. For example, eq. (10d) becomes dt=Qdt'.

Instead of dy=dy' and dz=dz' as in the LT [3], one has eq. (10d) to insure simultaneity. Dividing eqs. (10a-c) by dt=dt' (or dt=Qdt' in general) *leads to exactly the same velocity transformation as for the LT*, thereby insuring not only adherence to Einstein's second postulate but also agreement with a number of other key experimental results such as the Fizeau light drag effect and the aberration of light from stars that ultimately cemented the reputation of STR.

The ALT also frees one from the necessity of assuming that two clocks can both be running slower than one another, which is the prediction of the symmetry principle of STR [7]. Equations such as dt=γdt' and dt'=γdt can both be derived in a straightforward manner from the LT by inverting the transformation equations, whereas the only possibility for the ALT is dt=dt' (or again more generally, dt=Qdt'). As a result, one can add a fourth postulate that upholds another ancient principle:

4) The ratio of any two physical quantities of the same type is the same for all observers, independent of their state of motion and position in a gravitational field (Principle of Rational Measurement or PRM [6]).

The latter postulate is the antithesis of the symmetry principle of STR. The PRM is in perfect agreement with the results of the Hafele-Keating experiments [1] and is also one of the underlying assumptions, in addition to the third postulate above (simultaneity), that allows GPS navigation technology to produce reliable measurements of distance [8,9]. It becomes feasible to introduce "conversion" factors for relating the results of measurements in different inertial systems. In GPS technology, for example, one simply assumes that any measurement of elapsed time on a satellite can be adjusted so as to provide the corresponding value that would be measured by a clock on the ground. If there were disagreement about which clock runs more slowly, the one on the satellite or its counterpart on the ground, such a procedure would be groundless, or at least would require a quite different set of logical assumptions than are used in actual practice to achieve the desired results.

Finally, the conventional version of STR [3] assumes that one can simply use the LT and the related energy-momentum four-vector relations to derive information about the ratios of measured values for two observers in relative motion. In this case, a few additional remarks are necessary to clarify the situation when the ALT is used instead. Time dilation cannot be derived from the latter [8], that is, on the basis of the simultaneity condition of eq. (10d). The light speed hypothesis is sufficient to derive energy-mass dilation, but as discussed above, the speed to be used in applying Einstein's original formula, i.e. $E=\gamma(v)E'$, in this instance must be taken relative to a particular rest frame, the ORS [4]. In Sect. II evidence has been cited to show that elapsed times scale in exactly the same manner with speed relative to a given ORS as do energy and mass. The latter result does not follow directly from the first four postulates given above, however, and hence a fifth postulate is required to complete the framework of the theory:

5) The unit of time in a given inertial system changes in direct proportion to those of energy and inertial mass.

The PRM [6] plays a key role in the scaling relationships. Consistent with the Hafele-Keating experiments [1], once the change in units has been established for two rest frames S and S' relative to their common ORS as $\gamma(v)$ and $\gamma(v')$, respectively, it follows that the conversion factor between their own units is given by the corresponding ratio $R = \gamma(v)/\gamma(v')$. Measurement is completely rational and objective. There is no question about which clock is slower than the other on this basis, nor by what factor, exactly as is required in the GPS timing procedure.

No additional postulate is needed for the scaling of distances. They must also vary in direct proportion to elapsed times because of the light speed constancy [8,9]. This means that time dilation is accompanied by isotropic length expansion [12], however, and not Fitzgerald-Lorentz length contraction. The main experimental evidence for this conclusion comes from observations of the transverse Doppler effect [13]. They show unequivocally that the period of electromagnetic radiation varies in direct proportion to its wavelength, independent of the source's direction of motion relative to the observer. The increase in period is a clear example of time dilation in the rest frame of the light source, and the corresponding increase in wavelength is no less an experimental proof for isotropic length expansion. The only way to avoid this conclusion is again to allow for violations of the PRM, as is done in STR [3], but not in the above formulation of relativity theory (Postulate 4). A further consequence of the last two postulates is that the *relative* velocities of two objects must be the same for all observers (also forces because of the proportionality of the energy and length scaling factors), not just the speed of light in free space [5,10].

IV. Conclusion

The above set of five postulates allows for accurate predictions of the respective measured values of physical quantities for the same object obtained by different observers who are in relative motion to one another. The resulting theory is distinguished from STR [3] primarily because of its use of the ALT of eqs. (10a-d) [8,9] in lieu of the LT, and also because of its adherence to both the principle of simultaneity of events and the principle of rational measurement (Postulates 3 and 4). The result is a rationalized theory of relativity (RTR) that is free of the paradoxes that follow directly from the symmetry principle of STR [7]. It is no longer necessary to assume that two clocks can both be running slower than one another simply because they are in relative motion, for example. The latter assertion is clearly violated in GPS technology, where it is simply assumed that clocks on a satellite run more slowly than those on the ground because of time dilation.

One of the main objectives of the present work is to show that the formula for the kinetic energy of an object is ambiguous with regard to the relativistic symmetry principle. If one assumes that the effects of energy and time dilation vary in direct proportion to one another, the conclusion is that the increase in energy of an accelerated object is not described completely by the $K=mv^2/2$ term. Additional terms that are linear in v or independent of it entirely cannot be detected in low-energy collision experiments because of the requirements of momentum and mass conversation. An additional postulate is needed in order to incorporate this result into RTR because it is not possible to derive the time dilation effect from the ALT alone, contrary to what is assumed in STR based on the LT. It is emphasized, however, that both the time dilation and energy-mass dilation effects are the result of acceleration of the object due to an applied force in a particular rest frame (ORS [4]). All objects undergoing the same degree of acceleration change their properties in a perfectly uniform manner, so that an observer co-moving with them cannot detect any difference in any of their physical properties, consistent with the Relativity Principle.

The main (but not the only [15]) way to distinguish between the present formulation of relativity theory (RTR) and the conventional version based on the LT (STR) is through experiment. The most useful properties in this regard involve elapsed times. As already mentioned, the GPS technology relies on the satisfaction of the simultaneity and rational measurement postulates, which are rejected in the STR formulation. If events do not occur simultaneously, it would be impossible to synchronize clocks on a general basis. The Hafele-Keating experiments [1] on the contrary indicate that the relative rates of all Earth-based clocks can be determined exactly by simply knowing the altitude and rotational speed of the Earth at their respective locations. An extremely sensitive test of the Simultaneity Principle can be made on this basis. The clock rates themselves can be determined quite accurately in terms of the transverse Doppler effect, as discussed in detail elsewhere [14]. The faster a clock runs, the smaller will be the frequencies measured with it. Thus light signals exchanged via satellite carry the potential of demonstrating the accuracy of the Hafele-Keating formula, and with it, the viability of both the third and fourth postulates of RTR given in Sect. III. They also can be quite useful in quantitatively verifying the fifth postulate, which is ultimately the basis for the Hafele-Keating formula itself.

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