

Role of an Objective Rest System to Determine Energy and Momentum Relationships for Observers in Relative Motion

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Abstract

The energy-momentum four-vector transformation of relativity theory is derived from Hamilton's equation for the change in the energy of an object that is produced by an applied force. Einstein pointed out in his original work that two clocks in relative motion can be distinguished when one of them has been accelerated with respect to the other, and he used this as justification for his prediction of time dilation. An example is presented in which two airplanes are subjected to the same degree of acceleration so that they both attain the same speed relative to the ground while traveling in opposite directions. The experiment with circumnavigating airplanes carried out by Hafele and Keating shows that, for a hypothetical non-rotating planet, their respective onboard clocks will be running at the same rate despite the fact that they are in relative motion (Triplet Paradox). It is argued that this result is consistent with the fact that neither clock has been directly accelerated with respect to the other in this example, but rather each with respect to the surface of the non-rotating planet. It

is concluded that the conventional energy-momentum transformation does not hold under these circumstances , and therefore that the corresponding invariance relation ($E^2 - p^2 c^2 = E'^2 - p'^2 c^2$) is by no means of general validity. The concept of an objective rest system (ORS) from which objects are accelerated is employed to define a rational set of units for energy, time and mass for different inertial systems. Accordingly, two observers must always obtain measured values for these properties that are in the same proportion for any object. This procedure allows for a resolution of the Triplet Paradox consistent with Einstein's original conjecture, while avoiding the contradictions that arise when the above invariance relation for energy and momentum is assumed to be of general validity.

I. Introduction

One of the fundamental aspects of the special theory of relativity (STR) is the existence of four-vector relationships. The most basic of these involves the space-time four-vector that is associated with the Lorentz transformation (LT). Einstein argued in his original work [1] that an invariance condition (Lorentz invariance) must exist between these four variables in order to satisfy his second postulate (LSP). The latter states that the speed of light has a constant value of c ($2.99792458 \times 10^8 \text{ ms}^{-1}$) which is independent of the states of motion of both the source and the observer. More recently, it has been shown [2,3] that the LT is not the only transformation that satisfies the LSP, however. The alternative transformation (GPS-LT) satisfies a different space-time invariance condition, however.

In previous work [4, 5], attention has been called to an empirical formula referred to as the Universal Time Dilation Law (UTDL) that relates the rates of clocks to their speeds relative to a specific rest frame (objective rest system ORS [6]). Experiments with circumnavigating airplanes [7] have shown, for example, that the ORS for onboard atomic clocks is the set of "non-rotating polar axes." Correspondingly, the axis of the high-speed rotor employed in the transverse Doppler measurements carried out by Hay et al. [8], and later by Kündig [9] and Champeney et al. [10], is the ORS in this case. Einstein anticipated such relationships [1], predicting that a clock at the Equator would run more slowly than its identical counterpart located at one of the Poles. The symmetrical relationship normally expected between inertial systems according to STR was assumed to be inoperative since a force has been applied to one of them.

One of the implications of the UTDL, however, is that two clocks can have the same rate even though they are moving with respect to each other. This situation occurs whenever two clocks move at the same speed relative to their common ORS. This conclusion, which shall be referred to in the following as the "Triplet Paradox," implies that one cannot generally predict the relative rates of atomic clocks based merely on knowledge of their

respective speeds relative to a given observer, contrary to what is expected from STR [1].

The UTDL indicates further that two clocks cannot each be running slower than the other, for example, in opposition to what is expected on the basis of the Lorentz invariance condition of STR [1]. In the discussion below, it will be shown that the existence of such relationships also puts energy-momentum invariance in a new light.

II. Derivation of the Invariance Condition

The definition of energy or work E is given in terms of an applied force \mathbf{F} on an object. A key point in the classical theory of kinematics is that the object must move along the direction of the applied force in order for any work to have been done:

$$dE = \mathbf{F} \cdot d\mathbf{r}, \quad (1)$$

where $d\mathbf{r}$ is the vector distance the object moves. The change in energy dE can be related to the velocity $\mathbf{u} = d\mathbf{r}/dt$ of the object by introducing Newton's Second Law, $\mathbf{F} = d\mathbf{p}/dt$. The result is Hamilton's equation:

$$dE = d\mathbf{p} \cdot \mathbf{u} = u dp, \quad (2)$$

where $d\mathbf{p}$ is the change in momentum caused by the force \mathbf{F} in time dt . The increase in energy ΔE (kinetic energy = $\frac{1}{2} m u^2$) is obtained in the classical theory by substituting $d\mathbf{p} = m d\mathbf{u}$ in this equation and integrating (m is the inertial mass of the object).

The Galilean transformation (GT) expresses the relationship between measurements of space and time relative to two different origins that are moving with a fixed velocity relative to one another. If the direction of relative motion is along the x axis, then

$$dx = dx' + u dt'. \quad (3)$$

The unprimed quantities refer to the measurements made by a "stationary" observer relative to his fixed origin, whereas the primed values are obtained relative to the "moving" origin. A

similar equation holds for the corresponding energy and momentum measurements because of eq. (2):

$$dE = dE' + u dp_x', \quad (4)$$

where the primed values refer to the measurements of an observer for whom the “moving” origin is fixed. The other pairs of primed and unprimed quantities are assumed to be equal ($dt=dt'$, $dy = dy'$, $dp_y = dp_y'$ etc.) in the GT.

The above equations are inconsistent with Einstein’s second postulate regarding the constancy of the speed of light in free space. The LT results by demanding that $dr/dt = dr'/dt' = c$ independent of the relative speed u of the two origins. Again, an analogous set of equations can be derived for energy and momentum, in this case by demanding that $dE/dp = dE'/dp' = c$ when the object is a light pulse [$\gamma = (1 - u^2/c^2)^{-0.5}$]:

$$dE = \gamma (dE' + u dp_x') \quad (5a)$$

$$dp_x = \gamma [dp_x' + (u/c^2) dE'] \quad (5b)$$

$$dp_y = dp_y' \quad (5c)$$

$$dp_z = dp_z'. \quad (5d)$$

In this case, the condition of invariance for the (E, \mathbf{p}) four-vector is

$$dE^2 - dp^2 c^2 = dE'^2 - dp'^2 c^2, \quad (6)$$

in close analogy to that for the space-time variables of the LT,

$$dr^2 - c^2 dt^2 = dr'^2 - c^2 dt'^2. \quad (7)$$

There is an important special case for eq. (5), namely when the object is at rest with respect to the moving origin ($d\mathbf{p}'=0$). Under this condition,

$$dE = \gamma dE'. \quad (8)$$

If one interprets the differential quantities as infinitesimal parts of a macroscopic object and then integrates eq. (8), the relativistic equivalent of kinetic energy K is obtained, namely

$$K = E - E' = (\gamma - 1) E'. \quad (9)$$

The inverse of eq. (5b) is given below:

$$dp_x' = \gamma [dp_x - (u/c^2) dE] \quad (10)$$

If the $d\mathbf{p}'=0$ condition is again assumed, it is found [3] after integration and defining inertial mass m as the ratio of momentum p to speed u that

$$m \equiv p/u = p_x/u = (E/c^2), \quad (11)$$

which is the famous Einstein mass-energy equivalence relation [1]. This result also shows that the rest energy of an object is generally not zero since it must be proportional to the rest (or proper) mass $\mu = m'$, that is, the value measured *in situ* by any observer [3]. It also shows on the basis of eq. (8) that

$$m = \gamma m' \equiv \gamma \mu, \quad (12)$$

since the speed of light c is independent of u . Because of the definition of momentum \mathbf{p} in eq. (11), it also follows that

$$\mathbf{p} = \gamma \mathbf{p}'. \quad (13)$$

Einstein obtained an analogous proportionality relation for elapsed times from the LT that is the basis for time dilation in STR [1]:

$$dt = \gamma dt'. \quad (14)$$

The LT itself implies that events may not occur simultaneously for two observers in relative motion, but that is not expected on the basis of eq. (14) since it is not possible for dt to be equal to zero while dt' has a non-zero value. . This non-simultaneity prediction of STR is also contradicted by experience with atomic clocks employed in the Global Positioning System (GPS) technology, thereby speaking against the LT as a physically valid space-time transformation [2,3]. It will be shown below that similar objections arise for eqs. (8,12-13) for energy, inertial mass and momentum based on experiments with atomic clocks located on circumnavigating airplanes [7].

III. Limitations in Validity of E,p Invariance

There has been a broad consensus that all of the relativistic equations discussed above enjoy universal validity for inertial systems. The space-time LT invariance condition of eq. (7) implies that the speed of light in free space is always equal to c (LSP), for example, consistent with the results of numerous empirical investigations dating back to the Michelson-Morley [11] and Kennedy-Thorndike experiments [12]. Observations of the transverse Doppler effect [8-10, 13] also support the LSP[1] to a satisfactory degree of approximation. Indeed, the results of the Fresnel light-drag experiment carried out in the earth 19th century already suggested that the speed of light should not change with the velocity of a medium with refractive index of unity [14].

The above results also constitute strong evidence for Galileo's relativity principle (the first postulate of STR), namely that the laws of physics are the same in all inertial systems. Einstein pointed out in his original work [1] that this equivalence is destroyed whenever force is applied to an object such as an atomic clock, however. His prediction of time dilation was the subject of much debate because of uncertainty on this point [15, 16]. The experiments of Hafele and Keating [7] with circumnavigating airplanes provided explicit confirmation of Einstein's conjecture, but they also emphasized that care must be taken in applying eq. (14) to compute the amount of the time-dilation effect. They were only able to obtain satisfactory agreement with observed timing results by employing a hypothetical reference clock located on the Earth's polar axis. The speed u to be used to evaluate γ in eq. (14) had to be determined relative to this axis. The justification given for this procedure was that clocks located on the airplanes and on the Earth's surface were subject to acceleration due to rotation about this axis, which view is at least consistent with Einstein's original arguments [1].

It is not difficult to find a counter-example for this position, however. Consider the diagram in Fig. 1 that shows two airplanes (rockets) flying off in opposite directions. The prescriptions of relativity theory must also apply for the case when the airport from which the

planes leave is not subject to any kind of acceleration. When the airplanes reach a constant speed, there is thus no reason under these circumstances to disqualify them as non-inertial. One is then led to a clear contradiction in applying eq. (14) to this situation, since it would imply that the clocks on each airplane must run slower than those on the other (Triplet Paradox). What this example demonstrates instead is that the key point in applying the time-dilation formula is to identify a reference clock at the position where acceleration of the object has occurred (objective rest system or ORS [6]). Since the clocks on the two airplanes in Fig. 1 have each reached the same speed relative to the airport, *it follows from symmetry* that there is no way to distinguish them since all directions are equivalent under these circumstances. Just because the airplanes are inertial systems does not allow the use of their onboard clocks as a reference in applying eq. (14). Since their speed relative to the ORS in this application is the same, it therefore follows that their clock rates are exactly equal, consistent with the symmetry inherent in Fig. 1. As discussed elsewhere [2,3, 17, 18], this result shows that the Lorentz invariance condition of eq. (7) is not universally satisfied. Nonetheless, a different space-time transformation exists (the Global Positioning System-Lorentz transformation / GPS-LT) which is consistent with all experimental timing data and does not come into conflict with either of Einstein's two postulates of STR [1].

The Triplet Paradox for atomic clocks mentioned above is also relevant for measurements of energy and mass made by observers on the two airplanes, however. From the standpoint of an observer in the ORS (airport), identical objects on both planes have exactly the same properties. If the rest energy of these objects is E' , then according to eq. (8) *the ORS observer* will measure a value that is $\gamma(u)$ times larger for the energy E they possess on both airplanes when they reach the constant speed u relative to the airport. Again, this holds true independent of the direction traveled by either of the planes. Because of the relativity principle, the observers located on the airplanes will continue to measure the energy of the object co-moving with them to be E' , however. Since the energy of the identical

objects is the same on both airplanes for the ORS observer, however, it follows that *it is also the same for each of the observers on the airplanes*. One is therefore led to the conclusion that each airplane observer will measure the energy of *both* objects to be E' , *the one on his own airplane but also that on the other*.

The latter result stands in direct contradiction to the (E, \mathbf{p}) four-vector invariance relation of eq. (6), however. If the airplanes are not moving in the same direction, then they clearly have a non-zero speed relative to one another, which therefore implies on the basis of eq. (8) that $E > E'$ for the object carried onboard the “other” airplane in each case. The same argument holds for the inertial masses of these two objects. Since the two energy values are equal for both observers ($E = E'$), however, it is clear that eq. (12) does not hold for the observers on the two airplanes under these circumstances.

The results of the Hafele-Keating experiments [7] indicate instead that each of the eqs. (8, 12-14) holds for the *special case* when the object has been accelerated relative to an ORS. In each instance the primed quantity in these equations refers to the value measured by the observer in the ORS when the object is still at rest there (i.e. the proper mass, energy etc.), whereas the unprimed value refers to the corresponding measurement made *by the ORS observer* when the object has been accelerated to speed u relative to him.

What is clearly needed then are generalizations of the above equations that hold for all observers. This goal is made easier by the fact that the same factor γ appears in each them for the special case when the observer is at rest in the ORS from which the object of the measurement has been accelerated. One therefore must know the relative rates of clocks in the two inertial systems in which the object and the observer are at rest. For this purpose it is helpful to define a standard clock rate, say on the Earth’s surface. If clocks in the rest system of the object run α_M times slower than the standard clock, while those in the observer’s rest frame run α_O times slower than the standard, the following relationships hold between measured and proper values for the above quantities:

$$E = (\alpha_M/\alpha_O) E' \quad (15)$$

$$m = (\alpha_M/\alpha_O) \mu. \quad (16)$$

$$\mathbf{p} = (\alpha_M/\alpha_O) \mathbf{p}' \quad (17)$$

$$dt = (\alpha_M/\alpha_O) dt'. \quad (18)$$

These equations are clearly consistent with the relativity principle because $\alpha_M=\alpha_O$ for all *in situ* measurements, that is, the proper values for each of these quantities will be obtained in all cases when the object is not moving relative to the observer. When the object is accelerated relative to the observer's rest frame, then it is assumed that the ratio $\alpha_M/\alpha_O = \gamma$, so that the above equations then revert back to the special cases of eqs. (8, 12-14).

To obtain a direct comparison with the conventional interpretation of STR [1,19], let us assume that two objects, X_2 and X_3 , each with rest energy E' , are accelerated with respect to observer O_1 at the airport (Fig. 1). They always have the same speed relative to O_1 , but at the end of the acceleration period they are traveling in opposite directions to one another. Two other observers, O_2 and O_3 , travel with the objects (Triplet Paradox). An identical object X_1 stays behind with O_1 . Finally, at the end of the acceleration period, X_2 and X_3 are each traveling with constant speed u relative to O_1 .

The results obtained by the three observers for the energies of these three objects are given in matrix form in Table 1a for the conventional interpretation of relativity theory. The matrix is symmetric because it is assumed in STR [1, 19] that “everything is relative.” The value that O_2 obtains for X_3 is the same as O_3 obtains for X_2 . It is determined by their relative speed $2u$ (actually the speed will be somewhat less than this value when it is computed correctly with Einstein's velocity addition formula [1]). The key point is that the results in Table 1a are not internally consistent: O_3 supposedly finds the energy of X_2 to be larger than that of X_3 ($\gamma>1$ for all speeds), for example, whereas O_2 finds the energy of X_3 to be larger than that of X_2 . Moreover, O_1 finds the energy of both X_2 and X_3 to be equal, whereas both O_2 and O_3 find them to be different in each case. In short, there is no

proportionality among these results. More importantly, they are inconsistent with the results of the Hafele-Keating experiment [7], which show that O_2 's clock must be running at exactly the same rate as O_3 's and therefore imply that their respective units of energy are also the same.

On the other hand, Table 1b results if one insists upon employing a rational system of units to describe these measurements. Note that this matrix is no longer symmetric. This is unavoidable if the system of units is rational. If O_1 finds that X_2 has more energy than X_1 , the same must hold true for O_2 and O_3 . The various observers may have a different unit of energy, just as happens when one works in the mks system (J) and the other in the cgs system (erg), but this choice cannot affect their conclusions regarding the *ratio* of energies of any two objects. In Table 1b it is assumed that the unit of energy is γ times larger for both O_2 and O_3 than it is for O_1 , which is consistent with both STR and O_1 's actual measurements. This requires, however, contrary to the conventional interpretation of STR [1, 19], that both O_2 and O_3 agree that the energies of X_2 and X_3 are equal to the rest energy E' in each case, and also that the energy of X_1 is *less* than the rest energy E' .

One can summarize these results quite succinctly by stating that the standard units of time, energy and mass all vary in the same proportion from one inertial system to another. In order to compare the results of two observers for the same event, it is simply necessary to know the values of their respective clock-rate parameters α_M and α_O . If observer M measures a value of X (M) for one of these properties, then the corresponding value obtained by observer O must be

$$X(O) = (\alpha_M/\alpha_O) X(M). \quad (19)$$

This arrangement amounts to employing a *rational* set of units, that is, *the ratios of measured values for two observers are always in the same proportion for any given property*. It is important to see that eqs. (15-19) do not depend on the choice of a standard clock. Only ratios are involved in these equations, so they will not be affected by a change in standard.

For this reason it is helpful to use a single proportionality factor in each of them, which has been defined in earlier work [17, 18] as $Q \equiv \alpha_M/\alpha_O$. It appears explicitly in each of the GPS-LT equations, for example. One can look upon Q as a *conversion factor* between the units employed in the pertinent two rest frames. If the roles of the object and observer are reversed, the corresponding conversion factor in this direction is just the reciprocal ($1/Q$) of that in the forward direction. The same reciprocal relationship exists for conventional unit conversions, such as from m to cm in one case and cm to m in the other. This arrangement makes the resulting theory of measurement perfectly objective, as opposed to the case in the conventional STR interpretation. A more detailed discussion of this general subject may be found elsewhere [20, 21]. It is the antithesis of the “everything is relative” interpretation of STR [1, 19], which predicts that two observers will each think the other’s clock is running slower than his own. The latter position is unequivocally contradicted by the results of the Hafele-Keating experiments [7], and also those of the transverse Doppler studies using high-speed rotors [8-10].

IV. Conservation of Momentum and Relativistic Invariance

One of the most interesting consequences of the Triplet Paradox is the fact that it is inconsistent with the relativistic invariance relation of eq. (6) for the energy-momentum four-vector. It demonstrates that the energy of a moving object cannot always be computed by knowing its rest energy and its speed relative to the observer. It therefore also shows that the transformation of eqs. (5a-d) is also only valid under quite specific conditions, namely *when the object of the energy-momentum measurements has been accelerated due to an applied force in the observer’s rest frame*. This point is certainly consistent with the way it has been derived in Sect. II. The use of Hamilton’s equation in both eqs. (4, 5) is predicated on the existence of an applied force causing the object to be accelerated to its current speed u relative

to the observer. This condition does not hold for the two airplanes in Fig. 1, however. In this case both have been accelerated with respect to a common ORS, namely the airport. Their relative speed is not the result of one airplane being accelerated because of an applied force at the other's *current* location.

It is important to see that a different assumption is employed to obtain the LT and the alternative transformation (GPS-LT [2, 3, 20, 21]) mentioned in the Introduction. In this case it is completely immaterial how the object has reached its current speed. The resulting transformation simply relates the measurements of space and time for this object relative to two different origins that are in constant relative motion to one another [3]. It can therefore be used by observers on either airplane in the example of Fig. 1 to relate their measurements of elapsed time and distance traveled by the object with respect to these two different origins. Accordingly, each observer must measure the speed of a given light pulse relative to either of these origins to have the same constant value of c .

The results of the Triplet Paradox also require a reexamination of the concept of “translational energy.” The conventional definition of this term is the excess energy that an object possesses by virtue of its motion. One needs to qualify this definition by specifying the state of motion of the observer as well as the object. If the object has been directly accelerated relative to the observer, the amount of kinetic or translation energy is given directly by eq. (9); that is, as $(\gamma - 1) E'$, consistent with the derivation based on Hamilton's equation given in Sect. II. This value reduces to $\frac{1}{2} \mu v^2$ in the non-relativistic case. For any other observer, however, it is necessary to know the ratio of his clock-rate parameter α_O to that (α_{ORS}) of the inertial system in which the initial acceleration occurred. Using the same notation as above, this means that the definition of translational energy K becomes:

$$K = (\alpha_{ORS}/\alpha_O) (\gamma - 1) E'. \quad (20)$$

In order to obtain an experimental verification of eq. (20), it is necessary to study collisions of two objects from the vantage point of observers on the two airplanes. If both

objects have been accelerated with respect to the same ORS, it is clearly possible for an observer who is at rest in this inertial system to employ the laws of energy and momentum conservation from his perspective to obtain the correct solution. For any other observer to obtain consistent results for the same collision, he must compute the momentum of the various colliding objects *relative to the ORS*. In addition, he has to take into account the fact that his units of energy and momentum are generally not the same as in the ORS. In effect, this means scaling each conservation equation by the same factor $\alpha_{\text{ORS}}/\alpha_{\text{O}}$ on both sides. This procedure obviously does not destroy the corresponding equalities, so by employing it, each observer is assured that both energy and each of the momentum components are conserved from his vantage point, as well as for the ORS observer.

In practice there can be more than one ORS involved in a given collision. In this case the procedure must be modified as follows. Each object's momentum must be determined *relative to its own ORS*, i.e. by multiplying the inertial mass of the object (in the units of the ORS observer) with its velocity relative to the ORS. A different set of conversion factors is then required for each colliding object, that is, taking into account the values of the respective clock-rate parameters of each such ORS in order to obtain the correct conservation relations for energy and momentum *in the observer's system of units*.

V. Conclusion

The role of acceleration is more important than is generally recognized in relativity theory. A number of key relationships that are believed to be of universal validity actually only hold for the special case when the object of the measurement has reached its current velocity *as the result of an applied force in the observer's rest frame*. This fact is demonstrated by the experiments carried out with circumnavigating airplanes by Hafele and Keating [7]. They show that the standard formula for time dilation must be applied with respect to a reference clock located on the Earth's (non-rotating) polar axis. Clocks on the

airplanes and at the airport of departure have all undergone acceleration relative to this axis. As a consequence, the relative rates of clocks on the two airplanes cannot be obtained simply on the basis of their speed relative to one another.

This is true even if the two airplanes are inertial systems, as for example would be the case when they take off from an airport on the surface of a non-rotating planet and each reach constant velocities relative to the airport. If the airplanes have the same relative speed to the airport but are travelling in opposite directions (Triplet Paradox; see Fig. 1), their onboard clocks must be running at the same rate according to the Hafele-Keating analysis [7]. A similar conclusion holds for energy and mass, because these properties increase at the same rate as clocks slow down. An observer on one of the airplanes must measure the same energy $E = E'$ (the rest or proper energy) for an object carried onboard as for an identical object aboard the other airplane, even though the latter may be moving at high speed relative to him. The energy-momentum four-vector invariance relation of eq. (6) therefore does not hold in this situation. It is only valid when the object of the measurement has been accelerated by an applied force in the observer's rest frame, in which case $E = \gamma E'$.

One can trace back the limitation in the applicability of eq. (6), and also the corresponding energy-momentum transformation equations of eqs. (5a-d), to the way they are derived in STR [1]. Hamilton's equation is assumed to be valid, which in turn is derived from Newton's Second Law. Accordingly, an applied force at the object's original position causes a change in its energy dE [see eq. (1)]. In particular, *the speed u in this equation must be computed relative to the latter's rest frame*. The situation is qualitatively different than for the corresponding relativistic space-time transformation, which defines how the distance travelled by an object and the corresponding elapsed time vary when they are computed relative to two different origins in relative motion. In this case the relative speed of the two origins is required, independent of how the object of the measurement has been accelerated relative to a given observer.

The above discussion emphasizes the need for identifying an objective rest system (ORS) for an accelerated object in order to compare the measured values for its physical properties obtained by different observers in relative motion to one another. This concept allows one to define a rational set of units for each observer by employing the time-dilation formula of eq. (14) and the corresponding relations for energy and mass in eqs. (8,12, 13). Accordingly, the measured values for a given property will always be in the same ratio for two different observers for any object. The two airplanes in the Triplet Paradox have the same units, for example, and thus respective onboard observers must always agree perfectly on the values of their measurements for a given object. One therefore avoids the contradictions inherent in the “everything is relative” interpretation of STR whereby two observers each conclude that the other’s clocks run slower than his own.

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Table 1. Predicted energy values obtained by three different observers O_i for identical objects X_i in the example of Fig. 1 (Triplet Paradox) according to: a) the conventional interpretation of STR [1] and b) the ORS interpretation employing a rational set of units [6].

| | | | |
|-----------|-------------|------------------|------------------|
| a) | X_1 | X_2 | X_3 |
| O_1 | E' | $\gamma E'$ | $\gamma E'$ |
| O_2 | $\gamma E'$ | E' | $\gamma (2u) E'$ |
| O_3 | $\gamma E'$ | $\gamma (2u) E'$ | E' |
| b) | X_1 | X_2 | X_3 |
| O_1 | E' | $\gamma E'$ | $\gamma E'$ |
| O_2 | E'/γ | E' | E' |
| O_3 | E'/γ | E' | E' |

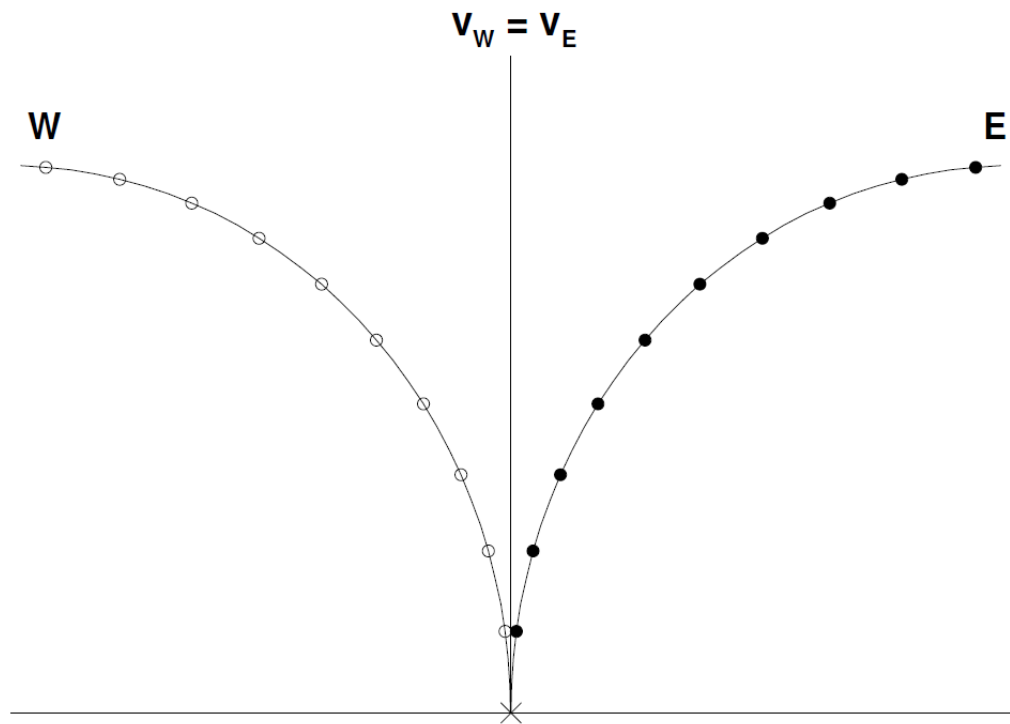


Fig. 1. Diagram showing two rockets leaving the same position in a gravity-free region of space from the surface of a hypothetical non-rotating planet. Their speed relative to the departure position is the same for both at all times, even though their respective directions of velocity are always different. The symmetric relationship of their trajectories indicates that the rates of their respective onboard clocks are always the same. This remains true for the termini of the trajectories shown, in which case the rockets are both a) inertial systems (each traveling at constant velocity) and b) in relative motion to one another at that point.

(Jan. 26, 2016)