Critique of Einstein's Light-speed Postulate

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Abstract

There are two conflicting interpretations of Einstein's light-speed constancy postulate, which he enunciated in his landmark 1905 paper on relativistic mechanics. The one that he proposed in his work was that the speed of light (c) in free space has the same value for all observers independent of their own state of motion (two years later he amended this position by stating that it only applied to cases where both the light pulse and the observer are at the same gravitational potential). A review of the relevant experimental data indicates, however, that there is a fundamentally more restrictive version of the light-speed constancy assumption which fits in better with the facts, namely that only the speed of light in free space relative to its source always has the same constant value, whereas the value relative to any given observer depends critically on the latter's state of motion. Particularly surprising is that Einstein's postulate leaves open the distinct possibility that the speed of light relative to its source can be as high as 2c. It is also interesting to note that, although Einstein's second postulate stands in direct contradiction to the predictions of the classical (Galilean) velocity transformation, there is no such conflict when the version which requires that the speed of flight in free space is only equal to c relative to the source from which it is emitted is used instead. Most telling is that it is found that the primary example Einstein used to illustrate his second postulate is actually inconsistent with the relativistic velocity transformation which he also derived in his original work on the basis of the same assumption.

Keywords: Einstein's light-speed constancy postulate, Galilean velocity transformation, relativistic velocity transformation, Lorentz transformation, Newton-Voigt transformation, Einstein's non-simultaneity conjecture
I. Introduction

When Einstein began to develop his theory of relativity [1], he had a difficult choice to make regarding the speed of light in free space, one that could not be decided on the basis of experiment alone. Either the light-speed value (c) is the same for every observer independent of his state of motion, or it only has that value relative to its source. There is a big distinction between these two choices that needs to be spelled out in detail. Extrapolation of the Fizeau/Fresnel light-drag experimental results [2] to the free-space value of 1.0 for the refractive index of light indicated that the former choice is the correct one because the light speed is found to be completely independent of the speed v of the source/transparent medium: c(v)=c.

The later "null" finding of the Michelson-Morley interferometer experiment [3] was ambiguous on this point, however, since it could be interpreted to indicate that the speed of light in free space only has the same value relative to the source, independent of the direction in which the light travels. It is a fact of history [4] that Einstein claimed to have not been influenced by this experiment in making his decision. By the same token, the mathematical result from Maxwell's theory of electromagnetism, namely that the speed of light in free space is the same in every rest frame, also does not force one to go beyond the conclusion that observers in the rest frame of a given light source must necessarily find that the light speed has a value of c for them.

Nevertheless, in 1905 Einstein came down squarely on the side of the broader interpretation, stating: "Every light beam from a 'rest' coordinate system moves with a definite value of the speed $V$, independent of whether this light beam is emitted from a stationary or a moving body." He was not the first to come to this conclusion. Voigt had used it in 1887 [5] to define an alternative to the classical (Galilean) space-time transformation. A decade later, Larmor and Lorentz [2,6] came up with a crucially revised edition which has come to be known as the Lorentz transformation (LT), and this version is identical to the one Einstein later derived in his work [1].

Einstein then used the LT to make numerous predictions, among them the phenomena of length contraction and time dilation [1]. He also derived a companion velocity transformation, which is clearly distinguishable from the Galilean version. Furthermore, he used the light-speed constancy assumption, which has come to be known as the second postulate of relativity, to prove on this basis that events which are simultaneous for observers in one rest frame will generally not occur at the same time for their counterparts moving relative to them. This prediction was clearly revolutionary. It ran counter to the teachings of
Newton and his contemporaries, which had been treated as dogma by physicists for more than two centuries beforehand. In the following discussion, Einstein's reasoning behind this ground-breaking conclusion will also be considered in detail.

II. Non-simultaneity of Events for Observers Moving Relative to Each Other

Einstein based his analysis on a light beam emitted from a source that is moving relative to the observer. For concreteness, it will be assumed that the latter observer is stationary on a platform and that the source of the light pulse is moving on a bar with constant speed $v$ parallel to the platform. Furthermore, he specifies that the clocks used to measure time differences located on both the platform and the bar are of the same kind ("gleich beschaffen"). In his example, a light beam is emitted from a source that is stationary on the bar. It travels from position A to B on the platform along a section of length $r_{AB}$. The initial time on a platform clock is $t_A$, whereas the final time when the beam reaches position B is $t_B$. All measurements are made from the vantage point of a stationary observer on the platform. The corresponding time difference is thus $t_B - t_A$. He uses his second postulate to then conclude that the distance the light travels relative to the platform observer to this point is $c(t_B - t_A)$. This leads to the following relationship, after taking account of the fact that the bar itself moves a distance of $v (t_B - t_A)$ relative to the platform observer:

$$c \ (t_B - t_A) = r_{AB} + v \ (t_B - t_A),$$

whereupon Einstein concludes [1] that the recorded time difference on the platform clock is

$$(t_B - t_A) = r_{AB} \ (c-v)^{-1}.$$

Upon reaching point B on the train, the light beam is reflected back to its original position on the bar, which is now located at point A' on the platform. The light beam is now traveling in the opposite direction as the train, and so the time difference in going from B to A' is computed, again using Einstein's second postulate, according to the following equation:

$$c \ (t_A' - t_B) = r_{AB} - v \ (t_A' - t_B),$$

whereupon he concludes [1] that the recorded time difference according to a platform clock is

$$(t_A' - t_B) = r_{AB} \ (c+v)^{-1}.$$

He then makes the statement, without further specification, that the above two time difference results measured on a platform clock show that "the observer moving with the bar therefore will not find that the two clocks are running synchronously" with one another. However, he leaves it up to the reader to justify this purported non-synchronous relationship.
The most obvious explanation is that Einstein meant that the observer moving with the bar will simply find the time difference in going from A to B and then back to A' to be \( \tau = 2\ c^{-1}r_{AB} \), which is numerically different than the sum of the above two time differences measured on the platform clock.

One needs to take into account, however, that the above time difference on the bar's clock is based on a different set of space and time units than the latter sum measured in the rest frame of the platform. This conversion can be done quite simply by taking note of two other predictions discussed elsewhere in Einstein's work [1], namely length contraction and time dilation. From the vantage point of the bar observer, the length of the bar is not \( r_{AB} \), but rather \( (1-v^2c^{-2})^{-0.5} r_{AB} = \gamma (v) r_{AB} \) because of length contraction in his rest frame. In addition, since his clock is running \( \gamma (v) \) times slower, there is an additional factor of \( \gamma (v) \) on this basis. The actual time difference measured on the bar is thus \( \gamma^2 \tau = 2\ \gamma^2 c^{-1}r_{AB} \) when converted to the units that are extant on the platform. On the other hand, the sum of the above two time differences measured directly on the platform clocks is seen to be \( r_{AB} (c-v)^{-1} + r_{AB} (c+v)^{-1} = 2c r_{AB} (c^2-v^2)^{-1} = 2\ \gamma^2 c^{-1}r_{AB} \), exactly the same value as obtained on the above basis after conversion to the platform units, proving that the claim of non-simultaneity is specious.

III. Velocity Transformation and Simultaneity

The above argument shows unequivocally that Einstein's conclusion that there is a non-synchronous relationship between the train and platform clocks is not supported by key aspects of his own theory. There is another straightforward way to reach the same judgment, namely by computing the distance traveled by the light beam in the forward direction from the vantage point of both the bar and platform rest frames. According to Einstein's second postulate, the light beam travels a distance of \( c (t_B-t_A) = c \Delta T \) between points A and B on the platform. At the same time, the bar itself moves a distance of \( v \Delta T \) relative to the platform. As a consequence, the platform observer finds that the light beam moved a distance of \( (c-v) \Delta T \) relative to the bar. This means that the speed of the light pulse relative to its source on the bar is \( c-v \). For example if \( v \) is close to \( c \), the light speed would be arbitrarily close to zero. However, this result stands in clear violation to his second postulate, since it states that the speed of light is equal to \( c \) independent of the state of motion of the observer. In the backward motion of the light pulse in Einstein's example, one finds according to the second postulate that the speed of the light pulse relative to its source is \( c+v \), in which case the speed
could be close to 2c relative to its source if v were close to c, again in obvious disagreement
with the prediction of the second postulate.

There is another path to the same conclusion, namely to make use of Einstein's velocity
addition theorem (see p. 906 of ref. 1). Accordingly, the speed of light relative to its source
on the bar is equal to \((c-v)(1-c^2vc)^{-1} = c\), i.e. since c is the speed of the light beam from the
vantage point of the platform and v is the corresponding speed of the source relative to the
platform. Again the result is c, not c-v.

This impasse can be avoided by making a simple change to the second postulate, namely
to remove the phrase "independent of the state of motion of the observer." Instead, the second
postulate can be reformulated to state that "the speed of light in free space when measured in
the rest frame of the light source is always equal to c." This amended version is in agreement
with all known experimental results. For example, it agrees with the null result of the
Michelson-Morley experiment [3]. It also fits in perfectly with the Fizeau/Fresnel light-drag
experiment [2]. It also agrees with the finding that no amount of acceleration of an electron
or other massive object can exceed a value of c. It also is perfectly consistent with the
prediction that results from Maxwell's theory of electromagnetism [7]. More importantly in
the present context, this reformulation of the second postulate eliminates all the
inconsistencies noted above with respect to the example of a light beam moving back and
forth along the direction of a bar moving relative to a platform. In particular, it leaves open
the distinct possibility that any pair of events that occurs simultaneously on the clocks in one
of these rest frames will also occur at the same time based on stationary clocks in the other.

Restricting the scope of the light-speed assumption is not the most important
consequence of the detailed analysis of the bar-platform example, however. Even more
cogent is the fact that it allows for the direct application of the classical (Galilean) velocity
transformation to describe these results. It needs to be recognized that the distance traveled
by the light beam between A and B from the vantage point of the platform observer is
necessarily obtained as the sum of the distance traveled by the bar itself \((v\Delta t)\) and that
traversed by the light beam in the rest frame of the bar \((c\Delta t)\), i.e. \((c+v)\Delta t\). As a result, one
concludes according to the newly formulated version of the light-speed postulate that the total
speed of the light beam relative to the platform in time \(\Delta t\) is \(c+v\). This is exactly the same
result as would be expected from the Galilean transformation.

Moreover, the analogous result must be expected if it were a conventional object
moving along the bar with a constant speed \(w\) which is substantially less than that of light in
free space. The total speed of the object would again be found to be, in complete agreement with the classical result, \( v + w \). In other words, all that happens in the present case is that the object moves with speed \( c \) relative to the rest frame of the bar, rather than some other value \( w \) which is typical for another type of object. There is accordingly no need to develop a complicated new theory of relativity to explain why the platform observer finds the light beam to have speed \( v + c \) while his counterpart in the rest frame of the bar finds it to move with a different speed, namely \( c \).

Another key example in which the amended light-speed assumption has a critical effect is the phenomenon of stellar aberration at the zenith [2]. In this case, the sun plays the role of the bar and the platform is the rest frame of the earth, which is rotating around the sun with speed \( v \). The previous distance analysis, when based on the amended light-speed postulate, finds that the light beam emanating from the sun travels a distance of \( c \Delta t \) relative to the sun in the \( y \) direction from the vantage point of the earth-bound observer, while the sun itself travels a distance of \( v \Delta t \) in a perpendicular direction (\( x \)) over the same time frame relative to the same observer. Therefore, from his vantage point, the light beam travels a distance of \((v^2+c^2)^{0.5} \Delta t\) while it moves only a distance of \( c \Delta t \) relative to the sun. The corresponding speed is \((v^2+c^2)^{0.5}\), which again is greater than the speed of the light beam relative to the sun, There is thus a deviation angle relative to the \( y \) axis of \( \tan^{-1} \frac{v}{c} \). All these calculations are consistent with the Galilean velocity transformation.

By contrast, adherence to Einstein's second postulate [1] leads one to conclude that the total distance traveled by the light beam relative to the earth-bound observer is only \( c \Delta t \) and the corresponding angle is \( \tan^{-1} \gamma \frac{v}{c} \). That conclusion stands in contradiction to the velocity transformation derived in Einstein's work [1], however, which indicates that the observer on earth must find the same value (\( c \)) for the speed of the light beam relative to the sun as a hypothetical observer who is stationary in the sun's rest frame. For more than a century now, physicists have generally looked upon the extra factor of \( \gamma \) relative to the classical value for the deviation angle as a triumph of Einstein's theory [2] without apparently realizing that this inconsistency exists. In practice, the value of \( \gamma \) differs from unity by less than a factor of one part in \( 10^8 \) in this application, so there is no way to settle this question experimentally. At the very least, this situation should be taken as an opportunity to reconsider Einstein's conclusion.

A key concept in the present discussion is that of "relative velocity," i.e. the velocity with which two objects separate from one another. According to the Galilean velocity transformation, this quantity has the same value for all observers, regardless of their own state.
of motion. For example, if an object moves with speed $w$ in the direction of the bar's motion, the platform observer will find that it moves with speed $v+w$ relative to his position. He can deduce the relative speed of the object to the bar using the Galilean transformation by simply subtracting $v$, the speed of the bar, from this value, thereby also obtaining $w$ as a result. The situation is no different for the relativistic velocity transformation. In this case, the speed of the object relative to the platform observer is $w^S = (v+w)(1+vw^2)^{-1}$. Making the calculation of the relative speed of the object is more complicated in this case, however, and is not just a simple matter of subtracting $v$ from $w^S$. Instead, the velocity transformation must be used explicitly as follows: $(w^S - v)(1-c^2vw^2)^{-1} = [(v+w)(1+vw^2)^{-1} - v][1-c^2v(v+w)(1+vw^2)^{-1}]^{-1} = (v+w - v - vw^2)(1+vw^2-c^2v^2 - c^2vw)^{-1} = w$. In complete analogy to the case using the classical transformation, one therefore finds that the relative speed of the object to the train which is deduced by the platform observer is exactly the same as that found explicitly by the observer moving with the bar. As a result, it would seem to be by far the most reasonable approach to follow this guide and assume that all observers must agree on the relative velocity of any two objects, independent of their own states of motion. This situation exists while, at the same time, it is clear that the only way two observers can agree on the velocity of any given object is if they are stationary in the same rest frame themselves. The bar-platform example is simply one illustration of both conclusions.

IV. A Litmus Test for Deciding Which Transformation to Use

The axiom about relative velocities discussed above leads to several surprising conclusions regarding Einstein's example [1] of a light beam moving from a source which is itself in relative motion to an observer in another rest frame. First, the Galilean velocity transformation actually explains the details of the relative motion in a quite convincing and straightforward manner. As soon as one accepts that each of the two observers agrees that the speed of the light beam relative its source is $c$, it immediately follows that the speed of light from the vantage point of the observer who is moving with speed $v$ relative to the source is $v+c$, exactly as one must conclude based on the classical transformation. Secondly, it is equally clear under the same assumption that the relativistic velocity transformation, which assumes that both observers find the light beam to be moving with the same speed $c$ relative to each of them, is definitely not up to the task of describing the true facts of Einstein's primary example.
On the other hand, von Laue [8] showed that the opposite situation holds for the Fizeau/Fresnel light-drag experiment. He derived the long-accepted formula for this interaction of light with matter in terms of the index of refraction of the medium through which light moves. He did so by avoiding the classical transformation, which cannot possibly explain the extrapolated \( c(v) = c \) result, but by using the relativistic version instead.

The critical question is then what is the distinguishing feature in any example of relative motion which requires the use of the Galilean velocity transformation, and what is by contrast the other set of circumstances which demands instead that one assume that the speed of light is the same in two different rest frames. In other words, is there a simple "litmus test" to make this distinction?

It is actually quite straightforward to make this determination. If there are two rest frames with different observers and a single object, as in Einstein's example of a light beam emanating from a source on the bar, one needs to ignore the relativistic velocity transformation because it leads to the false conclusion that the speed of the object/ light beam is the same for both observers. As the above discussion shows on the contrary, when one considers the respective distances traveled by the light source relative to the platform observer \( (v \Delta t) \) and by the light pulse relative to the same source \( (c \Delta t) \), the conclusion is that the speed of light relative to the platform observer is \( c + v \), not \( c \). That conclusion is perfectly consistent with Maxwell's theory of electromagnetism [7], which indicates that the speed of light in free space in any rest frame relative to the source from which it is emitted is always equal to \( c \), independent of the latter's state of motion.

The example of the Fizeau/Fresnel light-drag experiment does not fall in the same class because in this case there is only one observer who is stationary in a single rest frame and who is making measurements under two distinct conditions. In this case, the two conditions are the different speeds of the refractive medium under which the light-speed determination is made. The same laboratory observer makes both determinations. An analogous situation occurs in the unsuccessful experimental attempts to accelerate an electron to a speed greater than \( c \). In one situation the electron is found to move at its current speed relative to the laboratory and in the second, the electron is observed after a force has been applied to it; again one observer, two conditions. It is well established mathematically [9,10] that the relativistic velocity transformation cannot lead to a speed greater or equal to \( c \) upon addition of an accelerating speed \( v \) for a given object when its initial speed is less than \( c \). By the same token, it is clear that one can use the classical velocity transformation to show that an object
moving with a given speed \( w \) in one rest frame can be found to have a speed greater than \( c \) in some other rest frame which moves relative to the first with speed \( v \), i.e. \( v + w > c \). Therefore, clearly the classical transformation is inapplicable in this case, but the current example also fails to satisfy the critical test of having two observers for single object. It is the same observer in each case.

The Michelson-Morley experiment [3] presents a different sort of example. There is no need for either the classical or the relativistic velocity transformation to explain the null result of the interferometer procedure. The amended version of the light-speed postulate suffices, whereby it is simply assumed that the speed of light in free space relative to its source always has a value of \( c \) for an observer who is stationary in that rest frame. No velocity addition principle is required on this basis to arrive at the experimental result that the light speed is the same in all directions at all times of the year and all altitudes. One can change this arrangement by looking at the experimental set-up from the vantage point of an observer moving toward the laboratory at speed \( v \). Then the observed light speed will be \( v + c \) in the direction of relative motion in the forward sense and \( v - c \) in the opposite sense. The corresponding speeds in the perpendicular directions are \( (v^2 + c^2)^{0.5} \) in each case. As with the classical velocity transformation in general, however, the different speeds do not change the fact that both observers find that each light wave arrives back at the starting point at the same time. In the classical case, there is always a "compensating" effect on the two speeds of the different observers which guarantees the simultaneity condition in all cases. A similar change in conditions can obviously be applied in the Fizeau/Fresnel-light-drag and electron-acceleration experiments, with the same results.

V. The Lorentz Transformation and Time Dilation

The cornerstone of Einstein's theory of relativity [1] is the Lorentz transformation. It is identical to the space-time transformation introduced a decade earlier by Lorentz [11]. Larmor [6] had discovered the same set of equations shortly before Lorentz, and Voigt [5] had suggested a somewhat different version a decade before that. What has become known as Einstein's second postulate regarding the speed of light in free space was the basis for each of these transformations. The \( \Delta t = \Delta t' \) relationship assumed between elapsed times obtained by stationary observers in different rest frames which is foreseen in the classical space-time (Galilean) transformation is replaced therein by an equation which mixes the spatial and time coordinates in the following manner:
\[ \Delta t' = (1-v^2c^{-2})^{-0.5}(\Delta t-vc^{-2}\Delta x) = \gamma(v)(\Delta t-vc^{-2}\Delta x). \]

A simple example of two lightning strikes on a train moving with speed \( v \) along the x direction relative to the station platform suffices to define the variables contained therein. The distance separating the two strikes from the vantage point of a platform observer is designated to be \( \Delta x \) and \( \Delta t \) is the corresponding time difference recorded on synchronized stationary clocks located on the platform. If both \( v \) and \( \Delta x \) are not equal to zero, it follows according to this equation that the strikes cannot have occurred simultaneously for a stationary platform observer (\( \Delta t=0 \)) without them having occurred at different times (\( \Delta t'\neq0 \)) based on the stationary clocks on the train. This predicted relationship between the two elapsed times is generally referred to as remote non-simultaneity.

Einstein also used the above space-time equation of the Lorentz transformation to derive the following relationship:
\[ \Delta t' = (1-v^2c^{-2})^{0.5}\Delta t = \Delta t /\gamma(v). \]
It follows from this equation because of the fact that a stationary clock in one rest frame (the train in the present example) is found to change its position by \( \Delta x = v\Delta t \) in the other (platform) \[1,12\]. It is a perfectly general relationship, referred to as time dilation, which can be summarized by stating that it is always the "moving" clock which runs at a slower rate \[1,13\]. It has, however, been pointed out that the two equations are incompatible with one another \[14,15\]. This realization shows unequivocally that the Lorentz transformation, from which both the remote non-simultaneity and time dilation relationships are derived, is not a viable component of relativity theory.

There is a clear way to fix this problem, however, namely by rejecting Einstein's second postulate. One can instead make the perfectly plausible assumption that the rate of an atomic clock remains the same indefinitely until some external unbalanced force is applied to it (Law of Causality). Two such clocks of the same kind can have different rates, but it is nonetheless unavoidable that their ratio is constant as well. As a consequence, one expects the following relationship to exist between the respective elapsed times measured on the two clocks for the same pair of events:
\[ \Delta t' = \Delta t/Q, \]
where \( Q \) is the ratio of the above two clock rates. This proportionality relationship will be referred to as "Newtonian Simultaneity" in the following discussion. Simultaneity is an apt term because it is obvious from this equation that it does not allow for a situation in which two events occur at the same time for one observer (\( \Delta t=0 \)) but not (\( \Delta t'\neq0 \)) for some other who
is stationary in a different rest frame. That such a conclusion is designated "Newtonian" is simply in recognition of his firmly held view that everything occurs at the same time throughout the entire universe.

The question arises, however, as to whether this strict form of clock-rate proportionality can be made consistent with Einstein's two postulates of relativity [1]. The answer is very simple. One just has to combine this equation with the relativistic velocity transformation [16,17], which does satisfy both postulates itself. The result is:

\[
\begin{align*}
\Delta t' &= Q^{-1} \Delta t, \\
\Delta x' &= \eta Q^{-1} (\Delta x - v \Delta t) \\
\Delta y' &= \eta (\gamma Q)^{-1} \Delta y \\
\Delta z' &= \eta (\gamma Q)^{-1} \Delta z,
\end{align*}
\]

whereby \( \eta = (1 - c^{-2} v \Delta x / \Delta t)^{-1} \). Note that the latter quantity already appears in the relativistic velocity transformation itself; in Einstein's notation [1], \( U = (v + w) (1 + c^{-2} vw)^{-1} = \eta (v + w) \).

In order to satisfy the first postulate, the Galilean Relativity Principle, it is necessary to place a condition on the inverse transformation, in which \( Q \) is replaced by \( Q' \), namely \( QQ' = 1 \). The latter set of equations must be obtained by what will be referred to as "Galilean inversion," i.e. by interchanging of the respective variables and changing the sign of \( v \), thereby simulating the actual exchange of the positions of the different observers in the two rest frames.

There is a key equality that involves Galilean inversion which is useful in obtaining the proof that the relativistic velocity transformations satisfies the second postulate, namely \( \eta \eta' = \gamma^2 \), which is obtained using one of the equations of the transformation itself [18]. The above transformation can be aptly named the Newtonian-Voigt transformation because of the key role the Newtonian Simultaneity condition plays in its definition and the importance of the space-time mixing conjecture of Voigt [5,19] in arriving at it.

The \( Q' = 1/Q \) relationship arises quite naturally from the nature of these parameters, namely as "conversion factors" between the units of time in the two rest frames. Thus, as for all other physical quantities, the conversion factor in the reverse direction is simply the reciprocal of that in the forward direction. This is key distinction between the Lorentz transformation and the Newton-Voigt transformation, since the former predicts in its version of time dilation [1] that the effect is symmetric, i.e. that each observer finds it is the other's ("moving") clock which has the slower rate [13]. Newtonian Simultaneity, by
contrast, foresees an asymmetric relationship between any two such clock rates in different rest frames.

The parameter Q in the Newton-Voigt transformation is an experimental quantity that is necessarily specific for each pair of rest frames. The first definitive test of the rates of atomic clocks was carried out by Hafele and Keating by carrying them onboard circumnavigating airplanes [20,21]. They found that the rate of a given clock, after making a correction for the effects of gravity on the basis of Einstein's equivalence principle, was inversely proportional to its speed relative to the earth's center of mass (ECM). Einstein had made a specific prediction in his 1905 paper [1] that was in general agreement with this finding. The empirical result of the Hafele-Keating experiment is given below in the form of the following proportionality relationship for the elapsed times Δt and Δt' for a given portion of their flight; results were obtained to order (v/c)^2:

\[ \Delta t' \gamma(v') = \Delta t \gamma(v), \]

where \( v' \) and \( v \) are the respective speeds of the clocks relative to the ECM. Comparison with the Newtonian Simultaneity relation then allows one to directly compute the value of Q as:

\[ Q = \frac{\gamma(v')}{\gamma(v)}. \]

Moreover, one can also compute the value of the inverse parameter Q' by Galilean inversion as:

\[ Q' = \frac{\gamma(v)}{\gamma(v')} = \frac{1}{Q}, \]

which, in accord with the Relativity Principle, is just the reciprocal of Q.

The operation of the Global Positioning System (GPS) makes direct use of the above relationships. The goal is to have the atomic clock on a given orbiting satellite run at exactly the same rate as its counterpart on the earth's surface. To this end, a "pre-correction" procedure [22,23] is applied to the rate of a satellite clock prior to launch. The rate of slowing down of the clock is predicted using the above Q parameter, and Einstein's red shift formula [24] is used to estimate the corresponding increase in rate caused by the change in altitude.

There are two points that need to be emphasized in this respect. First, it is clear that time dilation is asymmetric. The assumption is clearly that the effect of increasing the speed of the clock is to slow it down. The adjustment is based on an objective theory of measurement, not Einstein's idea [1] that the question of which clock runs faster or slower is purely a matter of the perspective of the observer. Secondly, the GPS methodology makes clear that events
anywhere in the universe occur at the same time on the satellite as on the ground. Otherwise it would be pointless to make the adjustment at all [25].

The Hafele-Keating experiment with atomic clocks was not the first to lead to the conclusion that time dilation is asymmetric. The results of the Hay et al. study [26] of the dependence of light frequencies on the speed of a high-speed rotor, on which both an x-ray detector and source were mounted, present evidence which is consistent with exactly the same phenomenon. The inverse proportionality between elapsed times and $\gamma(v)$ mentioned above is also found in the rotor investigation. The speed $v$ to be inserted in that formula is $R\omega$, where $\omega$ is the frequency of the rotor and $R$ is the distance of either the detector or the x-ray source from its axis. In particular, it was found that the measured frequency of light emanating from the x-ray source is greater when the detector is located farther from the axis than the source (blue shift). Hay et al. did not describe this result in these terms, however, claiming instead that everything they found is in complete agreement with Einstein's relativity theory [1], even though the latter makes the opposite prediction of a red shift [13]. Sherwin [27] quickly disputed this claim, however, and pointed out that it is exactly the observed asymmetry of their findings which is the most significant result of the experiment. A slightly modified version of the rotor experiment was carried out by Kuendig [28] and Champeney et al. [29], with exactly the same asymmetric result. An attempt was made to explain the blue shift in terms of Einstein's equivalence principle [24], but the Hafele-Keating experiment rules out such an explanation since it shows that kinetic and gravitational effects on clock rates are completely independent of one another.

When all is said and done, the experimental law of an inverse proportionality between $\gamma(v)$ and elapsed times/periods of radiation is the most significant result of both investigations. In recognition of this fact, the latter equation has been designated as the Universal Time-dilation Law (UTDL) [30]. In order to apply it, it is necessary to designate a rest frame (objective rest system or ORS [31]) from which the speeds in the $\gamma(v)$ factors are to be determined. In both cases, the ORS is the same for both clocks, either the ECM [20,21] or the rotor axis [26,28,29], but it easy to extend it to applications involving clocks moving relative to different ORS, such as the earth and the moon, for example. In that case, the relative rates of clocks in the two rest frames must also be provided.

The derivation by Thomas [32,33] of the precession of the spin of an electron or other elementary particle is a particularly interesting case in which Einstein's second postulate has been successfully applied. Again, one has the situation where a single observer makes two
separate observations, namely the spin of the electron's orbit around the nucleus at different times. The main point of interest in this connection is that the derivation makes explicit use of the Lorentz transformation in arriving at its result. This might be construed as confirmation of the validity of that space-time transformation, despite the fact that it makes self-contradictory predictions of proportional time dilation and remote non-simultaneity. Such a conclusion is incorrect, however, since the same result of the derivation is obtained with the use of the Newton-Voigt transformation. The salient point is that these two transformations differ by a single factor in each of their four equations, namely \( \eta(\gamma Q) \). Since both the forward and inverse Lorentz transformations are used consecutively in the derivation, the corresponding factors when the Newton-Voigt transformation is used instead simply cancel one another out.

VI. Conclusion

The second postulate introduced by Einstein in 1905 [1] assumes that the speed of light in free space has the same value independent of the state of motion of the observer or the source. In the example he gave, the light source moves away from the rest frame of the observer with speed \( v \) at the same time that it emits a light pulse in the same direction. Accordingly, at the end of a given time \( \Delta t \), the observer finds that the pulse has traveled a distance of \( c\Delta t \) relative to his position at the same time that the source has moved a distance of \( v\Delta t \). As a consequence, one finds by subtraction that the light pulse has moved a distance \( (c-v)\Delta t \) relative to its source, i.e. at a speed of \( c-v \) relative to its source during this time. This is indeed a strange result when it is realized that if the source moves relative to the observer at a speed close to \( c \) in the direction the light pulse is emitted, this means that the light speed relative to the source can be arbitrarily small, equal to that of a bicycle, for example. Not only is this counter-intuitive, it also stands in conflict with the prediction of Maxwell's theory of electricity and magnetism, which holds that the speed of light in free space has the same value of \( c \) in any and all rest frames throughout the universe. Moreover, in the reverse direction in Einstein's example, the analogous calculation finds that the speed of the light pulse relative its source is \( c+v \), i.e. it could be almost as large as \( 2c \) if \( v \) is close to \( c \). This is truly an absurd result when one thinks about the original intention of the second postulate, which was to limit the speed of any object to a value less than \( c \).

More directly, the \( c-v \) and \( c+v \) conclusions run contrary to the result deduced from Einstein's own relativistic velocity transformation. It states [1] that the speed of the light pulse relative to the source in his example is \((c-v)(1-v/c)^{-1}=c\), which is again the value expected.
from Maxwell's theory. On this basis, one concludes that the speed of light relative to the rest frame of the observer is actually \(v+c\), i.e. the sum of the speeds of the source relative to the observer and that of the light pulse relative to its source. As surprising as this may seem after hearing quite the opposite conclusion for more than a century, that value stands in complete agreement with the prediction of the classical (Galilean) velocity transformation, and not that of the relativistic velocity transformation, which instead predicts a value of \(c\) for the same quantity.

Einstein's second postulate is not applicable to any case in which observers in two different rest frames compare their measurements of the speed of the same object. The classical velocity transformation works in its place in every instance, and not just for the light pulse specifically under discussion. On the other hand, the relativistic velocity transformation, which is based on the second postulate, is required in place of the classical transformation whenever a single observer is to make measurements for the same object under two different conditions. This is the case for the Fizeau/Fresnel light-drag experiment, for example, in which the same observer measures the speed of light before and after the speed of the medium is altered. The classical transformation cannot successfully describe this relationship, so both transformations are needed to cover all the possibilities.

The second postulate also leads to a false conclusion regarding the supposed lack of simultaneity in Einstein's example in the two observers' measurements of the arrival of the light pulse back at its original position at the light source. In order to obtain the true relationship between the two elapsed times, it is necessary to take account of the effects of length contraction and time dilation which are predicted by Einstein's theory. When this is done properly, it is found that the two observers do indeed agree on the moment at which the light pulse returns to the position of the source.

This is a symptom of a broader misunderstanding in Einstein's theory regarding non-simultaneity. Although it is easy to find cases in which the theory predicts that a pair of events occurs simultaneously for one observer but not for the other, in no such situation is the result compatible with the time-dilation prediction of the same theory. This is because the latter conclusion always implies that the two time differences are related by a strict proportionality, i.e. \(\Delta t'=X\Delta t\), something which clearly rules out the possibility that one of them could have a zero value without the other doing so as well.

The contradiction between the remote non-simultaneity and time-dilation predictions is not the fault of Einstein's light-speed postulate, but rather the mixing of space and time
coordinates implied by the Lorentz transformation. Recognition of the characteristics of inertial systems allows for a simple remedy for this problem, namely that the rates of inertial clocks must be constant. As a result, time differences measured for the same pair of events by two such clocks must always occur in a fixed ratio, i.e. $\Delta t' = \Delta t/Q$, thereby eliminating any possibility of the "space-time mixing" predicted by the Lorentz transformation. At the same time, it also rules out the possibility of remote non-simultaneity, since the above equation requires that either both $\Delta t'$ and $\Delta t$ be equal to zero (mutual simultaneity) or both not (mutual non-simultaneity).

This proportional relationship between elapsed times is referred to as Newtonian Simultaneity. By combining it with the relativistic velocity transformation, one obtains a different space-time transformation, one which also satisfies both of Einstein's postulates of relativity, but avoids any conflict between its predictions of space-time relationships in general. The constant Q can be obtained by comparison of experimental timing relationships, as, for example, those that are available from the Hafele-Keating study of circumnavigating atomic clocks. The resulting set of equations is referred to as the Newton-Voigt transformation. *The Newton-Voigt transformation can be used successfully in any example for which the Lorentz transformation leads to predictions which agree with experiment.*

The overriding conclusion of the present study, however, is that any relationship between the measurements of two observers of the velocity of a given object can be obtained accurately by using the classical (Galilean) velocity transformation. The Newton-Voigt transformation and the relativistic velocity transformation, on the other hand, are essential for the description of the relationships obtained by a single observer for the velocity of a given object under two different sets of conditions, such as those that occur in the Fizeau/Fresnel light-drag experiment.

**References**


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