Changes in the Relativistic Theory of Electromagnetism Prescribed by Experience with the Global Positioning System

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Abstract

One of the main principles on which the Global Positioning System (GPS) technology is based is the strict proportionality between the rates of clocks in relative motion. In recent work it has been demonstrated that relativity theory can be modified to incorporate this principle without violating Einstein's postulate of the constancy of light in free space, also assumed in GPS. In the present work attention is centered on Einstein's relativistic treatment of electromagnetic interactions. For example, it is pointed out that forces are assumed to be invariant in the revised theory, whereas Einstein concluded that transverse components of the Lorentz electromagnetic force have different values in the rest frame of accelerated electrons than in that of the stationary frame from which they are observed. It is shown that the necessary modifications of relativity theory can be achieved by eliminating an undeclared assumption in Einstein's original derivation of the Lorentz transformation (LT) and replacing it with the GPS axiom of the strict proportionality of clock rates in different rest frames. An assumption about the form of the inverse transformation of the electromagnetic field components also has to be changed. The example of two electrons being repelled as they move alongside each other in the laboratory is used as an application of the revised theory.

Keywords: Lorentz force, uniform scaling, Global Positioning System (GPS), Lorentz transformation, Maxwell equations

I. Introduction

It is has been shown in recent work [1] that the procedures used by the Global Positioning System (GPS) can be used to measure the changes in the dimensions of objects that occur as a result of being accelerated. Clocks on the satellites are slowed by time dilation (after correcting for the effects of the gravitational red shift), but because of the relativity principle no change in rate will be noted onboard. The same holds true for length measurements, but the elapsed times for light to traverse the objects will be greater on clocks on the earth's surface than they were prior to launch. Since the speed of light does not change on the satellite, in accord with Einstein's postulate [2] of the constancy of the speed of light in free space, it follows that the objects' dimensions have expanded isotropically as a result of the acceleration. This result runs contrary to the predictions of the length contraction effect of special relativity (SR [2]) and also of the equivalence principle of general relativity (GR), which holds that lengths are not changed upon acceleration.

There are two other aspects of the GPS methodology than are in conflict with SR. First, there is no ambiguity with regard to which clock runs slower. The rates of satellite clocks are pre-corrected so as to compensate for time dilation. Secondly, it is assumed that events on the satellite occur at the same time for observers on the earth's surface. Otherwise, there would be no point it making the clock-rate adjustment. On the contrary, SR [2] claims that there is no such thing as remote simultaneity for clocks in relative motion and that it is impossible to say with certainty which of them has the slower rate.

Each of the above three conflicts with experiment is removed by eliminating one of the assumptions in Einstein's original paper [2] in the derivation of the Lorentz transformation (LT) and replacing it with the strict proportionality of clock rates which is the basis of the GPS pre-correction technique. However, one of the main purposes of SR was to construct a relativistic theory that is compatible with the Maxwell electromagnetic field equations. Einstein's two postulates of SR were chosen specifically for achieving this goal. The question is thus how making what amounts to an additional postulate in the revised theory affects the treatment of electromagnetic interactions. In particular, it has been found that assuming strict proportionality between the rates of clocks in relative motion implies that all other physical properties are subject to similar relationships. Since time, energy and distance have equal proportionality constants for observers in different rest frames, it follows that there must always be complete agreement on their respective values for forces and relative velocities. This conclusion again runs contrary to SR since it holds that the values of forces in directions that are transverse to the direction of the relative velocity of the two rest frames will be different for the respective observers, although the corresponding parallel components will be the same.

A review of the relativistic theory of electromagnetic interactions will be undertaken in the following discussion. Emphasis will be placed on the way the revised mechanical theory based on the GPS methodology differs from conventional SR, and the implications this has on the treatment of Maxwell's electromagnetic field equations.

II. Review of Einstein's Theory of Electromagnetism

The starting point in the discussion is the Maxwell field equations in free space:

$$
\begin{aligned}
c^{-1}\partial E_x/\partial t &= \partial B_z/\partial y - \partial B_y/\partial z & -c^{-1}\partial B_x/\partial t &= \partial E_z/\partial y - \partial E_y/\partial z \\
c^{-1}\partial E_y/\partial t &= \partial B_x/\partial z - \partial B_z/\partial x & -c^{-1}\partial B_y/\partial t &= \partial E_x/\partial z - \partial E_z/\partial x \qquad (1)\\
c^{-1}\partial E_z/\partial t &= \partial B_y/\partial x - \partial B_x/\partial y & -c^{-1}\partial B_z/\partial t &= \partial E_y/\partial x - \partial E_x/\partial y,
\end{aligned}
$$

where E_i and B_i (i=x, y, z) are the respective electric and magnetic field components, x, y, z and t are the spatial and temporal coordinates and c is the speed of light in free space. These equations are valid in all inertial systems, and in accord with the relativity principle they must have the same form in each. For this condition to be satisfied, it is necessary that the above equations be invariant to a suitable coordinate transformation. Maxwell also showed that the speed of light must be the same in all inertial systems in order for the equations to have general validity, thus ruling out the classical Galilean transformation for this purpose. The Lorentz transformation (LT) does satisfy this criterion, but it is only possible to specify the exact form of the corresponding equations to within a normalization factor φ [3] without additional information other than the constancy of the light speed [v is the relative speed of the two rest frames S and S' and $\gamma = (1-v^2c^{-2})^{-0.5}$]:

$$x' = \gamma\varphi\,(x - vt) \tag{2a}$$

$$y' = \varphi y \tag{2b}$$

$$z' = \varphi z \tag{2c}$$

$$t' = \gamma\,\varphi(t - vxc^{-2}). \tag{2d}$$

Since speed is a ratio of the change in one of the spatial coordinates to the corresponding elapsed time, the same result for the light speed holds for any value of φ. Einstein was aware of this lack of specificity [2], but he argued that φ could only be a function of v and then went on to show on the basis of symmetry that $\varphi = 1$ was therefore the only acceptable solution. However, there is no reason that φ cannot also depend on the *velocity* ***u*** *of the object of the measurement* for the observers in the different rest frames. It has been shown elsewhere [4, 5] that the GPS assumption of the proportionality of the rates of clocks in relative motion can be incorporated into an alternative form of the LT by setting t' in eq. (2d) to Qt, where Q is the value of the proportionality constant used in the GPS "pre-correction" procedure. On this basis the value of φ is found to be $\eta Q\gamma^{-1} = (1 - vx/c^2t)^{-1}Q\,\gamma^{-1} = (1 - vu_x/c^2)^{-1}Q\,\gamma^{-1}$, which does depend on the velocity component u_x as well as v.

In order to apply the LT in any form to the Maxwell equations given in eq. (1), it is first necessary to obtain the corresponding relations between partial derivatives of the four coordinates:

$$\partial/\partial x = \gamma\varphi\, (\partial/\partial x' - vc^{-2}\partial/\partial t') \quad (3a)$$

$$\partial/\partial y = \varphi\, \partial/\partial y' \quad (3b)$$

$$\partial/\partial z = \varphi\, \partial/\partial z' \quad (3c)$$

$$\partial/\partial t = \gamma\varphi\, (-v\partial/\partial x' + \partial/\partial t'). \quad (3d)$$

Because of the homogeneous form of the Maxwell equations in eq. (1), it is clear that the value of the function φ in the above equations is immaterial for the outcome of applying this transformation to obtain the corresponding relations in S'. This fact demonstrates that the GPS form of the LT in eq. (2), with $\varphi = \eta Q\gamma^{-1}$, is just as suitable for carrying out the transformation between different rest frames as Einstein's original version [2]. It is therefore not possible to distinguish between the two versions of relativity theory on this basis.

The next step in the derivation is to find a set of relations coupling the respective electromagnetic field components in the two rest frames. The relationship must be such that when the corresponding substitutions are made in the transformed version of eq. (1), the result is an equivalent set of Maxwell field equations in S'. Einstein succeeded [2] in finding the desired relationships, but as before with the derivation of the LT, it is found that they also come with an undetermined normalization constant, which he referred to as ψ (v):

$$E_x' = \psi(v)\, E_x \qquad B_x' = \psi(v)\, B_x \quad (4a)$$

$$E_y' = \gamma\psi(v)\, (E_y - vc^{-1}B_z) \qquad B_y' = \gamma\psi(v)\, (B_y + vc^{-1}E_z) \quad (4b)$$

$$E_z' = \gamma\psi(v)\, (E_z + vc^{-1}B_y) \qquad B_z' = \gamma\psi(v)\, (B_z - vc^{-1}E_y). \quad (4c)$$

In this case, he argued that ψ (v) must be an even function of v and **also be equal to its inverse function ψ^{-1} (v)**. As before with φ, his conclusion on this basis was that the only permissible value is $\psi(v) = 1$.

In order to deduce the transformation properties of the law of motion for the electromagnetic interaction it is necessary to also specify how the components of acceleration transform. Einstein's procedure [2] was to use the LT for this purpose. He did not give complete details of this derivation, but the result is given below:

$$a_x = d^2x/dt^2 = \gamma^{-3}d^2x'/dt'^2 = \gamma^{-3}a_x'$$
$$a_y = d^2y/dt^2 = \gamma^{-2}d^2y'/dt'^2 = \gamma^{-2}a_y' \qquad (5)$$
$$a_z = d^2z/dt^2 = \gamma^{-2}d^2z'/dt'^2 = \gamma^{-2}a_z'.$$

At least from mnemonic standpoint, one can arrive at the same result by using the Fitzgerald-Lorentz contraction effect (FLC) and time-dilation formulas from the LT: $x'=\gamma x$, $y'=y$, $z'=z$ and $t'=\gamma^{-1}t$. A more detailed discussion of the derivation of eq. (5) is given by Sard [6]. Einstein obtained the following equations of motion for the stationary observer (μ is the rest mass of the electron and e is its electric charge) by combining eqs. (4) and (5):

$$\mu\,\gamma^3 d^2x/dt^2 = e\,E_x = e\,E_x'$$
$$\mu\,\gamma^2 d^2y/dt^2 = e\,\gamma\,(E_y - vc^{-1}B_z) = e\,E_y' \qquad (6)$$
$$\mu\,\gamma^2 d^2z/dt^2 = e\,\gamma\,(E_z + vc^{-1}B_y) = e\,E_z'$$

He assumed that the equations of motion in the inertial frame (S') in which the electron is at rest can be obtained on the basis of the non-relativistic (F= ma) version of Newton's Second Law. He also assumed thereby that that the electronic charge e is invariant and that only electric fields need to be considered in S' since the electron is not in motion there.

The above theoretical development can be criticized on several grounds. For example, consider eq. (5) that Einstein used for the transformation of the acceleration components. The most straightforward means of obtaining these relationships is to start with Einstein's relativistic velocity transformation (VT), which is obtained by division of the variables in eq. (2):

$$u_x' = x'/t' = dx'/dt' = \eta\,(u_x - v)$$
$$u_y' = y'/t' = dy'/dt' = \eta\,\gamma^{-1}u_y \qquad (7)$$
$$u_z' = z'/t' = dz'/dt' = \eta\,\gamma^{-1}u_z.$$

In the case under discussion, $u_x = x/t = v$, and thus $\eta = (1 - vx/c^2t)^{-1} = \gamma^2$. More interestingly, however, **u'** *has constant null value* by virtue of the definition of S' as the frame in which the electron is always at rest. This means that the right-hands sides of all three equations are not actually determined, so **the VT is not a suitable starting point for obtaining the desired relationships between the corresponding acceleration components.** The fact is that the derivative of a constant null vector, which **u'** is by definition, can never give any result other than null. This negates Einstein's derivation of eq. (6), irrespective of whether the resulting equation of motion is valid or not. Nonetheless, eq. (5) can be obtained from eq. (7) by ignoring this fact and then proceeding with the differentiation with respect to $t' = t/\gamma$ under the assumption that γ has a constant value.

Consistent with the above criticism, Planck suggested a different way to derive eq. (6) and this approach was readily acknowledged by Einstein to be superior in a later paper [7, 8]. The general version of Newton's Second Law (F=dp/dt) was used in conjunction with the relativistic definition of momentum, $\mathbf{p} = \gamma\mu\mathbf{v}$, with the result below:

$$F_x = d/dt\,(\gamma\mu v_x) = \mu\gamma^3 d^2x/dt^2 = \mu\gamma^3 a_x$$
$$F_y = d/dt\,(\gamma\mu v_y) = \mu\gamma d^2y/dt^2 = \mu\gamma a_y \qquad (8)$$
$$F_z = d/dt\,(\gamma\mu v_z) = \mu\gamma d^2z/dt^2 = \mu\gamma a_z.$$

Einstein's eq. (6) then results by introducing the expression for the Lorentz force, $\mathbf{F} = e\,(\mathbf{E} + \mathbf{v}c^{-1}\mathbf{x}\,\mathbf{B})$, with **v** parallel to the x axis:

$$F_x = e\,E_x\,,$$
$$F_y = e\,(E_y - vc^{-1}B_z) \qquad (9)$$
$$F_z = e\,(E_z + vc^{-1}B_y).$$

On the basis of the latter derivation, it no longer seems appropriate to distinguish between a "longitudinal mass" ($\gamma^3\mu$) and a "transverse mass" ($\gamma\mu$) in the equation of motion [2]. The factors multiplying the rest mass evolve in a straightforward way as a result of differentiating the relativistic momentum to obtain eq. (6).

III. The Revised Theory Based on the GPS Methodology

The two derivations (Einstein's original [2] and that suggested to him by Planck [7]) of the equations of motion in eq. (6) differ in a significant way with regard to the relation between the electromagnetic force measured by the two observers. In the latter version the Lorentz force is simply set equal to d**p**/dt, whereas in the original, Einstein combined the acceleration relations of eq. (5) (note the extra factor of γ in the transverse components relative to the result derived from use of d**p**/dt) and the electromagnetic field transformation of eq. (4). In the latter case [2], an asymmetric relationship is assumed between the respective Lorentz force components in S and S':

$$F_x' = F_x$$
$$F_y' = \gamma F_y \qquad (10)$$
$$F_z' = \gamma\, F_z,$$

whereas each of the respective force components in the two rest frames is assumed to be equal in the later derivation [7]. In the following we will consider how eq. (6) is derived within the framework of the revised version of relativity theory which is based on the principles of the GPS methodology. Special attention will be paid to the relationship between the force components in the two rest frames to see if it complies with eq. (10).

The first step in Einstein's original derivation [2] is to apply the transformation of the derivatives in eq. (3) to Maxwell's equations given in eq. (1). It has already been pointed out in the previous section that the value of φ is immaterial in this procedure, and thus the GPS version of the LT which is compatible with the assumption of the strict proportionality of clock rates ($\varphi = \eta Q \gamma^{-1}$) also ensures that the form of Maxwell's equations is equivalent in the two rest frames. However, it should be noted that the latter version of the LT does not satisfy [5] the Lorentz invariance condition,

$$x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2, \quad (11)$$

and thus *it is incorrect to claim that the latter relation is essential in the relativistic theory of electromagnetic interactions.*

The GPS version of relativity theory also differs from SR in the way it obtains relationships between respective measured values of properties in different rest frames. This distinction shows up first and foremost with the assumed proportionality of clocks rates in the GPS methodology:

$$t' = Q\, t. \quad (12)$$

Similar proportionalities are assumed for other quantities, however, to maintain consistency with available experimental information. Because the speed of light is the same in all inertial systems, for example, it follows that distances r must satisfy the analogous relation [1]:

$$r' = Q\, r. \quad (13)$$

In addition, Bücherer's observations [9] of the increase in the inertial mass m_I of electrons indicate that the same proportionality constant should be used for this property, and as a consequence of Einstein's mass/energy equivalence relation, the same is true for energy E:

$$m_I' = Q\, m_I \quad (14)$$

$$E' = Q\, E. \quad (15)$$

Since forces are ratios of energy and distance, it follows from eqs. (13) and (15) that their measured values are the same for observers in different rest frames. Hence, it is clear that **Einstein's asymmetric force relations in eq. (10) are in disagreement with the predictions of the GPS-compatible version of relativity theory**. The question is thus how to amend Einstein's original derivation to avoid the above asymmetry and still obtain the correct equations of motion in eq. (6).

The answer lies in the definition of ψ (v) in eq. (4). Einstein assumed [2] that ψ must be equal to its inverse $\psi^{-1}(-v)$. This would necessarily be the case if the two rest frames were indistinguishable, but the fact is that such an assumption is not justified. By definition, S' is the rest frame co-moving with the accelerated electron, whereas S is an arbitrary rest frame in which the observer makes his measurements of the electromagnetic field quantities of interest. Therefore, it is not necessary to assume that $\psi^2 = 1$, but rather only that ψ is an even function of v [2]. As a result, one can use the condition of invariance of forces in the two rest frames to fix the value of ψ. For example, based on eq. (4b), we find that

$$F_y' = e'E_y' = e'\gamma\psi(v)(E_y - vc^{-1}B_z) = e'\gamma\psi(v)F_y e^{-1}. \qquad (16)$$

After equating F_y' and F_y and solving, we obtain

$$\psi(v) = \psi_y(v) = e(e'\gamma)^{-1}. \qquad (17)$$

(Note that the electric charges e and e' in S and S' have not been set equal, contrary to Einstein's original convention [2]; we will return to this point shortly in Sect. IV.)

Since the quantities in eq. (4) are components of a vector, it is not correct to assume without further information that ψ (v) is the same for all three directions; hence, a label has been included in eq. (17). However, one knows from symmetry that the corresponding value for the z-component must have the same value ($\psi_y = \psi_z$). Nonetheless, the situation is different for the x-component. In that case, based on eq. (4a), we obtain:

$$F_x' = e'E_x' = e'\psi_x(v)E_x = e'\psi_x(v)F_x e^{-1}, \qquad (18)$$

and therefore,

$$\psi_x(v) = e\,e'^{-1}. \qquad (19)$$

Substitution of these values in eq. (4) then leads to the following field transformation:

$$E_x' = ee'^{-1}E_x \qquad B_x' = ee'^{-1}B_x \qquad (20a)$$

$$E_y' = ee'^{-1}(E_y - vc^{-1}B_z) \qquad B_y' = ee'^{-1}(B_y + vc^{-1}E_z) \qquad (20b)$$

$$E_z' = ee'^{-1}(E_z + vc^{-1}B_y) \qquad B_z' = ee'^{-1}(B_z - vc^{-1}E_y). \qquad (20c)$$

This result is seen to be identical with Lorentz's original transformation [3]. It clearly satisfies the requirement that the Lorentz force components are the same in every rest frame, as was to be shown.

Eliminating Einstein's requirement that ψ (v) = 1 in all cases has consequences for the inverse transformation of eq. (20). It is not permissible to obtain this result by simply interchanging the primed and un-primed symbols and changing the sign of v. Instead, one has to replace ψ by ψ^{-1} in obtaining the inverse, while carrying out the other changes in the usual manner. This *caveat* is clearly unimportant for the x-components, where the normal procedure could be used without change, but it is critical for the transverse components. The correct inverse transformation is thus:

$$E_x = e'e^{-1} E_x' \qquad B_x = e'e^{-1} B_x' \tag{21a}$$

$$E_y = \gamma^2 e'e^{-1} (E_y' + vc^{-1}B_z') \qquad B_y = \gamma^2 e'e^{-1} (B_y' - vc^{-1}E_z') \tag{21b}$$

$$E_z = \gamma^2 e'e^{-1} (E_z' - vc^{-1}B_y') \qquad B_z = \gamma^2 e'e^{-1} (B_z' + vc^{-1}E_y'). \tag{21c}$$

The fact that the equations for the transverse field components do not have equivalent forms in eqs. (20) and (21) is permissible (and necessary) because, as mentioned above, S and S' are distinguishable due to the unique role played by the accelerated electron.

IV. Uniform Scaling of Properties

The theoretical treatment discussed in the preceding sections makes a definite assumption about the relative rates of clocks in S and S', specifically that the one in S' moving with the electron runs γ times slower than its counterpart in S. An important aspect of the GPS-compatible version of relativity theory is that the ratio of clock rates can take on any value, consistent with the proportionality axiom of eq. (12). Thus, $Q = \gamma^{-1}$ in the above discussion. Since the revised theory is objective, it is necessary to assume that the inverse relationship holds from the standpoint of the observer in S, namely

$$t = Q^{-1} t' = Q' t'. \quad (22)$$

A simple way to think about these relationships is ascribe a different unit of time in the two rest frames. Because of the relativity principle, both observers assume that their local unit is standard (1 s), but in reality only one (or neither) of them can make this claim. In other words, the standard has to be defined in a specific rest frame mutually agreed upon to be fully consistent with observation. In the case of the GPS methodology it is natural to define the standard unit of 1 s to a clock at a particular location on the earth's surface. The corresponding unit on the satellite (excluding the effects of the gravitational red shift as usual in this discussion) based on this standard would then be Q' s. This means that the numerical value of an elapsed time for an event anywhere in the universe will be Q' times greater at the standard location than that measured locally on the satellite, consistent with eq. (22). In other words, Q' can be looked upon as a *conversion factor* that must be used in order to predict elapsed times on the ground based on measurements carried out with an identical (proper) clock located on the satellite. Analogous considerations hold for all other physical properties [see eqs. (13-15), for example]. It needs to be emphasized that $Q=Q'^{-1}$ varies from one rest frame to another and thus in the general case must be treated as a variable quantity in the theoretical discussion.

A good example for illustrating the above points can be given with reference to the force equations in eq. (8). It is important to recognize that μ is the rest mass of the electron and thus not the value of the inertial mass m of the electron when observed in a rest frame other than S'. The general formula for this quantity depends on the value of the conversion factor in the rest frame in which the measurements are to be made. It helps to avoid confusion by rewriting the equations in terms of the variable $m = Q'\mu$; for example, $F_y = ma_y = Q'\mu a_y$. It is then understood that both m and a_y are to be determined by an observer in his units. If the observer is at rest in S', then $m = \mu$, since $Q'=1$ there.

The reason why it is important to make the above distinction is that it has an important effect on the equation of motion for F_x due to the γ^3 factor in eq. (8). The latter factor arises because of the differentiation of the momentum using Newton's Second Law. In particular, γ^2 is seen to derive from $d\gamma/dt$, whereas the other factor of γ comes from the value of the inertial mass of the electron in the observer's units. Thus, in light of the above remarks, it is preferable to rewrite the equation in terms of m, namely as:

$$F_x = \gamma^2 m a_x. \qquad (23)$$

This conclusion is of special interest when evaluating eq. (22) in S', i.e. the rest frame of the electron: $F_x = \gamma^2 \mu a_x' = \gamma^2 \mu d^2x'/dt'^2$. In deriving eq. (5), Einstein assumed to the contrary that $F_x = \mu a_x'$, but in doing so, he overlooked an important aspect of the problem at hand. By construction, the electron is under constant acceleration along the x-axis, and thus its momentum is changing with time. Einstein's formula is only valid for the case in which the speed v_x is constant *until the force is applied* to produce the acceleration.

To illustrate how these conclusions work out in practice, let us next consider the case of two electrons moving in parallel along the x-axis. For an observer in an arbitrary rest frame, the equation of motion in S using the Lorentz force in the y-direction is:

$$m a_y = Q'\mu d^2y/dt^2 = F_y. \qquad (24)$$

The formula for the time-dependence of the change Δy in the location of one of the electrons is therefore:

$$\Delta y = F_y t^2/2Q'\mu. \qquad (25)$$

Eq. (13) states that the value in S' must be $\Delta y' = Q'^{-1} \Delta y$. The corresponding equation of motion is [with $t'=Qt=Q'^{-1}t$ from eq. (12)]:

$$\mu a_y' = \mu d^2y'/dt'^2 = F_y' = F_y, \qquad (26)$$

from which it follows that

$$\Delta y' = F_y t'^2/2\mu = F_y t^2/2Q'^2\mu = Q'^{-1} \Delta y. \qquad (27)$$

In the GPS example, S' is the rest frame of the satellite and Q'=γ. Thus the distance values measured locally on the satellite are always smaller than in S on the ground.

Next we consider the equation of motion in the parallel direction:

$$m\gamma^2 a_x = Q'\gamma^2 \mu d^2x/dt^2 = F_x. \quad (28)$$

The formula for the time-dependence of the change Δx in the location of one of the electrons is therefore:

$$\Delta x = F_x t^2/2Q'\gamma^2\mu. \quad (29)$$

Eq. (13) states that the value in S' must be $\Delta x' = Q'^{-1}\Delta x$. The corresponding equation of motion is therefore:

$$\mu\gamma^2 a_x' = \mu\gamma^2 d^2x'/dt'^2 = F_x' = F_x, \quad (30)$$

from which it follows that

$$\Delta x' = F_x t'^2/2\gamma^2\mu = F_x t^2/2Q'^2 \gamma^2\mu = Q'^{-1}\Delta x. \quad (31)$$

The scaling is consistent, with times, distances and inertial masses changing in tandem and forces and relative speeds being invariant (note that acceleration scales as $\mathbf{a}=Q'^{-1}\mathbf{a'}$).

There is a separate scaling of properties connected with the gravitational red shift. It is based on a quantity S that plays essentially the same role when the gravitational potential of the observer and/or object of measurement changes that Q plays when the state of motion is varied. More details concerning the general topic of gravitational scaling may be found elsewhere [10]. The value of S increases with altitude just as Q increases with the speed of the rest frame relative to the appropriate reference. For example, $S = 1 + ghc^{-2}$ when the altitude of the rest frame changes by h and the acceleration due to gravity is g. It is thus a generalization of the scaling quantity introduced by Einstein in his 1907 paper [7]. Accordingly, energy and force scale as S, time and inertial mass as S^{-1}, and distance and momentum are invariant (S^0). This is the reason behind Einstein's conclusion that the speed of light changes with altitude, since velocities change in the same proportion as energies, forces and light frequencies. It is relatively easy to incorporate the effects of gravitational scaling into the framework of the GPS-compatible version of relativity theory. In particular, one has to include the effect of the gravitational red shift in computing the pre-correction factor for clocks on the GPS satellites [32].

Finally, we return to the question of how electric charges vary between different rest frames that was raised in Sect. III in connection with eqs. (16) and (17). The convention in discussing the transformation of the Maxwell equations has been to assume that charge is invariant, but this position overlooks an important detail concerning the way this quantity is defined in the treatment of electromagnetic interactions. The only essential requirement is that the ratio e^2/ε_0 in Coulomb's Law varies as Q^2, since otherwise electric forces will not be invariant because of eq. (13) for the scaling of distances. This can obviously be arranged in a variety of ways [11], without insisting that charge be invariant. For this reason, the ratio ee'^{-1} has been included explicitly in the field transformation of eqs. (20-21), leaving open the option of setting it to unity as the most convenient choice.

V. Conclusion

The relativistic theory of electromagnetic forces has been re-examined in light of the working assumptions of the GPS methodology. In contrast to Einstein's original theory (SR), it is assumed that measurement is perfectly objective and that the rates of clocks in different rest frames are always in strict proportion to one another. It has been shown on this basis that the lengths of objects increase uniformly in all directions in inverse proportion to the rates of clocks, rather than decrease anisotropically as predicted by the FLC of SR. An alternative LT has been derived which eliminates the mixing of space and time coordinates long thought to be essential in relativity theory and replaces it with the strict proportionality of clock rates (t'

= Qt) found to hold within the framework of GPS. This is accomplished by making a change in Einstein's original derivation of the LT which still leaves his two postulates of relativity in force. It is shown that Maxwell's equations are invariant to the alternative LT, thus satisfying the main requirement for a space-time transformation for a suitably relativistic description of electromagnetic forces.

The GPS-compatible theory requires that the values of forces are the same in all rest frames because energies and distances scale in the same manner as the state of motion is varied. As with the derivation of the LT, it is found that the failure of SR to obtain this result lies in an incorrect assumption. Because the two rest frames can be distinguished on the basis of their relationship to the accelerated electron, it is not necessary for the inverse transformation of the electromagnetic field components to have the same form as that in the forward direction. Moreover, it is also not necessary to assume that the transformation equations for the parallel and transverse field components must satisfy the same normalization condition, contrary to what is assumed in Einstein's original derivation. Instead, the required normalization factors can be determined separately so as to guarantee the condition of invariant force components. In essence, it is only necessary to use Newton's Second Law and require that d**p**/dt be equal to the Lorentz force in all rest frames. The resulting theory has also been shown to be consistent with the general conclusion of the GPS version of relativity theory that the measured values of distances and elapsed times scale in the same manner in going from one rest frame to another.

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