On The Relationship Between Relativistic Length Variations and Time Dilation

Robert J. Buenker

Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42119 Wuppertal, Germany

Phone: +49-202-439-2509/2774

Fax: +49-202-439-2509

E-mail: buenker@uni-wuppertal.de

Abstract

An experimental procedure is considered to measure the change in the length of an object when it is accelerated. The same theoretical assumptions that form the basis of the methodology for the Global Positioning System (GPS) are used in order to predict the results of the experiment. It is concluded that the dimensions of the test object increase uniformly in all directions in the same proportion as the rates of co-moving clocks are slowed.

Keywords: time dilation, Fitzgerald-Lorentz contraction, Global Positioning System (GPS), definition of the meter

I. Introduction

Two of the most famous conclusions of Einstein's original paper [1] on the theory of special relativity are time dilation and Fitzgerald-Lorentz length contraction. Although there have been many well-established verifications of the slowing down of clocks upon acceleration, no corresponding observation of length contraction has ever been reported. For a long time it was argued that the latter phenomenon would be impossible to observe because of the requirement of simultaneous measurement of the locations of both ends of an object in relative motion to the observer. Yet distance measurements on the earth's surface based on timing measurements made on orbiting satellites have become routine in recent years due to the development of the Global Positioning System (GPS). In the following an experiment is described to show how measurements of the length of an object carried onboard a GPS satellite can be used to obtain a definitive test of relativistic length contraction.

II. Theoretical Assumptions and Results

The success of the GPS procedure for measuring distances is predicated on a series of assumptions that have their basis in relativity theory. The foremost of these is that the rate of a clock carried onboard a satellite is slowed due to the phenomenon of time dilation. However, there is a complication because of the increase in clock rate due to the gravitational red shift [2]. Knowledge of the speed of the satellite and its altitude above the center of the earth allows one to predict the fractional change in the rate of a clock as a result of being accelerated into a nearly circular orbit from its original position on the earth's surface. Distance determinations are ultimately based on another assumption of relativity theory, namely Einstein's second postulate of the constancy of the speed of light ($c=2.99792458 \times 10^8$ ms^{-1}) in free space. There is an additional complication due to gravity in this case as well because the speed of light increases with altitude, as Einstein pointed out two years after his first paper on relativity theory [3]. The important point in the present context is that it has proven effective to employ both of the above relativistic assumptions in obtaining accurate distance measurements with GPS. This experience therefore provides ample evidence for their validity in other applications such as the one described below.

One of the most basic features of the GPS methodology is its use of atomic clocks that are "pre-corrected" so that they run at the same rate on the orbiting satellite as their counterparts left behind on the earth's surface. To isolate the effects of motion on the length of objects, we will proceed in the following as if all gravitational effects on clock rates have been accounted for quantitatively. This is the same as assuming that the accelerated clock remains at the same altitude throughout the experiment, which is at least theoretically possible and is also sufficient to illustrate the effects of interest in the present discussion. Let us assume then that the clock on the satellite will be slowed by a factor of Q by the time it reaches its final constant velocity relative to its original position ($Q>1$). We need three identical clocks in order to conduct the experiment, along with one object whose length is to be measured. One clock (A) stays behind while the other two remain on the satellite as it is accelerated to its final speed. Of these the rate of one of the clocks (B) is uncorrected on the ground, while that of clock C is pre-corrected by speeding it up by the above factor of Q.

Prior to acceleration all three clocks are used to measure the elapsed time for a light pulse to pass over the length of the aforementioned object *in both directions*. If the length of the object is L m, as determined independently using conventional means (i.e. by laying it out relative to a standard meter stick), then the following timing results must be found: $2Lc^{-1}$ s for both clocks A and B and $2QL\ c^{-1}$ s for clock C with the adjusted rate. It is worth noting that the use of elapsed times is the *primary* means of measuring lengths according to the modern definition of the meter. All three results correspond to a measured length of L m since the "effective" speed of light based on the elapsed time on clock C is actually cQ^{-1} m/s because of its adjusted rate.

Once the satellite reaches its final velocity, the length measurements are repeated. In accord with the relativity principle all results are the same as found on the ground. The elapsed times for passage of the light pulse along the object are still $2Lc^{-1}$ s and $2QL\ c^{-1}$ s using clocks B and C, respectively. The comparison with the onboard meter stick also finds the length of the object to be L m. At this point it needs to be recalled that the rates of both clocks B and C have nonetheless decreased by a factor of Q by virtue of their acceleration. As a consequence, in full accord with the assumptions of the GPS procedure, clock C is now running at exactly the same rate as clock A which has remained stationary on the earth's surface. Consistent with Einstein's second postulate, the speed of light on the satellite is still c m/s for the ground observer. The latter therefore can measure the length of the object on the satellite in the same way as previously when it was on the ground by multiplying the elapsed time on his clock (inferred from clock C) by c and dividing by two. Therefore, the observer on the ground finds that the length of the object is now $2QLc^{-1}$ $(c/2) = QL$ m, i.e. it has increased in length $(Q>1)$.

The above result stands in contradiction to the expectations based on the Fitzgerald-Lorentz contraction effect [1]. The latter predicts that the length of an object which is moving relative to the observer along the x axis of the coordinate system with speed $v<c$ will contract by varying amounts depending on its orientation. The amount of the contraction has its maximum when the measurement is made along the x axis, with the observed fractional decrease equal to $(1-v^2/c^2)^{0.5} \equiv \gamma^{-1}$. If the measurement is made in a direction that is perpendicular

to that of the relative motion, however, no change will be observed. In summary, one expects anisotropic length contraction to occur. On the contrary, the analysis of the above experimental procedure leads unequivocally to the conclusion that *isotropic length expansion* occurs as a result of acceleration of the object on the satellite. The orientation is immaterial in this case because the length measurement is based solely on elapsed times read from local clocks. Furthermore, the greater the fractional decrease in the rate of the onboard clocks as a result of the relativistic time dilation, the longer it will take for a light pulse to traverse the object there. Hence, the measured value for the length must be greater (QL m) when the object is located on the orbiting satellite than it previously was on the earth's surface (L m).

When a prediction is demonstrated to be false, there is no recourse but to modify the theory on which it is based. Belief in Fitzgerald-Lorentz contraction is an unavoidable consequence of the Lorentz transformation (LT) introduced in Einstein's original paper [1], so it is clear in the present case what needs to be amended. Moreover, there is another prediction based on the LT which is clearly violated in the GPS experimental procedure, namely the supposed symmetric nature of the time dilation effect itself. The unadjusted (proper) clock B on the satellite is known to run slower than its stationary counterpart (A) on the earth's surface, even though the LT forces one to assume the opposite relationship must hold. Similar violations of the LT predictions have been noted in connection with transverse Doppler investigations carried out on high-speed rotors [4]. In that case it has been concluded that the problem with the LT is that it is only valid for uniformly translating systems [5].

Einstein's equivalence principle between the effects of gravity and acceleration [3] does lead to the observed Doppler frequency shifts in all cases, and so there has been a wide consensus that this alternative theory is universally valid when the object of the measurement is not in uniform translation. However, there is no reason to assume that the satellite in the above example is not moving at constant velocity when the length measurement is made on the satellite. Moreover, the arguments in favor of the equivalence principle invariably focus on the effects of time dilation alone and thus ignore questions about the way velocities and lengths change upon acceleration. As already mentioned above, Einstein [3] pointed out that the speed of light increases with gravitational potential. The reason is that light frequencies increase with altitude, whereas the corresponding wavelengths do not. The GPS experiment under discussion indicates instead that lengths increase with acceleration while the speed of light is unchanged. As shown in the Appendix, the same conclusion of isotropic length expansion results when one includes the effects of the gravitational red shift explicitly in the arguments. Thus, the results of the experiment are seen to be neither consistent with the Lorentz transformation nor the equivalence principle. Yet they are based on exactly the same theoretical assumptions that form the basis for the well-established GPS methodology.

III. Conclusion

An experiment has been described to measure the length of an object located on a satellite in relative motion to the observer on the earth's surface. By using the standard assumptions underlying the GPS procedure, it is found that the length expands by the same factor by which an onboard clock slows down due to its acceleration. Since the elapsed time for light to traverse the object is independent of the orientation of the object, it follows that the expansion is the same in all directions. Both effects run contrary to the predictions of the Fitzgerald-Lorentz contraction effect of special relativity.

This result uncovers a fundamental inconsistency in the latter theory. Since the speed of light is the same for two observers in relative motion according to Einstein's second postulate, as confirmed by experiment, it follows that the elapsed time for light to traverse a given object is a measure of its length. This is the basis for the modern definition of the meter as the distance travelled by a light pulse in c^{-1} s. A clock slowed by time dilation has a longer period and the unit of time in its rest frame is increased by the same fraction. However, in order for the speed of light to be unaffected by this change it follows that the unit of velocity does not change either. The only way this can occur in a consistent manner is if the unit of distance in the same rest frame, which is the meter (or meter stick in conventional terminology), increases by the same fraction as the unit of time, and *in all directions equally*. The Fitzgerald-Lorentz contraction effect also follows from Einstein's postulates through the LT. The only way to distinguish between the two predictions (isotropic vs. anisotropic, expansion vs. contraction) is by experiment. The required information to settle the issue definitively was not available in 1905, but the situation has fundamentally changed with the development of atomic clocks in general, and the GPS methodology in particular. The practical success of the latter leaves no doubt as to which prediction of Einstein's original theory is the correct one.

Appendix

Gravitational effects on clock rates and the speed of light have not been considered in the above example, but it is easy to include them in the theoretical discussion. In the actual GPS technology a correction is made for the gravitational red shift on the rate of the onboard clock as well as for the effects of time dilation. The two effects oppose each other and that of the gravitational red shift is stronger. Thus, two correction factors are needed in adjusting the rate of clock C in the above example: Q>1 for time dilation as before, and S>1 for the effects of gravitation, with S>Q. The pre-correction factor is thus QS^{-1} instead of simply Q above. The factor S also determines the value of the speed of light measured on the satellite by the observer on the earth's surface: the value is Sc in that case, i.e. it increases with the altitude of the satellite.

The elapsed time of $2Lc^{-1}$ s read from the proper clock B on the satellite is unaffected by a change in the gravitational potential of the satellite because this measurement is completely local. Because of the different pre-correction factor, however, the corresponding elapsed time measured with clock C now has a smaller value of $2QL(Sc)^{-1}$ s. As before, this value is equal to the elapsed time on clock A at the earth's surface, consistent with the standard assumptions underlying the GPS procedure. The speed of light on the satellite measured from the earth's surface (Sc m/s) is then multiplied with the latter elapsed time, which upon division by two leads to the same value for the length of the object on the satellite as obtained in Sect. II (QL m). The latter result is thus seen to be completely independent of gravitational effects, as was to be shown.

References

1) A. Einstein, Ann. Physik **17**, 891 (1905).
2) C. M. Will, *Was Einstein Right?* (Basic Books Inc., U.S, 1993), p. 272.
3) A. Einstein, Jahrb. Radioakt. u. Elektronik **4**, 411 (1907).
4) H. J. Hay, J. P. Schiffer, T. E. Cranshaw, and P. A. Egelstaff, Phys. Rev. Letters **4**, 165 (1960).
5) C. W. Sherwin, Phys. Rev. **120**, 17 (1960).

(April 26, 2011)